A CRITERION FOR DT GAS THERMONUCLEAR IGNITION BY A FOCUSING SPHERICAL SHOCK WAVE

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Conditions for thermonuclear ignition of deuterium-tritium (DT) gas by a focusing ideally symmetric shock wave are estimated. The wave is focused to the center and then reflects. In so doing a higher-pressure zone is produced following the front of the shock wave reflected from the center; if the wave is intensive enough, the DT gas thermonuclear ignition can occur in the zone. The ignition criterion can be written as $\rho_0 r_0 u_0^{22} \geq 1$, where ρ_0 is initial DT gas density, $[\rho_0]$ = g/cm^3, u_0 is mass velocity following the converging shock wave front of radius r_0, $[r_0]$ = cm, $[u_0] = 10^7$ cm/sec.

PACS: 28.52.Cx

The preceding paper of this issue [1] demonstrates the expediency of development of double-pulse lasers for studying the thermonuclear laser target performance based on moderate fuel pre-compression and subsequent ignition of the center by the focusing shock wave. Purpose of this communication is to estimate the conditions for thermonuclear ignition of deuterium-tritium (DT) gas by a focusing ideally symmetric shock wave.

A focusing spherical shock wave in homogeneous gas with adiabatic exponent γ tapers off to a self-similar regime near the focusing center [2]. This means that density ρ, mass velocity u, and sound speed c in the shock-compressed material can be expressed as

$$\rho = \rho_0 \delta(\tau), \quad u = \frac{r}{t} U(\tau), \quad c = \frac{r}{t} C(\tau), \tag{1}$$

where ρ_0 is initial gas density (density before the front of the converging shock wave), r is radius, t is time, $\delta(\tau), u(\tau), c(\tau)$ are functions of one self-similar variable

$$\tau = \frac{1}{\xi_0} t r^m, \tag{2}$$

ξ_0 is a constant, m is a constant dependent on the equation of state of gas following the shock front. For DT mixture, which we consider as ideal completely ionized gas with adiabatic exponent γ=5/3 (following the wave front near focusing), *m* is ([2]):

$$m = -1{,}45268. \tag{3}$$

Evaluate ξ_0. To do this, note that at the front of the self-similar converging shock wave [2]:

$$\tau = 1, \text{ that is } \xi_0 = t r^m, \; r = \left(\frac{\xi_0}{t}\right)^{\frac{1}{m}}, \; t = \frac{\xi_0}{r^m}, \tag{4}$$

$$u = \frac{r}{t} U(1) = \frac{r^{m+1}}{\xi_0} U(1),$$

$$\xi_0 = \frac{r^{m+1}}{u} U(1) = \frac{r_0^{m+1}}{u_0} U(1).$$

Conditions for thermonuclear ignition of deuterium-tritium (DT) gas by a focusing ideally symmetric shock wave are estimated. The wave is focused to the center and then reflects. In so doing a higher-pressure zone is produced following the front of the shock wave reflected from the center; if the wave is intensive enough, the DT gas thermonuclear ignition can occur in the zone. The ignition criterion can be written as

$$\rho_0 r_0 u_0^{22} \geq 1,$$

where ρ_0 is initial DT gas density, $[\rho_0]$ = g/cm^3, u_0 is mass velocity following the converging shock wave front of radius r_0, $[r_0]$ = cm, $[u_0] = 10^7$ cm/sec. Here u_0 is mass velocity following the converging shock wave front of radius r_0. From relations (1) and (4):

$$\rho = \rho_0 \delta(\tau), \; u = u_0 \frac{U(\tau)}{\tau U(1)} \left(\frac{r}{r_0}\right)^{m+1}, \qquad c = u_0 \frac{C(\tau)}{\tau U(1)} \left(\frac{r}{r_0}\right)^{m+1}. \tag{5}$$

Estimate pressure P and temperature T in the material compressed by the self-similar shock wave:

$$c^2 = \gamma \frac{P}{\rho} \tag{6}$$

Hence,

$$P=\frac{\rho c^2}{\gamma}=\frac{\rho_0\delta(\tau)}{\gamma}\frac{r^2}{t^2}C^2(\tau)=$$
$$=\frac{\rho_0 u_0^2}{\gamma}\frac{\delta(\tau)C^2(\tau)}{\tau^2U^2(1)}\left(\frac{r}{r_0}\right)^{2(m+1)}. \tag{7}$$

Having used the equation of state for completely ionized ideal DT gas,

$$P=2\frac{Nk}{A}\rho T, \tag{8}$$

where N = $6{,}02\cdot10^{23}$ is Avogadro number, k=1.38 10^{-16} erg/deg = $1.602\cdot10^{-9}$ erg/keV is Boltzmann constant,A is average atomic weight of DT gas (if the number of deuterium atoms, D, is equal to that of tritium atoms, T, then A=2.5), we find

$$T=\frac{Au_0^2}{2Nk\gamma}\frac{C^2(\tau)}{\tau^2U^2(1)}\left(\frac{r}{r_0}\right)^{2(m+1)}. \tag{9}$$

By expressing velocity u_0 in terms of 10^7 cm/sec and temperature, T, in terms of keV, we should take k = $1.602\cdot10^{-9}(10^{-7})^2=1.602\cdot10^{-23}$ in the formula.

Estimate the self-similar shock wave front velocity. At the wave front, τ = const; hence, from (2) we obtain

$$D=\left(\frac{dr}{dt}\right)_{\tau=const}=-\frac{1}{m\tau U(1)}u_0\left(\frac{r}{r_0}\right)^{m+1}. \tag{10}$$

To find out the conditions, under which the DT gas thermonuclear ignition by the shock wave is possible, it is necessary to include the heat losses and thermonuclear energy release. We are interested in the conditions that ensure the gas ignition following the front of the shock wave reflected from the center for the least wave intensity. The thermonuclear energy release at the converging shock wave front can be therewith neglected. In any case, additional energy can only strengthen the converging wave. In contrast, the heat losses can attenuate the converging wave in comparison with the self-similar one. If the heat losses are small, the wave may be considered the self-similar wave as before. It can be shown that the heat losses due to radiation can be neglected for the converging spherical shock wave, if the shock-wave intensity, $u_1=ur^{-(m+1)}$ (where u is mass velocity following the converging wave front of radius r; for the self-similar wave this value is constant, that is independent on the front radius r and equal to the mass velocity for wave radius r=1 cm), meets condition

$$u_1\le\frac{10}{A^{0.8}\rho_0^{0.07}},\ [u]=10^7\frac{cm}{c},$$
$$[r]=cm,[\rho_0]=\frac{g}{cm^3} \tag{11}$$

We will consider the condition met. Electron heat conduction can be neglected, if the temperature at the converging shock wave front is not higher than

$$Te\approx10\frac{\left(\rho_0r_0u_0^{2.2}\right)^{0/3}}{A^{0.13}} \tag{12}$$

$[Te]=kev,\ [\rho_0]=g/cm^3,\ [r_0]=cm,\ [u_0]=10^7cm/S$

We will consider the condition as met too.

Now consider gas volume, V, behind the front of the shock wave reflected from the center and write the law of conservation of energy for that volume with account for heat losses and thermonuclear reaction:

$$\frac{d}{dt}(E+E_k)=A+E_T-E_u-E_e. \tag{13}$$

Here E is internal energy of the gas:

$$E=\int_0^v\frac{p}{\gamma-1}dV, \tag{14}$$

E_k is kinetic energy of the gas:

$$E_k=\int_0^v\frac{\rho u^2}{2}dV, \tag{15}$$

A is work per unit time done by pressure forces on gas with taking into account the energy contributed to the volume along with the material through the shock front:

$$A=-4\pi r^2\left[Pu-(D-u)\left(\frac{p}{\gamma-1}+\frac{\rho u^2}{2}\right)\right], \tag{16}$$

where r is radius, D is reflected shock front velocity; all the quantities are taken at the shock front; E_T is thermonuclear energy released in gas volume V per unit time:

$$E_T=\int_0^v\frac{(3-A)(A-2)}{A^2}N^2\rho^2\varepsilon\frac{\varphi-4g}{5}f(T)dV, \tag{17}$$

where f(T) is thermonuclear DT-reaction rate [2], for which the following interpolation relation is valid:

$$f(T)=\frac{2.6296\cdot10^{-19}\left(1+0.23220T^{3/4}\right)}{T^{2/3}\sqrt{1+0.94150\cdot10^{-4}T^{3.25}}}e^{-\frac{19.9826}{T^{1/3}}} \tag{18}$$

$$[T]=kev,[f(T)]=10^7cm^3/\sec$$

ε is energy released in DT gas in one thermonuclear reaction event: $\varepsilon=2.83\cdot10^{-19}$ in units of 10^{14} erg = 10 MJ, φ and q are functions including thermonuclear alpha particle and neutron escape from volume V; for homogeneous spherical volume the functions have been determined by E. Pavlovskii:

$$g=\frac{1-(1-Y)}{1+\frac{5}{2}(1-Y)},$$

$$1-Y=\frac{3}{x}\left\{\frac{1}{2}-\frac{1}{x^2}\left[1-(1+x)e^{-x}\right]\right\},$$

$$x=2\left(\frac{3}{4\pi}\right)^{1/3}V^{1/3}\rho\frac{N}{A}\left[(3-A)\sigma_D+(A-2)\sigma_T\right],$$

$$\sigma_D=0.8\cdot10^{-24}cm^2,\quad\sigma_T=0.63\cdot10^{-24}cm^2 \tag{19}$$

$$\varphi=\begin{cases}1-\frac{1}{4Z}\left(1-\frac{1}{40Z^2}\right) & for\ Z\ge\frac{1}{2},\\ \frac{3}{2}Z-\frac{4}{5}Z^2 & for\ Z\le\frac{1}{2}\end{cases}$$

$$Z = 13.1\rho V^{1/3} \frac{2.28 + \lg T - 0.5 \lg \rho}{T^{3/2}} \quad for\ T > 1 keV,$$

$$[\rho] = g/cm^3, [T] = keV, [V] = cm^3,$$

E_u is energy lost by the gas due to bremsstrahlung (volume V under discussion is assumed to be transparent to bremsstrahlung):

$$E_u = \int_0^v a \frac{\rho^2}{A^2} \sqrt{T} dV, \tag{20}$$

where

$$a = 1.75 \cdot 10^3, [E_u] = 10^{21} \frac{erg}{\sec} = 10 \frac{MJ}{10^{-7}\sec},$$

E_e is the energy lost by the gas in volume V due to electron heat conduction, which can be written as

$$E_e = 3(\frac{4\pi}{3}) \quad \chi_e a_e T^{7/2} V^{1/3}, \tag{21}$$

where $a_e = 2 \cdot 10^{-2}$, $[E_e] = 10^{14}$ erq/10^{-7} sec=10 MJ/10^{-7} sec, x_e is a correction factor that includes proper temperature distribution over radius.

Having introduced the relevant profile factors for the other quantities (14,15,17,20), write them as

$$\begin{aligned} E &= x_E \frac{P}{\gamma - 1} V, \\ E_k &= x_k \frac{pu^2}{2} V, \\ E_T &= x_T \frac{(3-A)(A-2)}{A^2} \quad N^2 p^2 \varepsilon \frac{\phi - 4q}{5} f(T) V, \\ E_u &= x_u a \frac{p^2}{A^2} \sqrt{T}\, V. \end{aligned} \tag{22}$$

The values at the front of the shock wave reflected from the center appear in all relations (21 and 22).

Eq. (13) serves for determining temperature T following the front of the shock wave reflected from the center with account for heat losses and thermonuclear reaction. Density, ρ, mass velocity, u, and velocity, D, of the front of the shock wave reflected from the center also appear in this equation. All the values, with taking into account heat losses and thermonuclear reaction, generally speaking, differ from those at the front of the self-similar shock wave reflected from the center. To determine them accurately, besides Eq. (13), the equations of conservation of mass, momentum, and energy at the front of the (not self-similar) shock wave reflected from the center can be used. The relevant values before the front of the shock wave reflected from the center, that is behind the converging shock wave front, appear in the equations. We consider the converging wave to be self-similar, as both thermonuclear energy release and heat losses for it can be neglected, and conclude that all the values before the front of perturbed shock wave (with taking into account heat losses and thermonuclear energy) depend on a single self-similar parameter, τ, for which the relevant equation can be obtained. This equation along with Eq. (13) solves the set-up problem of estimation of all physical quantities following the front of the shock wave reflected from the center with taking into account heat losses and thermonuclear energy release. It is convenient to write Eq. (13) for temperature and the equation for self-similar parameter τ in the dimensionless form. To do this, take unperturbed temperature at the front of the shock wave reflected from the center, that is self-similar temperature (9) (instead of t), for the argument, where by τ is meant the value at the front of the self-similar wave reflected from the center:

$$\tau = \tau_1 = -1.558\ (for\ \gamma = 5/3) \tag{23}$$

Hereinafter we use the self-similar exponent m, self-similar variable τ, and functions $\delta(\tau), U(\tau), C(\tau)$ determined for adiabatic exponent γ = 5/3 by Yu.D. Bogunenko and E.A. Karpovtsev.

The equation for τ will be therewith written in simple form:

$$\frac{d\tau}{dT^i_a} = \frac{m}{2(m+1)} \frac{Q-1}{Q} \frac{\tau}{T_a}, \tag{24}$$

where Q is some dimensionless function, which velocity D (10) of the self-similar shock wave reflected from the center [with τ = τ_1 (23)] has to be multiplied by to obtain the perturbed shock wave velocity.

To transform Eq. (13) to variable T_a, take into consideration relations (9) and (10), then:

$$\frac{d}{dt} = \frac{d}{dT_a} \frac{dT_a}{dt} = 2(m+1) \frac{T_a}{r} D \frac{d}{lT_a}. \tag{25}$$

Here we simplify the problem by assuming that the ignition occurs at temperature T close to self-similar temperature T_a and take the values of all other parameters at the front of the self-similar shock wave reflected from the center. Then, in particular, function Q ≡1; from Eq. (24) it follows that τ = cosnt = -1.548 (the value before the reflected self-similar wave front); the relevant values of the self-similar functions are

$$\begin{aligned} &\delta(\tau) = 18.88; \quad U(\tau) = -0.2306; \\ &C(\tau) = 0.6140 \quad for \quad \tau = -1{,}548; \\ &U(1) = 0.5163; \\ &\delta(\tau_1) = 32.28; \quad U(\tau_1) = 0.1510; \\ &C(\tau_1) = 0.7498 \quad for \quad \tau_1 = -1.45268, \end{aligned} \tag{26}$$

at the reflected self-similar wave front.

Eq. (13) along with expressions (16,21,22) as well as (5 and 8) with taking into account (25) and (26) is transformed to the following equations for perturbed temperature T:

$$\begin{aligned} \frac{dT}{dT_a} &= 1.062 \frac{1 - 0.0752 x_k}{1 - 0.0241 x_k} \frac{T}{T_a} - 0.062 \frac{1 - 0.894 x_k}{1 - 0.0241 x_k} - \\ &\frac{13.6 x_u}{1 - 0.0241 x_k} \cdot \frac{\beta}{T_a^{2,6}} \left[2.9 \cdot 10^{24} \frac{x_T}{x_u} (\phi + 4q) f(T) - \sqrt{T} \right] + \\ &+ \frac{0.0022 x_e}{1 - 0.0241 x_k} \cdot \frac{1}{\beta T_a^{0.4}} \frac{T^{35}}{1.53 + lqT - 0.5 lqp_o}, \end{aligned} \tag{27}$$

which depends on parameter β:

$$[p_0] = q/cm, [\gamma_0] = cm, [u_0] = 10^7 cm/s, \tag{28}$$

and weakly depends (in the logarithmic manner) on initial density ρ_0.

Coefficient x_E is expressed in terms of x_k from the condition that for the unperturbed case, where $T \equiv T_a$, and $x_u = x_e = x_T = 0$, Eq. (27) should be satisfied identically.

Eq. (27) without inclusion of heat losses and thermonuclear energy release (when $x_u = x_e = x_T = 0$) is written as

$$\frac{dT}{dT_a} = a_1 \frac{T}{T_a} - a_2,$$
$$a_1 = 1.062 \frac{1 - 0.0752 x_k}{1 - 0.0241 x_k}, \qquad (29)$$
$$a_2 = 0.062 \frac{1 - 0.894 x_k}{1 - 0.0241 x_k}.$$

Clear that for $T=T_a$ the condition

$$a_1 - a_2 = 1 \qquad (30)$$

is met.

Eq. (29) with taking into account (30) has solution

$$T = T_a + (T_0 - T_{a0}) \left(\frac{T_a}{T_{a0}} \right)^{a_1}. \qquad (31)$$

Hence, with decreasing T_a (that is with increasing time) the perturbed temperature T will tend to the unperturbed T_a, if $a_1>1$. In fact:

$$a_1 = \begin{cases} 1.062 & for \quad x_k = 0, \\ 1.007 & for \quad x_k = 1, \end{cases} \qquad (32)$$

that is tendency $T \to T_a$ will proceed quite slowly. If the initial values of T and T_a are the same ($T_0 = T_{a0}$), then $T=T_a$.

Since by the ignition is meant a situation, where temperature T behind the front of the shock wave reflected from the center begins to increase (with decreasing self-similar temperature T_a), it can be considered that $\frac{dT}{dT_a}$ vanishes at the ignition time. Having assumed that the perturbed temperature T is therewith the same as the unperturbed (self-similar) temperature T_a, from Eq. (27) with taking into account (30) we obtain the equation for determining ignition temperature $T_{\cdot}=T_a$:

$$1 - \frac{13.6 x_u}{1 - 0.0241 x_k} \frac{\beta}{T^{2.6}} \times$$
$$\times \left[2.9 \cdot 10^{14} \frac{x_T}{x_u} (\varphi + 4q) f(T) - \sqrt{T} \right] + \qquad (33)$$
$$+ \frac{0.0022 x_e}{1 - 0.0241 x_k} \frac{1}{\beta} \frac{T^{3,1}}{1.53 + 1gT - 0.51g\rho_o} = 0.$$

From this equation we can find (for example, with the method of trials) the value of T such, with which the value of β will be minimal. This minimal value of β is just the critical value of the parameter, with which the ignition occurs, and the relevant value of T determines the ignition temperature. The critical values of β and T depend on the profile factors z_k, x_u, x_e, and x_r. In particular, with unit values of the profile factors the critical values of β and T are equal:

$$\beta \simeq 1.2; \quad T \simeq 8.5 keV. \qquad (34)$$

A change in the initial gas density, ρ_0, leads only to a slight change in the term that includes the gas electron heat conduction and is equivalent to some change in the profile factor x_e. Eq. (27) with parameter β, minimum necessary for the ignition, gives the dependence of T on T_a: if at the initial time $T_0=T_{0a}$, then with decreasing T_a the value of T first becomes less than T_a (due to heat losses), with some $T=T_i$ (ignition temperature) $\frac{dT}{dT_o}$ vanishes, and then (with further decrease in T_a) temperature T begins to increase due to thermonuclear energy, which is just responsible for the DT gas burst. With β less than minimal necessary no increase in T with decreasing T_a is observed and no ignition occurs. The minimum value of β, with which T still increases with decreasing T_a (beginning with some values of T and T_{ol}), is just the ignition criterion. The critical values of β as well as T and T_0 depend, besides the profile factors, on the choice of the initial value of $T_0=T_{a0}$ in the integration of Eq. (27). The minimum value of β and the associated critical values of T and T_a are

$$\beta \approx 0.85; T \approx 3.5 keV; \\ T_a \approx 2 keV; \\ for\ T_o = T_{ao} \approx 8 keV. \qquad (35)$$

With increasing or decreasing initial values of $T_0=T_{a0}$ the values of β and T_a increase and T remains about constant. Thus,

$$\beta \simeq 1.5; T = T_a = 3.5 keV \ with\ T_o = T_{ao} \simeq 4 keV,$$
$$\beta \simeq 2; T \simeq T_a \simeq 3.5 keV \ with\ T_o = T_{ao} \simeq 30 keV. \qquad (36)$$

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