1. INTRODUCTION

The undulator type radiation, UR, emitted by a beam of relativistic charged particles, undulating in the alternating wakefields induced by this beam when it is moving in the periodic structure, is interesting for practical applications of accelerators and FEL physics which recently has received development. Two new radiation mechanisms have been proposed for generating ultra-short wavelength light. In papers [1, 2] the image wave range where the wave diffraction can be neglected is given in [4, 5]. As follows from the theory, in the case of incoherent UR, the charge is small parameters $\omega_{\text{res}} < \omega$ is given for dipole approximation in the form [4]

$$P_{\text{UR}} = \frac{e^2}{16e} \int_0^\infty \int_0^\infty \int d^3p \sum_{n=1}^{\infty} \left[ \frac{k_f}{p_f} \right] \left[ 1 - \sin^2 \phi R(\alpha, \beta, \gamma) \right]$$

$$\times \left[ \delta \left[ \beta_n \cos \theta - 1 \right] - \frac{\gamma}{\epsilon} \right] \left[ \delta \left[ \beta_n \cos \theta + 1 \right] - \frac{\gamma}{\epsilon} \right]$$

(1)

Here $R(\alpha, \beta, \gamma) = \left[ 1 - \frac{\omega}{\Omega} \cos \alpha \right] - \frac{\omega}{\Omega} \cos \phi$ is the angle between the wave vector $k$ and the longitudinal axis $OZ$, $\omega$ is the frequency, $\omega_{\text{res}}$ is the electron plasma frequency of the waveguide metal, $\beta$ is the angle between the axis $OZ$ and the XOY is the plane projection of $k$, $\epsilon = \frac{2e^2 u_{\text{sp}}}{mc^2}$ is small parameters $| \alpha | < 1$, $u_{\text{sp}}$ is a transverse component of the $p^\text{th}$ harmonic of the wave function defined as $u_{\text{sp}} = w_{\text{sp}}(\gamma^2 + p^2)^{1/2}$, where $w(p)$ is

$$w(p) = \frac{D_{\text{eff}}}{4c^2} \sum_{n=1}^{\infty} \left[ g_{\text{sp}}^{(n)} \right] \left[ g_{\text{sp}}^{(n)} \right]$$

(2)

In the present paper, we will consider UR spectral-angular characteristics for relativistic charged particles interacting with transverse components of the nonresonant spatial harmonics of the self-wakefields excited in a periodic structure. Also, we will analyze the conditions when the incoherent UR power can exceed the loss power caused by exciting wakefields.

2. RADIATION BY SINGLE-PARTICLE

As a periodic structure, we will consider a vacuum corrugated waveguide with a metallic surface. Let a particle with the ultrarelativistic longitudinal velocity $v_0$, the charge $e$, and the mass $m$ moves through the structure with the period, $D$. The UR power emitted by the particle in the spectral region where the wave diffraction can be neglected is given as $\omega_{\text{res}} < \omega$ is given for dipole approximation in the form [4]

$$P_{\text{UR}} = \frac{e^2}{16e} \int_0^\infty \int_0^\infty \int d^3p \sum_{n=1}^{\infty} \left[ \frac{k_f}{p_f} \right] \left[ 1 - \sin^2 \phi R(\alpha, \beta, \gamma) \right]$$

$$\times \left[ \delta \left[ \beta_n \cos \theta - 1 \right] - \frac{\gamma}{\epsilon} \right] \left[ \delta \left[ \beta_n \cos \theta + 1 \right] - \frac{\gamma}{\epsilon} \right]$$

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In the present paper, we will consider UR spectral-angular characteristics for relativistic charged particles interacting with transverse components of the nonresonant spatial harmonics of the self-wakefields excited in a periodic structure. Also, we will analyze the conditions when the incoherent UR power can exceed the loss power caused by exciting wakefields.
$$P_{\beta} = \frac{2\pi}{4c^2} \left[ \sum_{\mu=\pm1} \int_{\sigma} \frac{d\theta}{\sigma} \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}} \right]$$

$$+ \frac{\sin^2 \beta}{\beta_0 \cos^2 \beta} \int_{\sigma} \left[ \frac{1}{\sigma} \frac{1}{\beta_0} \frac{1}{\beta} \right] \left[ \frac{1}{\beta_0} \frac{1}{\beta} \right]$$

$$+ \frac{\sin^2 \beta}{\beta_0 \cos^2 \beta} \int_{\sigma} \left[ \frac{1}{\sigma} \frac{1}{\beta_0} \frac{1}{\beta} \right] \left[ \frac{1}{\beta_0} \frac{1}{\beta} \right]$$

(3)

$$\theta_s = \arccos \left( \frac{1}{\sigma} \frac{1}{\beta_0} \right)$$

Here,

$$P_{\beta} = \left\{ \begin{array}{ll}
1, & \text{if } |1 - \frac{\beta^2}{\sigma}| \geq 1 \\
\text{integer of } |1 - \frac{\beta^2}{\sigma}|, & \text{otherwise}
\end{array} \right.$$

The number of harmonics in the sum is defined by the dipole limit resulting in $p < p_{\text{max}} = 2\pi / \max \{ \omega_n \}$.

Integrating Eq. (1) over $\theta$ and $\phi$, the spectrum distribution of the UR power is obtained in form

$$P_{\beta} = \frac{2\pi}{4c^2} \left[ \sum_{\mu=\pm1} \int_{\sigma} \frac{d\theta}{\sigma} \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}} \right]$$

$$+ \frac{\sin^2 \beta}{\beta_0 \cos^2 \beta} \int_{\sigma} \left[ \frac{1}{\sigma} \frac{1}{\beta_0} \frac{1}{\beta} \right] \left[ \frac{1}{\beta_0} \frac{1}{\beta} \right]$$

(4)

3. RADIATION BY BUNCH

Dimensions $\sigma$ of bunches accelerated in the high-energy RF lines satisfy the relation $D \gamma^2 < \sigma$ as well as $\omega_p < 2\Omega \gamma$. So, it is of interest to consider characteristics of incoherent hard radiation of real beams. In this case, the UR power may be written as [5]

$$P_{\beta} = \frac{4\pi}{3\omega_0} \left[ \sum_{\mu=\pm1} \int_{\sigma} d\theta \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}} \right]$$

$$+ \frac{\sin^2 \beta}{\beta_0 \cos^2 \beta} \int_{\sigma} \left[ \frac{1}{\sigma} \frac{1}{\beta_0} \frac{1}{\beta} \right] \left[ \frac{1}{\beta_0} \frac{1}{\beta} \right]$$

(5)

where $f_s(\tau_s, \tau) / V_0$ is the normalized function of charge-density structure, $S_\tau$ is the cross-section of the periodic density structure, $\tau = t - z / V_0$,

$$u[p](\tau) = \frac{1}{\sigma^2 N_\theta} \left( \int d\tau f_0(u) f_0(z) \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}}{\beta_0 \beta} \right)$$

(6)

$F(p, \tau, \tau_o)$ is the $p$th spatial harmonics of the force, produced by the point charge $e N_\theta$ (with the transverse coordinate $\tau_o$) acting on the charge $e$ (with the transverse coordinate $\tau$) moving at the distance $V_0 \tau$ after the point charge.

For analytical calculations it is convenient to consider a monochromatic filamentary uniform bunch of the length $l$, moving in a weakly corrugated circular waveguide of the radius

$$b(z) = b_0 [1 + \epsilon(z)] = b_0 \left[ 1 + \epsilon \left( \sum_{\mu=\pm1} \int_{\sigma} d\theta \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}}{\beta_0 \beta} \right) \right]$$

(7)

Here $\epsilon(z) < 1$ is the relative depth of the corrugation, $b_0$ is the average radius of the waveguide.

The UR power lost by the bunch moving at the distance $r_s$ from the waveguide axis is given by [5]

$$P_{\beta} = \frac{4e^6}{3\omega_0^2} \left[ \sum_{\mu=\pm1} \int_{\sigma} d\theta \frac{\sqrt{1 - \frac{\beta^2}{\sigma}}}{\beta_0 \beta} \right]$$

$$+ \frac{\sin^2 \beta}{\beta_0 \cos^2 \beta} \int_{\sigma} \left[ \frac{1}{\sigma} \frac{1}{\beta_0} \frac{1}{\beta} \right] \left[ \frac{1}{\beta_0} \frac{1}{\beta} \right]$$

(8)

Let us consider the sinus-type corrugated waveguide of a radius $b(z) = b_0 [1 + \epsilon(z)]$, with $\epsilon(z) = 0.05$. Let an electron bunch, with the typical for SLAC Linac parameters, $l_0 = 500$ nm, $N_\theta = 4 \times 10^{10}$, and $\gamma = 10^5$ moves in a sub-millimeter structure with $b_0 = D = 0.3$ mm. The bunch distance from the axis is chosen equal to $r_s = 0.9 b_0$ to estimate the maximal values of the UR power.

The distribution of the synchronous harmonic of the longitudinal wake function $u_\beta^{(n)}$ and the $(+1)^n$ harmonic transverse wake function $u_r^{(+1)}$ along the bunch are represented in Figs. 1 and 2, respectively. In calculations 120 resonant modes are taken into account.

![Fig. 1. The synchronous harmonic of longitudinal wake function](image1)

![Fig. 2. The nonsynchronous harmonic of the transverse wake function](image2)
As shown in Fig. 1, the bunch head, up to 50 µm, basically excites WF, while UR is predominantly emitted by the next part after head of the bunch (see Fig. 2).

\[\text{Fig.3. The UR and WF power versus the bunch length}\]

Eq.(5) shows that in the range of ultra-sort wavelength light the UR power grows as a square of particle energy. Therefore, we can expect that for very high-energy particles the UR power can exceed the WF power which, as well-known, is independent on the particle energy. Figs. 1, 2 indicate to increasing the UR power fraction with lengthening the bunch. This is confirmed by Fig. 3. The incoherent UR power reaches the WF power at the bunch length 200 µm.

The incoherent UR power and WF power for the bunch of length 500 µm versus electron energy are represented in Fig. 4. As seen from this figure the UR predominates at energies above 50 GeV.

\[\text{Fig.4. The incoherent UR and WF power}\]

Fig. 5 represents the dependence of a number of electrons in the bunch of length 500 µm on their energy when the incoherent UR power equals to the WF power. This dependence determines the limit above which the incoherent UR power exceeds the WF power.

\[\text{Fig. 5. The number of bunch electrons versus the electron energy when UR power equals the WF power}\]

4. CONCLUSIONS

The spectral–angular characteristics of undulator-type radiation of the ultrarelativistic charged particle undulating in nonsynchronous spatial harmonics of the self-wavefields in the periodic structure are obtained. It is shown numerically that whereas the beam power losses fall with lengthening the bunch, the relative fraction of the UR power increases. The conditions when the incoherent UR power becomes comparable with the WF power are defined.

REFERENCES


ХАРАКТЕРИСТИКИ ОНДУЛЯТОРНОГО ИЗЛУЧЕНИЯ СГУСТКА ЗАРЯЖЕННЫХ ЧАСТИЦ В КИЛЬВАТЕРНОМ ПОЛЕ

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Получены характеристики ондукаторного излучения густка релятивистских заряженных частиц, взаимодействующих с несинхронными гармониками кильватерного поля в периодической структуре.

ХАРАКТЕРИСТИКИ ОНДУЛЯТОРНОГО ВИПРОМІНЮВАННЯ ЗГУСТКОМ ЗАРЯДЖЕНИХ ЧАСТИНОК В КІЛЬВАТЕРНОМУ ПОЛІ

А.М. Опанасенко

Одержано характеристики ондукаторного випромінювання густком релятивістських заряджених частинок, які взаємодіють з несинхронними гармоніками кильватерного поля в періодичній структурі.