ON THE FW ABSORPTION IN MULTI-ION COMPONENT PLASMAS AT THE FUNDAMENTAL ICR HARMONIC IN THE CASE OF QUASIAPERPENDICULAR PROPAGATION

S. S. Pavlov* and F. Castejon**

*Institute of Plasma Physics, National Science Center

“Kharkov Institute of Physics and Technology”, 61108 Kharkov, Ukraine

**Asociacion Euratom-Ciemat para Fusion, Ciemat, 28040 Madrid, Spain

1. Introduction

In the work [1], devoted to the study of influence of the strong space dispersion upon the absorption of fast wave (FW) in the range of fundamental harmonic of ICR, it was shown that the formulae for the calculation of the absorption that were obtained during the 60s’ are not exact enough in the case of the quasiperpendicular propagation and IBW has the form:

\[ N^2 = \frac{1}{\omega^2} \sqrt{\pi} Z_{\alpha\alpha} \rho_{\alpha}^4 \frac{\rho_{\omega}^4}{C^4} \omega^2 \cdot \omega_{-\alpha} + N^2_{\perp} - N^2_\perp \cdot N^2_{\parallel} \cdot (N^2_{\parallel})^2 = 0, \]  

(3)

where \( N^2_{\parallel} \) is the square of refractive index for FW in the cold approximation, \( Z_{\alpha\alpha} = \alpha \beta \sqrt{2} k_Z V_{T \alpha} \), \( W_{-\alpha} = W(Z_{-\alpha}) \) is the plasma dispersion function of the argument \( Z_{-\alpha} \) \( = (\alpha - \alpha \beta) / \sqrt{2} k_Z V_{T \alpha} \), \( V_{T \alpha} = T_{\alpha} / m_{\alpha} \) is the thermal velocity of resonant ions, \( W = \alpha \beta \alpha \beta \) is the roots of equation (3) describe FW properties (exactly for the resonance case, where \( Re(W^{-1}) \geq 1 \) and any Bernstein wave (with less level numerically but, moreover, one can deal with it analytically. Considering the size of the several terms that appear in (2) and doing some approximations, as keeping only the main terms, we obtain the following dispersion equation for FW which is valid for any value of \( N^2_{\parallel} \)\( \geq (k_{\perp} \rho_{\alpha})^2 \):

\[ N^2_{\perp} = \frac{1}{\omega^2} \sqrt{\pi} Z_{\alpha\alpha} \rho_{\alpha}^4 \frac{\rho_{\omega}^4}{C^4} \omega^2 \cdot \omega_{-\alpha} + N^2_{\perp} - N^2_{\perp} \cdot N^2_{\parallel} \cdot (N^2_{\parallel})^2 = 0, \]

(3)

Such calculations were made and they gave corrections to the absorption coefficient and the propagating properties of FW in single-ion component plasmas in the case of quasitransverse propagation.

In the present report the results of the work [1] are developed with higher exactness and generalized to the case of two-ion component plasmas.

2. Calculation of the FW absorption taking into account the strong space dispersion.

We shall look for the absorption of FW by means of the solution of dispersion equation, that connects components of dielectric tensor with refractive index.

The full dispersion equation describing SW, FW, and IBW has the form:

\[ \begin{align*}
\epsilon_{\parallel} N^2_{\parallel} + 2N_{\perp} \epsilon_{\perp} N^2_{\perp} &+ \epsilon_{\perp} (\epsilon_{\perp} - N^2_{\parallel}) - \\
\epsilon_{\parallel} (\epsilon_{\perp} - N^2_{\parallel}) + \epsilon_{\perp} &- \epsilon_{\perp} - N^2_{\parallel} + 2N_{\perp} \epsilon_{\perp} (\epsilon_{\perp} - N^2_{\parallel}) + \epsilon_{\perp} = 0,
\end{align*} \]

(1)

where \( \epsilon_{\parallel} \) and \( \epsilon_{\perp} \) are the components of dielectric tensor that can be written as expansions in the parameter \( k_{\perp} \rho_{\alpha} \) (index \( \alpha \) denotes resonant sort of ions). The equation (1) can be transformed into a double quadratic equation by taking into account that \( \epsilon_{\perp} \) and \( \epsilon_{\parallel} \) are proportional to \( N^2_{\perp} \). It is easy to show that the terms with \( \epsilon_{\perp} \) and \( \epsilon_{\parallel} \) are mutually cancelled. The FW and SW branches are easily detached in the ICRF range because of the relation \( \epsilon_{\perp} = \epsilon_{\parallel} \) and \( \epsilon_{\perp} = \epsilon_{\parallel} \). As a consequence, the usual equation for this branch in the ICRF frequency range can be used to calculate the FW absorption:

\[ (\epsilon_{\perp} - N^2_{\parallel}) N^2_{\parallel} + (\epsilon_{\perp} - N^2_{\parallel}) (\epsilon_{\perp} - N^2_{\parallel}) - \epsilon_{\parallel}^2 = 0. \]  

(2)

The solution of this equation can be found, of course,
For the typical plasma parameters of “T-10” tokamak (B0=3T, n0=7·1013cm−3, Te=Tp=3KeV), one obtains that the absorption is enhanced about 2 times as compared with “cold” case at N,θ=3. When the value of N,θ decreases the absorption sharply increases and at about N,θ=0.1 it has ImN,θ~ ReN,θ. Furthermore, when the value of N,θ goes to 0, then ImN,θ goes to 0 too, in accordance with expression (5). From the expression (5) it follows that additional absorption appears when 1+2nβzβ/nαzα(1+zβzα/k0)2 (here ξ=zβm/k0m0), since Im{(N,θ)2}=Re{(N,θ)2}(1+2nβzβ/nαzα(1+zβzα/k0)2). Thus, if the concentration of resonant ions, nα, is more or order of the concentration of nonresonant ions, nβ, then additional dissipation appears when Zα=1/nαzαn0z0>1 or N,θ<1/k0(1)β3C/V. This condition coincides with analogical one for single-ion plasmas [1].

To evaluate the increasing of absorption that this calculations provide in the case of nα is larger or of the order than of nβ, it is useful to compute the optical thickness τ=Imk⊥dR. This quantity is estimated assuming that the magnetic field B0 changes as 1/R. ICR absorption is proved to be strong only in a narrow domain ∆R=R0/Zα in this case. For the cold plasma, we have:

\[ \tau=\text{Im}(k_\perp)=\text{Re}(k_\perp)/(4\sqrt{\pi}Z_0)\approx k_\perp R_0/(4\sqrt{\pi}Z_0). \]  

(6)

For the case of "hot" plasma it is necessary to use the expression (4) to calculate Imk⊥. Then we have

\[ \tau=\text{Im}(k_\perp)=\frac{\text{Im}N^2_\perp}{\text{Re}N^2_\perp} \approx k_\perp R_0/(2Z_0)k_\perp R_0/(4\sqrt{\pi}Z_0). \]  

(7)

This formula is simpler for the extreme cases:

a) N,θ>>1 (d<<1), \[ \tau=\text{Im}(k_\perp)=k_\perp R_0/(4\sqrt{\pi}Z_0). \]  

(8)

b) N,θ<1 (d>>1), \[ \tau=\text{Im}(k_\perp)=k_\perp R_0/(2Z_0). \]  

(9)

Thus when N,θ >>1, the optical thickness does not depend on N,θ and therefore, it may be estimated using formula (6) for "cold" plasma at the value of N,θ for which the full absorption and "cold" one are approximately the same. This value of N,θ is easy to obtain from the equation Zα(k,θ)=1. The calculations for the same plasma parameters as above give for a middle device of the type of tokamak “T-10” a small value of optical thickness τ=10⁻⁴. However for large devices of the type of ITER tokamak one has τ=0.5 and therefore, RF heating by FW at first harmonic resonance becomes rather interesting as a heating regime. Note also the fact that the optical thickness does not depend on N,θ except for very small values of N,θ, makes easier the task of injecting high frequency power into plasma due to the possibility of using smaller values of N,θ.

3. The physical nature of the "additional" FW absorption

To find out the nature of appearing of the FW additional absorption in the case of quasiperpendicular propagation, let us calculate FW polarisation in "cold" and "hot" cases. In the "cold" case, it can be readily obtained from the wave equation:

\[ \frac{E_Y}{E_X} = \frac{\epsilon_1 - \epsilon_2^{-1}N^2_\parallel}{i\epsilon_2} = i(-1 + \frac{3}{2\pi\sqrt{\xi}}) \]

\[ -\frac{1}{Z_0} = (-1)^{\frac{1}{2}} \frac{[1 + 2n_\beta z_\beta/n_\alpha z_\alpha(1 + \xi)]}{\sqrt{\pi}} \]

where the value of the imaginary part corresponds to circular polarisation of FW and its sign corresponds to the direction of rotation of the electrical field vector, against the direction of ion rotation. Since one has that Zα=1, the real part of the polarisation introduces a weak (if nα≥nβ) ellipticity and is responsible of the existence of weak absorption of order \[ \frac{E_Y}{E_X} \approx \frac{1}{\pi Z_0} \approx \frac{1}{\pi Z_0}. \] In the "hot" case the polarisation is given by:

\[ \frac{E_Y}{E_X} = i(1 - x + ... - \frac{[1 + 2n_\beta z_\beta/n_\alpha z_\alpha(1 + \xi)]}{2\pi\sqrt{\xi}}Z_0). \]

where x=(k,θp)². From this expression it follows that in the real part there are not practically changes in comparison with the "cold" case, because x<<1 for FW. However it appears a small addendum in the imaginary part which is proportional to x. This term introduces additional ellipticity in the polarisation and, consequently, enhances the FW absorption. This effect is due to the fact that the absorption is proportional to x²=(k,θp)⁴. Comparison of the additional "hot" absorption with "cold" one gives

\[ \frac{D_{\text{hot}}}{D_{\text{cold}}} = \pi(k_\perp p_\perp)^4 Z_0^2 \frac{1}{[1 + 2n_\beta z_\beta/n_\alpha z_\alpha(1 + \xi)]^2} \]

From that appreciation it follows that when N,θ decreases the additional "hot" absorption may become stronger than the "cold" one (if nα≥nβ). Thus, if the value of N,θ is small enough the "hot" addition in the polarization will enhance the FW absorption. The calculation of this absorption using the more strict formula gives, in terms of the main parameter x=(k,θp)², the following result

\[ D = \frac{\omega}{8\pi} \left| E \right|^2 \sqrt{\pi N_\alpha z_\alpha Z_0} \exp(-Z_0^2) \]

\[ \left\{ \frac{1}{4} + \frac{1 + 2n_\beta z_\beta/n_\alpha z_\alpha(1 + \xi)}{2\pi Z_0^2 \exp(-Z_0^2)} \right\} \]

where the first term in brackets is connected with the "hot" absorption and the second term with the "cold" one.

References