KHARKOV DATA ON PION PHOTOPRODUCTION AND Δ^+(1232) RESONANCE PARAMETERS

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We present the analytic review of π^+ and π^0 mesons photoproduction off the proton target as the only source of information about the Δ^+(1232) resonance parameters. The review focuses at the estimation of the influence of different contributions to the experimental database on determination of the E2/M1 ratio for $\gamma p \to \Delta^+$ transition and the resonance parameters, with discussion of the previous Kharkov results.

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1. INTRODUCTION

The successful experiment on lithium splitting on 10 October 1932 was the initiating event followed by the entire chain of long-term developments in the atomic science in Kharkov and Former Soviet Union. As one of the consequences, 33 years later the 2 GeV electron linac was built in Kharkov, under direction of K.D. Sinel'nikov and A.K. Val'ter – two participants of the "high-voltage brigade" which had split up the lithium nuclei. Unfortunately the following pulse stretcher ring project left unrealized and performing the coincidence experiments turned out to be very complicated. As the result the single pion photoproduction on proton happened to be practically the only one elementary process thoroughly studied in Kharkov. In particular, in the region of excitement of the first nucleon resonance there were carried out numerous measurements of asymmetry Σ in reactions with linearly polarized photons [1,2]. The first round of successful experiments using the polarized proton target in addition [3,4] (see also article [5] in this issue) was the most important result of these measurements. The new information obtained in these experiments made it possible to solve problem of the unique solution in the energy independent multipole analysis using some plausible stabilization procedure [6], and next without any additional conditions [7].

The most interesting and theoretically important result of such a multipole analyses is determination of the resonance radiative decay amplitudes, in particular the ratio *EMR* of the electric quadroupole *E2* and magnetic dipole *M1* amplitudes. The deviation of *EMR* from 0 is evidence in favor of the existence of color magnetism due to the gluon exchange between quarks. The *EMR* value plays a role of the litmus paper for hadron models that predict this ratio to be quite small with values ranging between –0.5% and –6% [8]. From the Kharkov data *EMR* was obtained with the range of –(1.2 ... 1.3)% [9].

However, later on and especially in the last decade the database has considerably expanded. In addition, the *EMR* happened to be very sensitive to some inconsistencies in the database [10]. The present-day *EMR* value is determined in many works [11], approximately with the results being at least twice-larger then those obtained in [9] (in recent work [12] *EMR* = =–[3.07 ± 0.26(stat + syst) ± 0.24(model)]%). The reason of this contradiction can be understood from article [10] where the model dependence and the influence of choice of database in extracting the Δ(1232) electromagnetic transition amplitudes were investigated. In particular, the crucial correlation between the Bonn π^0 cross section data [13] and the *EMR* was demonstrated. The strong influence of this data on the extracted EMR has also been confirmed by the LEGS group [14,12].

The mass splitting of the different Δ charge states is another problem associated with the Kharkov data. Indeed, the πN scattering analyses has given the Δ^0 and Δ^{++} masses, characteristic values being 1233.6±0.5 MeV and 1230.9±0.3 MeV respectively [11] (PDG)). The missing Δ^+ mass can be obtained only from the single pion photoproduction reactions on proton, by the resonance fitting of the multipoles leading to the final π N state with isotopic spin $T = 3/2$ and total momentum J=3 /2 at condition that the Watson's theorem is not used. The Δ^+ mass value 1234.9±1.4 MeV was obtained by authors of work [15] (MIROSHNIC 79 in PDG) from the results of their preceding energy independent analysis with 'free' imaginary part of the magnetic dipole resonant amplitude [16]. But it has been a concern that this Δ^+ mass is inconsistent with Δ^{++} and Δ^0 masses cited above. In particular, this issue was the subject of a special publication [17], where it has also been emphasized that a similar problem arises in any analysis starting from multipoles determined in a separate analysis.

The purpose of this mini-review is to shed some light on the problems with both the Kharkov *EMR* and Δ^+ mass value, with paying a special attention to the Kharkov data. We have invoked a realistic resonance model approbated in our contribution to [18] and fulfilled a series of retrospective fits beginning from the data since 1990 and scaling down by degrees the year of the data involved. It turned out that the Bonn's cross sections mentioned above have a strong influence not only on the *EMR*, but also on the Δ^+ mass.

2. REMARKS ON THE DATABASE

Several years ago the full collection of the Kharkov data on single pion photoproduction was uploaded to the public-available compilation SAID [19,20]. In connection with this we have ceased replenishment of our own database using the results of the vast work undertaken by the authors of SAID. Now the Kharkov data are well accessible and up to this time they are used in the multipole analyses, for example, [12,21]. An essential feature of the SAID compilation is that it contains all accumulated data being verified by direct contacts with numerous authors, without any preliminary selection according to somebody's preferences. As a result, the database contains some conflicting measurements, but there is a possibility to investigate the influence of these discrepancies on the results of any specific multipole analysis.

3. FORMALISM IN THE Δ(1232) REGION

Our analysis was carried out in terms of the Walker's helicity multipoles $M^I_{l\pm} \equiv A^I_{l\pm}, B^I_{l\pm}$ with the following isospin structure of the π^0 and π^+ photoproduction on proton [22]:

$$A^{1/2} = 1/3\, A(\pi^0) + \sqrt{2}/3\, A(\pi^+), \tag{1}$$

$$A^{3/2} = A(\pi^0) - 1/\sqrt{2}\, A(\pi^+). \tag{2}$$

The starting point of the model used is the following expression for the resonant multipoles obtained in the K-matrix approach [23] as result of the unitary merging of the background and the 'pure' resonance:

$$M^{3/2}_{1+} = B^{BORN}_M \cos\delta_{33} e^{i\delta_{33}} + R^{PURE}_M \sin\delta^R_{33} e^{i\delta_{33}}. \tag{3}$$

In Eq. (3) B^{BORN}_M is the background function (in [23] this is the Born contribution to the resonant multipole), R^{PURE}_M is responsible for the resonance photoexcitation. The full phase shift δ_{33} for the P_{33} wave in the πN scattering is the sum of the background δ^B_{33} and the 'pure' resonance phase shift δ^R_{33}:

$$\delta_{33} = \delta^B_{33} + \delta^R_{33}. \tag{4}$$

To fully demonstrate the dependence on δ_{33}, which is the main focus of our interest, Eq. (1) can be written as

$$M^{3/2}_{1+} = B_M \cos\delta_{33} e^{i\delta_{33}} + R_M \sin\delta_{33} e^{i\delta_{33}}, \tag{5}$$

with

$$B_M = B^{BORN}_M - R^{PURE}_M \sin\delta^B_{33}, \tag{6}$$

$$R_M = R^{PURE}_M \cos\delta^B_{33}. \tag{7}$$

Eq. (5) is the base of our treatment of the resonant multipoles. The elastic background phase shift appears not only as a component part of δ_{33} but also has an additional influence on the redefined excitation functions (6,7). These corrections are expected be small and smooth, keeping in mind evaluations of the background phase shift for the mixed charge δ_{33}. There is no possibility to determinate the elastic background phase shift by using Eqs. (6,7) because the corresponding trigonometric functions are multiplied by other unknown smooth functions. So, the whole function B_M was parameterized through the Lagrange interpolation formula (E_γ being the laboratory photon energy):

$$B(E_\gamma) = \sum_{i=1}^{i=4} B(E_\gamma^{(i)}) \prod_{j=1\,(j\neq i)}^{j=4} (E_\gamma - E_\gamma^{(j)}) \Big/ (E_\gamma^{(i)} - E_\gamma^{(j)}), \tag{8}$$

with four knot energies and with coefficients being the knot values of the function. In a special test it has been compared to the relevant cubic spline without the first derivatives specified at the ends, and Eq. (8) happened to be preferable for extrapolating out of the energy interval restricted by the extreme knots.

The full phase shift δ_{33} in (5) is chosen in accordance with the standard Breit-Wigner formula:

$$tg\,\delta_{33} = \frac{W_0 \Gamma(W)}{W_0^2 - W^2} \tag{9}$$

with

$$\Gamma(W) = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \frac{X^2 + q_0^2}{X^2 + q^2}, \tag{10}$$

where q is the c.m. momenta of the pion, q_0 is the corresponding quantities at W_0. Here W_0 (the mass) is the value of the total c.m. energy W at which δ_{33} passes though 90°, and the width is $\Gamma_0 = 2/(d\delta_{33}/dW)\big|_{W=M0}$ ('experimental' values). Eq. (9) corresponds to the approach without introduction the explicit background at all, and a precaution about B_M seems to be excessive, as well. Accordingly, these definitions are the same as in πN phase shift analysis [24] (KH80) at determination of the Δ^{++} and Δ^0 parameters, where "some guess for uncertainty due to the non-resonant background are simply added to the quoted errors".

The last term in Eq. (5) corresponds the to resonance contribution. It is introduced according to Walker [22]:

$$M^{3/2,R}_{1+}(W) = C_M(W_0)\sqrt{\frac{k_0 q_0}{kq}}\, \frac{W_0\sqrt{\Gamma\ \Gamma_\gamma}}{W_0^2 - W^2 - iW_0\Gamma}. \tag{11}$$

Here $C_M \equiv \mathrm{Im} M^{3/2,R}_{1+}(W_0)$ is the resonance constant,

$$\Gamma_\lambda(W) = \Gamma_0 \left(\frac{k}{k_0}\right)^3 \frac{X^2 + k_0^2}{X^2 + k^2}, \tag{12}$$

where k is the c.m. momenta of the photon, k_0 is the corresponding quantities at W_0.

Expression (8) is used also to describe the real parts of the variable non-resonant multipoles, other background multipoles up to l=3 are taken as full Born approximation, and the corresponding imaginary parts are calculated according to the Watson's theorem:

$$\mathrm{Im}\, M^I_{l\pm} = \mathrm{Re}\, M^I_{l\pm}\, tg(\delta_{2I,\,2(l\pm)}). \tag{13}$$

The resonance constant in Eq. (11) refers to the experimental resonance in the meaning discussed above. For parameterization with some realistic non-zero background phase shift the arguments can be provided in such a way that the resonance constant would be very close to the 'pure' one (see [9] and discussion of this work in [25]). Especially, we can expect that for the resonance ratio *E2/M1* where reduction of the energy dependence is also expected.

4. RETROSPECTIVE MULTIPOLE ANALYSIS

To concentrate calculations in the Δ^+ region we have treated the data at the photon energy interval 260-420 MeV. The energies 280, 320, 360, and 400 MeV were taken as knot energies in Eq. (8). The necessary for Eq. (13) πN scattering phase shifts were calculated according the recent πN analyses [26] (SM02 in SAID).

The data were fitted by minimizing the standard χ^2 without introducing rating factors for any type of observable. The main set of independent variables included the Δ^+ mass M_0 and width Γ_0 (parameter X was fixed at the Walker's value 185 MeV) and the knot values from Eq. (8) used for the following functions: (a) the background functions B_A, B_B in Eq. (5) for the resonant multipoles; (b) the real parts of the non-resonant multipoles A_{0+}, A_{1-} with isotopic $I = 1/2,3/2$, A_{1+}, B_{1+} with $I = 1/2$ (*s,p* waves), and A_{2-}, B_{2-} with $I = 1/2$ to account the possible 'tail' of the second resonance. The results obtained for several series of fits with the data from different years are presented in Table 1. The first fit was obtained using the new data obtained from 1990. This data set practically corresponds to the BRAG low energy set [18]. As in [18] we were unlucky to describe the preliminary π^0 cross section HA97MA* from Mainz (our data labels contain two letters of the first author and the laboratory name being separated by the reduced year). In addition, there is a problem with the LEGS differential cross sections (BL01LE, π^0 and π^+ production). The corresponding $\chi^2_{dp} \equiv \chi^2/N$ (N is the number of points) are too large, and that needs special consideration. In preliminary calculations the normalization factors being introduced as additional free parameters for the both LEGS cross sections were about 0.9, but only for the π^0 production it was possible to get reasonable $\chi^2_{dp} \approx 2.6$. Consequently we have omitted these data in the initial data set (group I in Table 2, χ^2 in this table are calculated according to our final solution discussed below).

Table 1. *Δ^+ Parameters for different variants of the resonance model*

№	Variants	Year	Excluded data	*EMR*, %	M_0, MeV	Γ_0, MeV	N	χ^2_{df}
1	Main set	1990	I	-2.2 ± 0.2	1232.3 ± 0.8	117.1 ± 2.9	1339	1.59
2		1985	I	-2.2 ± 0.2	1232.7 ± 0.8	117.2 ± 2.9	1343	1.60
3		1980	I	-2.2 ± 0.1	1231.5 ± 0.6	113.5 ± 2.1	1762	1.80
4		1980	I,II	-2.2 ± 0.1	1232.0 ± 0.6	114.1 ± 2.2	1758	1.78
5		1975	I,II	-2.7 ± 0.1	1232.0 ± 0.5	111.3 ± 1.9	2152	2.35
6		1975	I-III	-2.6 ± 0.1	1231.7 ± 0.5	112.1 ± 2.0	2113	2.15
7		1970	I-III	-1.6 ± 0.1	1234.7 ± 0.4	117.3 ± 1.7	2997	2.30
8		1970	I-IV	-1.6 ± 0.1	1234.7 ± 0.4	117.5 ± 1.7	2992	2.28
9		1970	I-V	-2.5 ± 0.1	1232.0 ± 0.5	111.6 ± 1.7	2660	2.05
10		1960	I-VI	-2.5 ± 0.1	1232.1 ± 0.4	113.0 ± 1.7	3225	2.24
11	*s,p*	1960	I-VI	-2.2 ± 0.1	1232.4 ± 0.5	117.7 ± 1.8	3225	2.41
12	*s,p,d*	1960	I-VI	-2.6 ± 0.1	1233.0 ± 0.5	113.0 ± 2.1	3225	2.13

Table 2. *Characteristics of the deleted data*

G.	R	O	Label	N	E_γ, MeV	ϑ, deg	χ^2_{dp}
I	π^0	σ	HA97MA*	51	283-402	10.0-170.0	13.5
	π+	σ	BL01LE	48	265-322	20.0-170.0	25.7
	π^0	σ	BL01LE	49	265-334	70.0-130.0	12.5
II	π+	G	BL84KH	4	320-380	65.0-80.0	9.9
III	π+	Σ	GN76KH	32	280-420	25.0-140.0	10.2
	π^0	Σ	GB77KH1	4	280-400	75.0-120.0	43.5
	π^0	σ	JU76BO	2	373-416	89.4-90.9	7.6
IV	π+	Σ	ZD72ST	2	390-408	135.0	19.3
	π^0	σ	HE73TO	3	350-420	4.4-6.1	7.8
V	π^0	σ	GZ74BO1	332	260-420	10.0-160.0	6.9
VI	π+	σ	KN63UC	23	260-290	0-160	8.5
	π+	Σ	LU64ST	3	330	45-135	34.4

Note: *R – reaction, O – observable value*

Table 3. *χ^2 per point for different values for fit 10*

Reaction	dσ/dΩ	Σ	T	P
$\gamma p \to \pi^+ n$	2.26	1.82	1.66	1.35
$\gamma p \to \pi^o p$	2.55	2.47	3.31	1.93

By decreasing the initial year down to 1975 we exclude some other non-numerous data (groups II-IV in Table 2) with per degree of freedom exceeding 9. There were observed rather stable values of the resonance parameters with acceptable values of χ^2_{df}. But appearing in the current compilation of the numerous π^0 cross section data resulting from some experimental setups at Bonn [13] (GZ74BO) caused the striking effect. In particular the Δ^+ mass exceeding the Δ^0 value known from scattering has yielded (rows 7,8 in Table 1). Because of this we have omitted this old Bonn data and a subsequent involvement of the pioneer's photoproduction measurements has given our final solution, which seems to be the most realistic one (row 10). Corresponding values of χ^2_{dp} are placed in the last column of Table 2 (rejected data), in Table 3 (π^+ and π^0 production separately for cross section, Σ, *P*, and *T*), and in Table 4 for the Kharkov data from the final data set. In two last fits we have restricted the background variable parameters by the *s*, *p* waves and (row 11 in Table 1) and increased to vary the full set of the background *d* waves (row 12).

***Table 4.** Characteristics of the Kharkov data*

R	O	Label	E_γ, MeV	ϑ, deg	*N*	χ^2_{dp}
π^+	Σ	GE81KH	280-420	30-150	56	3.6
	P	GE81KH	280-420	30-150	54	1.4
	T	GE81KH	280-420	30-150	53	1.9
	T	GE80KH	340-340	30-150	7	2.2
	H	BL86KH	320-320	90-120	4	5.2
	H	BL84KH	320-380	65-80	4	1.0
π^0	Σ	BL83KH	280-420	60-135	38	3.6
	Σ	GN76KH1	300-420	60-135	35	4.7
	Σ	GB78KH	360-400	140	2	4.6
	P	BL83KH	280-420	60-135	38	1.4
	P	GB78KH	360-400	140	2	2.6
	T	BL83KH	280-420	60-135	38	3.9
	T	GB78KH	360-400	140	2	1.0

***Note:** R – reaction, O – observable value*

5. DISCUSSION

By going back from the last decade into the past and involving older data we observe a rather smooth and plausible variations of the Δ^+ mass and ratio *EMR* until stumbling at the old Bonn data on π^0 differential cross sections. As to the *EMR* the relevant jump corroborates the known effect discussed in the Introduction, but the rapid increase of M_0 and Γ_0 on about 2 and 4 MeV correspondingly is unexpected. For example, in our previous calculations [27] all fits were fulfilled with the data [13], but 'small' EMR = (–1.43±0.08)% accompanied by the mass of M_0 = 1232 ± 0.71 MeV appeared only in row 8 of Table 1 for the data up to maximal year 1984, and some comment seems to be necessary. The expression for the resonant multipoles used in [27] can be obtained from present Eq. (5):

$$M^{3/2}_{1+} = R^S_M \sin\delta_{33} e^{i\delta_{33}}, \tag{14}$$

with some function to parameterize the function

$$R^S_M = B_M \cot\delta_{33} + R_M . \tag{15}$$

Evident shortage of such a parameterization is that it has to describe the cot δ_{33} being a sufficiently strong function of independent resonance parameters. Used in [27] rigid parameterization was not relevant to reproduce this feather and that has influenced the resonance parameters.

Concerning the Δ^+ mass from [15] first of all one has to take into account the difference in definitions. With reference to the Olsson's work [28] the full magnetic resonant multipole is there proportional to the following construction (electric quadroupole was not treated):

$$\sin(\varphi_R(W) + \alpha(W))\exp(\varphi_R(W) + \beta(W)), \tag{16}$$

where the manifest notation of [15] are conserved. This block corresponds to one from Eq. (8) in [28], namely

$$e^{i\delta}\sin(\delta + \delta_P - \delta_e), \tag{17}$$

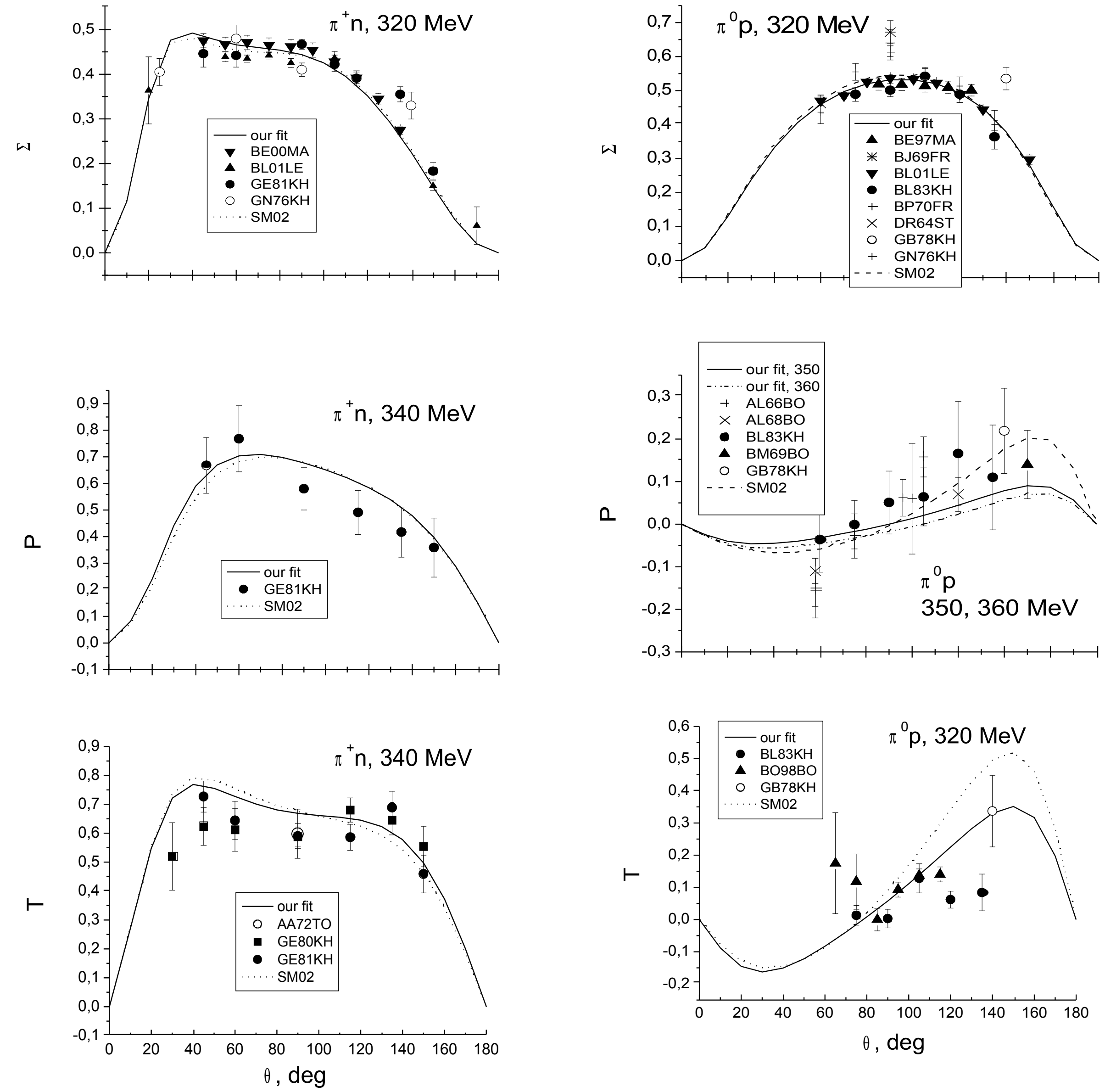
supposing the phase shift addition (our Eq. (4)). That is quite correct, as Eq. (8) in [28] is derived without any assumption about the low of the unitary merging of the resonance and background in scattering. (By the way, in [28] Olsson has been advocated the low with approximate subtraction of the resonance and the background phase shifts). However, the important point is that by introducing the background phase shift in parameterized form the authors of [15] are dealing with 'pure' resonance, with parameters being different from the "experimental' one discussed in Sect. 3. For example, the mass of the latter coincides with the energy at which the resonant photomultipole passes through zero. For involved in [15] analysis ([16], 1977, *s,p* waves are fitted) this is about 1240 MeV ($E_\gamma \approx$ 340 MeV). As we have previously seen to some extent that could be caused by to the Bonn data [13] already included in this analysis. It should be stressed that the main Kharkov data were absent yet and in any case could not have influence on this mass.

As to the old Bonn data it is not possible yet to reject them coming from the χ^2_{df} value. We prefer the solution obtained with more recent data and taking into account location of the Δ^+ mass relatively to the masses of the Δ^{++} and Δ^0 [24]. All these values can be compared using Table 5 (we only take from PDG the data with errors).

***Table 5.** The charge splitting of the* Δ(1232)

State	Mass, MeV	Width, MeV	Source	
Δ^{++}	1230.5±0.2	—	ABAEV	95
	1230.9±0.3	111.0±1.0	KOCH	80B
	1231.1±0.2	111.3±0.5	PEDRONI	78
Δ^+	1231.9±0.4	112.5±1.7	Table 1, row 10	
Δ^0	1233.1±0.3	—	ABAEV	95
	1233.6±0.5	113.0±1.5	KOCH	80B
	1233.8±0.2	117.9±0.9	PEDRONI	78

Some of the Kharkov data have got into groups II, III of rejected data. As it is clear now, the main reason is underestimation of the systematic errors. Nevertheless the overwhelming majority of this data has good or acceptable χ^2_{dp}. But time is coming, and now the polarization data *T*, Σ and *P* from Kharkov are considering as having the large statistical and systematic errors, especially for the $\gamma p \to \pi^0 p$ process [29]. In addition, the combined experiments with linearly polarized photons and polarized proton have not been repeated yet and remain to be unique. General situation with the polarization data in the both reaction at consideration is demonstrated in the figure below, where the Σ, *P*, *T* angle dependencies are presented at some energies convenient to compare with the up-todate polarization measurements at Bonn, Mainz, and Brookhaven. New points have appeared mostly for linear asymmetry, and the situation for *T* asymmetry and polarization *P* in the reaction $\gamma p \to \pi^0 p$ continue to be rather unambiguous. It is also easy to see that our final fit is in an excellent agreement with the last GWU solution [21] (SM02 in SAID) for the observables with looking reliable measurements. Some differences are seen only for two mentioned before *P* and *T*.



The Σ, P, and T vs pion c.m. angle θ for $\gamma p \to \pi^+ n$ and $\gamma p \to \pi^0 p$ reactions at E_γ = 320,...,360 MeV, with predictions according to the final solution (row 10 in Table 1, our fit) and GWU (SAID) solution SM02 [21]

6. SUMMARY

The basic points of the present analytical mini-review and its conclusions can be briefly formulated as follows:

- Our parameterization of the resonant photo-multipoles is the downright corollary of the expression obtained in the framework of the K-matrix formalism with multichannel two-particle unitarity [23], with using the Walker's model for the resonance term. The reliable presentation of the background multipoles was reached via the cubic polynomials for the real parts with using the Watson theorem for imaginary ones.
- The undertaken retrospective analysis reveals the significant influence of the Bonn π^0 differential cross sections [13] on the Δ^+ parameters: increase by ~ 3 MeV for the Δ^+ mass and about 2 MeV for the width. Such an effect for the EMR is the

same as in the analyses using the Watson theorem [10] (approximately dividing by 2). That fully explains the small value of the EMR obtained in [9] by using the Kharkov analysis and also can be a reason for observing very large Δ^+ mass in [15].

- The withdrawal of the Bonn data [13] allows to obtain the Δ^+ mass and width being in a reasonable accordance with the corresponding values for the Δ^{++} and the Δ^0 known from the πN scattering.
- Despite some criticism the overwhelming majority of the Kharkov data on pion photoproduction in the first resonance region preserve its scientific signifi cance and in some cases even the monopoly position.

As to the Bonn data the question is not so simple. They systematically cover the whole resonance region including the small and the large angles, where the new measurements yet are rather seldom and spread. Besides, coming from multipole analyses one can observe some 'suspicious' points and "derivations" in measurements of several laboratories, first of all at the edges of the energy or the angle intervals with measurements. Hence, the general conclusion is that the region of the first resonance needs to be explored more thoroughly. The systematic precision measurements of the differential cross section and polarization parameters using the relevant modern facilities would be an actual item in the program of a new modern middle energy accelerator.

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