

THE INFLUENCE OF NUCLEAR DEFORMATIONS ON THE PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS IN $1d_{2s}$ -SHELL NUCLEI

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To the blessed memory of E.V. Inopin

On the basis of a generalized model allowing for different deformations of a nucleus in its initial and ground states the analysis of the probabilities of electromagnetic transitions is performed. It is shown that the approach is the useful tool for investigations of the probabilities of $M1$ and $E2$ transitions between the states lying below the nucleon binding energy. The experimentally measured probabilities of $M1$ transitions between the analogue and anti-analogue states in the deformed $1d_{2s}$ -shell nuclei are in good agreement with the calculated ones.

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1. INTRODUCTION

During last decades, our knowledge on the nuclear structure substantially enriched due to both the wide amount of experimental data derived and the development and improvement of nuclear models reflecting important features of nuclear processes. (In the context, note the renaissance of the Nilsson model approach to light nuclei [1].)

The shell model built at the end of 1940th, having explained great amount of data associated with the ground and weakly excited states of atomic nuclei, faced substantial problems. Particularly, the measured values of quadrupole momenta for several nuclei appeared to be much higher the same values calculated due to the shell model. To tackle the problem, in 1950 J. Rainwater [2] proposed the following model. A nucleus should be considered as composed of the core containing nucleons from unclosed shells and a few outer nucleons. The interaction between the valence nucleons and the core leads to the equilibrium deformation of the latter. Even a small core deformation can condition the appearance of a substantial quadrupole momentum because a core contains the major part of nuclear substance.

Rainwater's hypothesis was the basis of the generalized model of a nucleus, developed further by A. Bohr and B. Mottelson [3]. However, as against Rainwater's model, the nuclear deformation in the generalized model is a dynamical variable. The explanation of the experimentally discovered rotational spectra was an outstanding success of the generalized collective model and a convincing proof for the existence of unspherical nuclei. At the moment, proved is the existence of several regions of mass number ($A \sim 25$, $150 < A < 190$, $A > 222$) where nuclei have stable deformations.

Early investigations on the nucleon motion in the spheroid potentials were carried out in refs. [4-7]. The most important role was played by the calculations accomplished by S. Nilsson [8], which provided the basis for successful classification of the wide amount of data concerning the spectra of deformed odd nuclei [7]. The similar classification for the range of heavy elements was done by the authors of ref. [9]. In Ref. [10] the validity of applying the generalized model in Nilsson's formulation to the light nuclei with $4 \leq A \leq 32$ was demonstrated.

2. GENERALIZED MODEL OF A NUCLEUS

According to the generalized model, the nucleons situated out of the closed shells cause the deformation of a nucleus, which depends on the number of valence nucleons and their quantum states. As a result, the mean effective field (potential) of a nucleus changes, altering the nuclear shape from the spherical to the ellipsoidal one, at least in the first approximation. The quantum characteristics for stationary states of the potential with axial symmetry differ from those ones of the spherically symmetric potential.

It is shown that the internal and collective motions of nucleons in the deformed nucleus are separated. Then the total wave function of a nucleus Ψ can be presented as a product of three independent wave functions each of which describes separately the rotation of a nucleus as a whole D_{MK}^I , its vibrational excitations ϕ_v and the internal excitations χ_Ω caused by individual nucleons:

$$\Psi = \sqrt{\frac{2I+1}{16\pi^2}} \phi_{\text{vib}} \left\{ \chi_\Omega D_{MK}^I + (-1)^{I-1} \chi_{-\Omega} D_{M-K}^I \right\} \quad (1)$$

The total Hamiltonian of such a system can be written as a sum of three terms:

$$H = H_{\text{int}} + H_{\text{rot}} + H_{\text{vib}}, \quad (2)$$

each of which corresponds to the three mentioned types of nuclear excitations [11, 12]: the first one describes the internal motion of a nucleon regarding the deformed core and is characterized by the quantum number Ω (Nilsson's model); the second one describes the rotation of a nucleus, keeping its form and internal structure; the third one describes collective vibrations of the nuclear surface regarding the equilibrium shape of a nucleus.

3. NILSSON'S MODEL

According to [8], the spherically symmetric potential of the shell model have the form:

$$V(r) = \frac{1}{2} m \omega^2 r^2 + C \mathbf{l} \cdot \mathbf{s}; \quad (3)$$

where the first term is the oscillatory potential and the second one is the potential of the spin-orbit interaction. The parameter ω characterizes the energy of the level which is

$$E_N = (N + 3/2) \hbar \omega, \quad N = 0, 1, 2, \dots \quad (4)$$

The diagram of levels generated by the potential (3) is shown on Fig. 1.

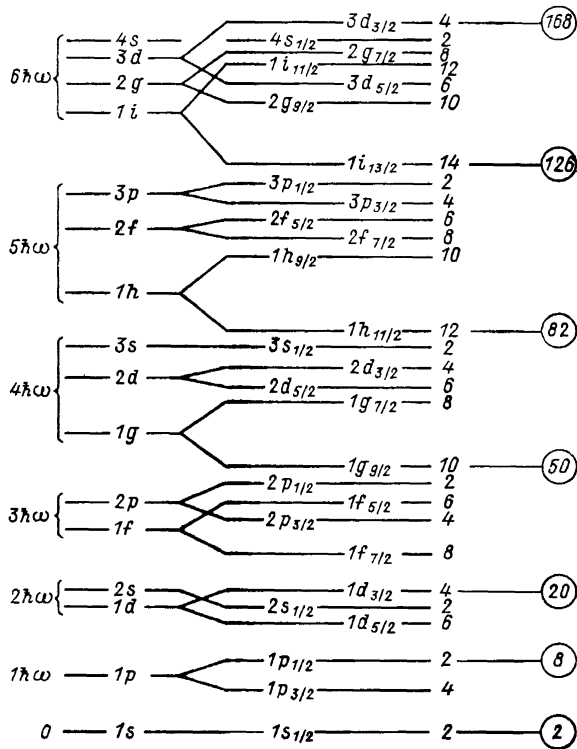


Fig. 1. The diagram of energy levels for protons and neutrons. The numbers of protons and neutrons on each level and the sums of these numbers are displayed at right. The oscillatory grouping of levels are shown at left

These levels are characterized by the following quantum numbers: the shell number N , the angular momentum l and the total momentum j . Due to the symmetry regarding the space rotations and the inversion at the origin, j and l (and the parity) of the nucleon are good quantum numbers, therefore, N , l , and j can be used to denote each level. So far many nuclei have more or less deformed shapes, the nucleons should be considered as moving in the deformed potential.

In Nilsson's model, the odd nucleon moves in the mean unspherical field of the rest of nucleons. It is also assumed that a nucleus is in its ground vibrational state. Then the collective motion is caused by the rotation of a nucleus around the axis perpendicular to the axis of symmetry. In this case, one introduces the one-particle Hamiltonian:

$$V_{\text{def}} = \frac{m}{2} [\omega_{\perp}^2 (x_1^2 + x_2^2) + \omega_3^2 x_3^2] + C(\mathbf{l} \cdot \mathbf{s}) + D l^2 \quad (5)$$

containing the shell-model Hamiltonian with ellipsoidal oscillator potential (first term), the conventional spin-orbit term and the correction interpolating between the square well and oscillator potentials.

The coefficients ω_{\perp} and ω_3 are related by the formulae

$$\omega_3 = \omega_0 \left(1 - \frac{2}{3} \varepsilon \right), \quad \omega_{\perp} = \omega_0 \left(1 + \frac{1}{3} \varepsilon \right), \quad (6)$$

where ε is the parameter characterizing nuclear deformation. When $\varepsilon \ll 1$ the coefficients ω_{\perp} and ω_3 satisfy the relation:

$$\omega_{\perp}^2 \omega_3 = \omega_0^3; \quad (7)$$

which is the condition on the nucleus volume to be constant regardless the deformation. The quadrupole momentum of a deformed nucleus is given by the formula

$$Q = \frac{4}{3} Z \langle r^2 \rangle \varepsilon \left(1 + \frac{1}{2} \varepsilon \right). \quad (8)$$

As it follows from Eqs. (5) and (6), the potential is determined by four parameters ω_0 , C , D and ε from which only ε strongly depends on nucleus shape. For the given nucleus, the parameter ε is usually found from the experimentally measured values of Q and $\langle r^2 \rangle$. The rest of the parameters (ω_0 , C and D) do not depend on the nucleus shape in the $\varepsilon \ll 1$ approximation and are derived from the nuclear spectra and the radii of spherical nuclei ($\varepsilon = 0$). The approximate values of these parameters are:

$$\hbar \omega_0 \approx 41 A^{-1/3} \text{ MeV}, \quad C \approx 0,1 \hbar \omega_0, \quad D \approx -0,02 \hbar \omega_0. \quad (9)$$

Nilsson's model assumes that all nucleons (except the last odd one) are paired and do not contribute to the nucleus momentum. In order to find the nucleus energy and wave function one should calculate the one-particle levels by solving the Schrödinger equation with the po-

tential (5). For the nuclei with small mass number the typical spectrum is shown on Fig. 2.

For $\epsilon = 0$ (the case of spherical symmetry) the calculated levels coincide with the corresponding levels of spherical harmonic oscillator, and they can be characterized by the quantum numbers N, j and l (N is the number of oscillator shell).

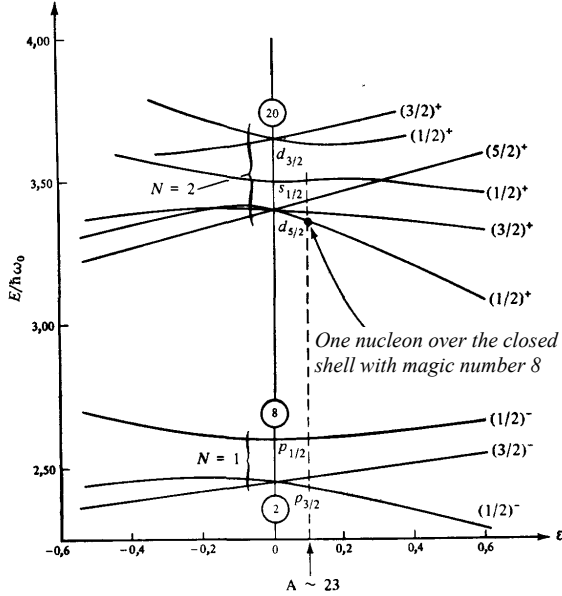


Fig. 2. Nilsson's model scheme of levels. Each level might be occupied by no more than two nucleons of the same type

In this case all the levels are degenerated $(2j + 1)$ times. As it is seen from Fig. 1, the deformation removes the degeneration. The level $p_{3/2}$ splits onto two ones, while the level $d_{5/2}$ - onto three separated ones. The nucleon level with the total momentum j in the spherical potential splits in the ellipsoidal one onto $1/2(2j + 1)$ different energy levels according to the values of $K = j, j - 1, j - 2, \dots, 1/2$. Here the coefficient $1/2$ accounts for the additional degeneration due to the symmetry of a nucleus regarding the plane perpendicular to its symmetry axis. The states with K and $-K$ have equal energies so the level with $|K|$ can be occupied by two nucleons of the same type.

The symmetry regarding the spatial rotations, except the rotations around the symmetry axis, is now violated so the momenta \mathbf{j} and \mathbf{l} are not conserved any more. In Nilsson's model conserved are the parity $\pi = (-1)^N$ and the projection of the total momentum on the symmetry axis K so each level is denoted as K^π .

Nilsson used the calculated one-particle eigenvalues and eigenfunctions to derive the total internal energy of nuclei, the equilibrium deformations, the one-particle excitation levels and also to determine the ground state spins, the magnetic moments and the probabilities of electromagnetic transitions $B(\sigma, l)$. The model of un-spherical nucleus automatically provides for the possibility of existence of the rotational levels in nuclei. The rotational bands can appear not only on the ground state

but on the excited ones (one-particle and vibrational) as well. In the nuclei with odd number of nucleons, the interaction between the one-particle and rotational motions prevails.

The calculations carried out in [10] showed that the generalized model in Nilsson's formulation successfully explained the properties of light nuclei with $4 \leq A \leq 32$: the equilibrium deformations, the spins, the magnetic and quadrupole momenta and the characteristics of rotational spectra. It turned out that the majority of light nuclei are strongly deformed and for several nuclei the ratio of ellipsoid half-axes characterizing the deformation even exceeds 1.5.

4. MODIFIED NILSSON'S MODEL

One of the most important problems is the investigation of a shape of a nucleus in the excited states. However, this task is not well tested, especially in the range of light nuclei. Up to now, the existence of deformation of light nuclei in both ground and excited states is experimentally proven. At the same time, it is usually assumed that the deformation does not change with the excitation energy.

In our opinion, this assumption conditioned the failure of attempts to explain the probabilities of electromagnetic transitions in the framework of generalized model. Bearing this in mind, we have proposed the new approach and developed the following method. Analyzing the probabilities of electromagnetic transitions, we treat the nuclear deformation as a variation parameter, meaning that the initial and final states are assumed to have different deformations. Thus, during the transition, the state of core nucleons changes alongside the state of the odd nucleon.

Using the $1d2s$ -shell nuclei, we have studied the influence of the changes in nucleus deformation on the probabilities of electromagnetic transitions. The initial and final states of a nucleus have been considered for different deformations and the contribution of one-particle part of wave function to the probabilities of electromagnetic transitions in light nuclei have been analyzed.

5. PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS IN $1d2s$ -SHELL NUCLEI

5.1. BOUND STATES

In order to determine the matrix element of the one-particle multipole transition operator

$$M = \sum_{i=1} \hat{t}(i) \quad (10)$$

we consider the systems of one-particle wave functions for the initial and the final states:

$$\varphi_1, \varphi_2, \dots, \varphi_A$$

$$\Psi_1, \Psi_2, \dots, \Psi_A.$$

Here A denotes the number of nucleons in the nucleus; indices $1, 2, \dots, A$ denote the numbers of occupied orbits in the initial and the final states.

The formula takes place:

$$\left(\Psi_{f, \mathbf{M}}, \Psi_i\right) = \sum_{s=1}^A \det B_{ij}^s, \quad (11)$$

where the determinant elements B_{ij}^s are as follows:

$$B_{ij}^s = \begin{cases} (\Psi_i, \hat{t} \varphi_j), & \text{если } i = s \\ (\Psi_i, \varphi_j), & \text{если } i \neq s. \end{cases} \quad (12)$$

In the case under study, the operator \hat{t} is:

$$\hat{t} = e \left[1 + (-1)^\lambda \frac{Z}{A^\lambda} \right] r^\lambda Y_{\lambda\mu}(\varphi, \Psi) \quad (13)$$

If $\lambda < K + K'$, then the reduced probability of electric multipole transitions between the initial and the final state with IK and IK' taken at different deformations η and η' , is equal to:

$$\begin{aligned} |M(E\lambda)|^2 &= \\ &= e^2 \left[1 + (-1)^\lambda \frac{Z}{A^\lambda} \right]^2 \left(\frac{\square}{M\omega_0} \right)^\lambda \frac{2\lambda + 1}{4\pi} \times \\ &\times \left| \langle \mathcal{L} KK' - K | I' K' \rangle \sum_{s=1}^Z \det(\Psi_i^s, \varphi_j) \right|^2 \end{aligned} \quad (14)$$

For $i = s$:

$$\begin{aligned} (\Psi_i, \hat{t} \varphi_j) &= \sum_{l'l'} \langle N'l' | r^\lambda | Nl \rangle \sqrt{\frac{2l+1}{2l'+1}} \langle \mathcal{L} 00 | l'0 \rangle \times \\ &\times \sum_{\Lambda' \Lambda \Sigma' \Sigma} \delta_{\Sigma' \Sigma} a'_{l'\Lambda'} a_{l\Lambda} \langle \mathcal{L} \Lambda K' - K | l' \Lambda' \rangle. \end{aligned} \quad (15)$$

while when $i \neq s$ we have:

Table 1. The nucleus deformation parameters in the region $21 < A < 27$, extracted via comparing the experimental and theoretical values of $B(\sigma l)$ for the case $L < K + K'$

Nucleus	$E_i \rightarrow E_f$, MeV	$J_i^\pi \rightarrow J_f^\pi$	$B(\sigma l)^{\text{exp}}$, W. u.	$B(\sigma l)^{\text{cm}}$, W. u.	η_i	η_f
M1 transitions						
21	0.432	$1/2^+ \rightarrow 3/2^+$	0.042(6)	0.028	2	4
Na	2.982 \rightarrow 0	$3/2^+ \rightarrow 3/2^+$	0.09(2)	0.014	2	4
	\rightarrow 0.440	$3/2^+ \rightarrow 5/2^+$	0.14(4)	0.022	2	4
E2 transitions						
23	0.008	$3/2^+ \rightarrow 3/2^+$	1.2(2)	0.00	2	4
25	0.151	$1/2^+ \rightarrow 5/2^+$	0.0(5)	0.00	2	4
25	0.186	$1/2^+ \rightarrow 5/2^+$	0.0(2)	0.00	2	4
27	0.010	$1/2^+ \rightarrow 5/2^+$	5.5(5)	5.55	2	4
27	0.670	$1/2^+ \rightarrow 5/2^+$	1.0(2)	0.00	2	4

Note: W.u. - Weiskopf units.

$$\left(\Psi_i, \varphi_j\right) = \delta_{N'N} \sum_{\Lambda} a'_{l'\Lambda'} a_{l\Lambda}. \quad (16)$$

For the case of magnetic multipole transitions, the sum of determinants over all orbits occupied by nucleons can be divided on two sums - over N orbits occupied by neutrons and Z ones taken by protons. Thus for $L < K + K'$ we find:

$$\begin{aligned} |M(M\lambda)|^2 &= \left(\frac{e\Pi}{2Mc} \right)^2 \left(\frac{\square}{M\omega_0} \right)^{\lambda-1} \frac{2\lambda + 1}{16\pi} \times \\ &\times \left| \langle \mathcal{L} KK' - K | I' K' \rangle \left[\det(\Psi_{i_p}, \varphi_{j_p}) \times \right. \right. \\ &\times \sum_{s_n=1}^n \det(\Psi_{i_n}^{s_n}, \varphi_{j_n}) - \det(\Psi_{i_n}, \varphi_{j_n}) \times \\ &\left. \left. \times \sum_{s_p=1}^Z \det(\Psi_{i_p}^{s_p}, \varphi_{j_p}) \right] \right|^2 \end{aligned} \quad (17)$$

The determinant elements for $i_n = s_n$ and $i_p = s_p$ correspond to the values G_{ML} for the neutron and the proton from Ref. [8].

In our previous works [13-18], using $M1$ and $E2$ transitions, we have investigated the influence of the changes in nucleus deformation on the probabilities of electromagnetic transitions in ^{21}Na , ^{23}Na , ^{25}Al , ^{27}Al nuclei (Table 1). The calculated value of the matrix element depends on two deformation parameters corresponding to the initial and the final states of the nucleus. Therefore, performing the comparison between the theory and the experiment, in the two-dimensional space of the deformation parameters we obtain the regions in which the theoretical and experimental matrix elements coincide. Analyzing the bands of transitions from one and the same level or onto one and the same state, we are able to reduce the regions of possible values of deformation parameters for some levels and sometimes even give exact values.

The joint analysis of the transition matrix elements allowing for deformations in the initial and final states and the other data on the low-lying levels depending on the deformation (the position, the quadrupole momenta etc.) also help to determine the values of deformation for these levels more exactly. The deformation parameters of the nuclei in the ground and excited states are usually extracted either from the data or from the theoretical calculations. In both cases the nucleus is treated as a deformed object so that the extracted information is model dependent.

The statements about the deformation parameters made due to the calculated probabilities of the transitions between the levels lying in the rotational bands witness that the deformation parameters are different not only for the transitions between one-particle states but also between the rotational levels in the band

5.2. ANALOG AND ANTIANALOG STATES OF NUCLEI

The developed method was applied for the analysis of the $M1$ transitions between the analog and antianalog states. Using the technique of Clebsh-Gordon coefficients, the AS wave function with the isospin $T_> = T_0 + 1/2$ and the projection $T_z = T_0 - 1/2$ can be expressed through the state of the core with the isospin T_0 and the projection T_{z0} and the state of the nucleon with the isospin t and the projection t_z :

$$|A\rangle = \sum_{T_{0z}, t_z} \langle T_0 T_{0z} t t_z | T_0 + \frac{1}{2} T_0 - \frac{1}{2} \rangle |T_0 T_{0z} t t_z\rangle. \quad (18)$$

The AAS wave function will have the form:

$$|\bar{A}\rangle = \sum_{T_{0z}, t_z} \langle T_0 T_{0z} t t_z | T_0 - \frac{1}{2} T_0 - \frac{1}{2} \rangle |T_0 T_{0z} t t_z\rangle. \quad (19)$$

In the case of the different initial and final deformations, the wave functions $|T_0 T_{0z} t t_z\rangle$ will have the form of Slater determinants composed from different sets of Nilsson wave functions describing $|nucleus(N, Z) + p\rangle$ and $|nucleus(N - 1, Z + 1) + n\rangle$ respectively. If one considers AS and AAS in the nucleus $(N, Z + 1)$ then their wave functions can written as:

$$\begin{aligned} |N, Z + 1\rangle_{T_>, T_{z<}} &= \frac{1}{\sqrt{2T_0 + 1}} |(N, Z) + p\rangle_{\eta_>} + \\ &+ \sqrt{\frac{2T_0}{2T_0 + 1}} |(N - 1, Z + 1) + n\rangle_{\eta_>} \end{aligned} \quad (20)$$

and

$$\begin{aligned} |N, Z + 1\rangle_{T_<, T_{z<}} &= \sqrt{\frac{2T_0}{2T_0 + 1}} |(N, Z) + p\rangle_{\eta_<} - \\ &- \frac{1}{\sqrt{2T_0 + 1}} |(N - 1, Z + 1) + n\rangle_{\eta_<} \end{aligned} \quad (21)$$

The indices $\eta_>$ and $\eta_<$ denote the deformation parameters of AS and AAS composed from the one-particle sets of Nilsson wave functions for $|N, Z + p\rangle$ and $|N - 1, Z + 1 + n\rangle$. In the considered scheme, the probability of $M1$ transition AS \rightarrow AAS is equal to:

$$\begin{aligned} &\left| \langle \bar{A}(N, Z + 1) | \mathbf{M}(M1) | A(N, Z + 1) \rangle \right|^2 = \quad (22) \\ &= \frac{2T_0}{(2T_0 + 1)^2} \left| \langle (N, Z) + p |_{\eta_>} | \mathbf{M}(M1) | (N, Z + p) \rangle_{\eta_>} - \right. \\ &\left. - \langle (N - 1, Z + 1) + n |_{\eta_>} | \mathbf{M}(M1) | (N - 1, Z + n) \rangle_{\eta_>} \right|^2, \end{aligned}$$

Table 2. The deformation parameters of AS and AAS in light nuclei

Nucleus	$E_{A \rightarrow \bar{A}}, \text{ MeV}$	$J_A^\pi \rightarrow J_{\bar{A}}^\pi$	$T_> \rightarrow T_<$	$B(M1)^{\text{exp}},$ W. u.	$B(M1)^{\text{sp}},$ W. u.	$B(M1)^{\text{cm}},$ W. u.	$^A \eta$	$\eta_{\bar{A}}$
^{23}Na	10.016 \rightarrow 3.913	5/2 ⁺ \rightarrow 5/2 ⁺	3/2 \rightarrow 1/2	0.27(7)	1.85	0.22	2	4
	10.872 \rightarrow 3.678	3/2 ⁺ \rightarrow 3/2 ⁺	3/2 \rightarrow 1/2	0.22(8)	1.59	0.29	2	4
^{27}Al	10.478 \rightarrow 6.479	7/2 ⁻ \rightarrow 7/2 ⁻	3/2 \rightarrow 1/2	0.7(2)	2.24	0.90	-4	-4
	10.586 \rightarrow 3.786	3/2 ⁺ \rightarrow 3/2 ⁺	3/2 \rightarrow 1/2	0.09(3)	0.05	0.07	-4	-4
	10.932 \rightarrow 4.154	5/2 ⁺ \rightarrow 5/2 ⁺	3/2 \rightarrow 1/2	0.03(1)	1.85	0.08	-4	-4
^{31}P	9.404 \rightarrow 4.431	7/2 ⁻ \rightarrow 7/2 ⁻	3/2 \rightarrow 1/2	-0.5(1)	2.24	0.8	-6	-6
^{35}Cl	7.194 \rightarrow 1.219	1/2 ⁺ \rightarrow 1/2 ⁺	3/2 \rightarrow 1/2	0.08(3)	1.95	0.20	-6	-4
	7.549 \rightarrow 3.162	7/2 ⁻ \rightarrow 7/2 ⁻	3/2 \rightarrow 1/2	1.8(6)	2.24	1.8	2	-2
	7.838 \rightarrow 4.177	3/2 ⁻ \rightarrow 3/2 ⁻	3/2 \rightarrow 1/2	0.7(2)	1.59	0.38	-6	-4
	8.208 \rightarrow 1.763	5/2 ⁺ \rightarrow 5/2 ⁺	3/2 \rightarrow 1/2	0.11(3)	1.85	0.06	-6	-4
	8.850 \rightarrow 2.645	7/2 ⁺ \rightarrow 7/2 ⁺	3/2 \rightarrow 1/2	0.07(2)	-	0.03	-6	-4
^{37}Cl	10.220 \rightarrow 3.103	7/2 ⁻ \rightarrow 7/2 ⁻	5/2 \rightarrow 3/2	1.7(4)	1.62	-	-	-

⁴¹ K	8.875→1.582	3/2 ⁻ →3/2 ⁻	5/2 ⁻ →3/2	0.21(6)	1.59	0.21	-2	-2
	9.366→2.144	3/2 ⁻ →3/2 ⁻	5/2 ⁻ →3/2	0.93(9)	0.05	0.35	-2	-2

where

$$\langle (N, Z) + p \left|_{\eta_A} \right. M(M1) \left. \right| (N, Z + p) \rangle_{\eta_A}$$

and

$$\langle (N - 1, Z + 1) + n \left|_{\eta_A} \right. M(M1) \left. \right| (N - 1, Z + n) \rangle_{\eta_A}$$

are calculated with the formula (17).

In Table 2 we present the theoretical and experimental values of the squared reduced matrix elements of $M1$ transitions for several nuclei.

6. DISCUSSION

The deformation of shape of atomic nuclei in the ground and excited states is one of the most interesting properties of nuclear matter. Even a small change in the state of the one nucleon can lead to a valuable distortion of the form of the nucleus surface and cause the rebuilding of the configuration of states of the rest of nucleons. In other words, even a microscopic change can cause the alteration of the macroscopic characteristics of a nucleus. That is why the investigations of the influence of the nucleus deformations in different states on the observable values are permanently in the focus.

The idea that both the one-particle and the rotational excitation states of a nucleus can be characterized by the dynamic deformation has appeared quite fruitful: the experimental data on the probabilities of electromagnetic transitions has been really explained. However, the question on whether the deformation parameters extracted are equilibrium, i. e. correspond to the experimentally observed states of nuclei, is still opened. Besides, there is also no clarity in the details of the mechanism via which the states with different deformations are excited. It is usually assumed that the one-particle excitation of a nucleus is caused by the transition of the last outer nucleon on one of the higher-lying energy levels. Such an event must cause the deformation of the nucleus shape and its self-consistent one-particle potential and, therefore, lead to the alteration of the whole one-particle spectrum. In other words, the transition must hold between the levels calculated in different potentials and, so far the final state appears unknown beforehand, from the point of view of quantum mechanics the transition itself becomes impossible. It seems, one should assume that the one-particle excitations are initiated by the alteration of the nucleus shape the result of which is the transition of the outer nucleon on one of the higher-lying orbits. However, this question also needs further study.

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