MODELING OF ROTATORY FLOWS IN THE PLASMA ACCELERATOR CHANNEL WITH LONGITUDINAL MAGNETIC FIELD

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The numerical model of two-dimensional flows in coaxial channels of plasma accelerators is considered in presence of the longitudinal magnetic field. Computations are based on the MHD-equations taking into account the Hall effect, electrical conductivity tensor. The analytical and numerical solutions are analyzed and compared with reference to steady-state plasma flow in case of the ion current transport.

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INTRODUCTION

Many factors, above all the current crisis [1] due to the Hall effect, impede the achievement of high velocities in plasma accelerators. In order to overcome the negative consequence of the Hall effect it is appropriate to go to systems of the quasi-steady plasma accelerator type proposed in [2]. The experiments generally confirmed the underlying ideas on the basis of which such plasma accelerators are designed. The experimental investigations [3-5] show a real opportunity to get flows of certain geometry.

The presence of a longitudinal magnetic field opens up new possibilities for controlling the dynamic processes in the accelerator channel and makes to realize transonic flow in a channel of certain geometry.

In [8] the effect of a longitudinal magnetic field on two-dimensional axisymmetric two-component plasma flows is determined analytically. The investigations are carried out within the framework of the smooth channel approximation for ideal magnetohydrodynamic equations. The longitudinal magnetic field complicates the flow. For example, it leads to plasma rotation about the axis of the system. In this case, an analysis of the most important properties of plasma flows showed that the Hall effect and the anode flow zone can be significantly reduced owing to the longitudinal field.

BASIC EQUATIONS

The numerical model of axial-symmetrical flow in plasma accelerator channels in the presence of a longitudinal magnetic field is elaborated. Computations are based on the MHD-equations taking into account the Hall effect ($V_e \neq V_i$), electrical conductivity tensor and transport coefficients in magnetic field depending on the $\omega_e \tau_e$ [9].

\[
\rho \frac{d V}{dt} + \nabla P = \frac{1}{c} \left[ j, H \right]
\]

\[
\rho \frac{d e}{dt} + P \text{ div } V = \frac{k}{e (\gamma - 1)} \left( j \cdot \nabla \right) T + \frac{P_e}{e} \text{ div } V
\]

\[
\frac{1}{c} \frac{d H}{d t} = - \text{rot } E
\]

\[
E = \frac{1}{c} \left[ V_e, H \right] - \frac{1}{en} \nabla P_e + \frac{1}{en} R
\]

\[
j = \frac{c}{4\pi} \text{rot } H = en \left( V_i - V_e \right)
\]

\[
\frac{d}{dt} T = \frac{\nabla}{\rho} \cdot \left( \sigma \left( p - c_v \right) T \right)
\]

\[
\varepsilon = 2c_v T
\]

Here $V_i = V_i$; $n_i = n_e$; $T_i = T_e$; $P$ - total pressure, $\rho = m$ - density of heavy particles, $j$ - electric current; $q$ - heat flux.

\[
R = R_j + R_T
\]

\[
Q = Q_j + Q_T = \frac{1}{en} \left( R, j \right)
\]

\[
R_j = \frac{e n}{\sigma} \left( A_1(x) j_{//} + A_2(x) j_{\perp} + \frac{A_3(x)}{H} \right) \left( H, j \right)
\]

\[
R_T = -k n \left( B_1(x) \nabla_{//} T + B_2(x) \nabla_{\perp} T - \frac{B_3(x)}{H} \right) \left( H, \nabla T \right)
\]

\[
j_{//} = \frac{1}{H^2} \left( j, H \right) H
\]

\[
j_{\perp} = \frac{1}{H^2} \left[ H, \left( j, H \right) \right]
\]

\[
q = q_e + q_i = q_e^* = \frac{k}{e} \left( B_2(x) \right) j +
\]
\[ \frac{B_1(x) - B_2(x)}{H} \left( j, H \right) H + \frac{B_3(x)}{H} \left( H, j \right) \]  
\[ \sigma = \frac{e^2 n_e \tau_e}{m_e} \sim T^{3/2} \]  

\[ A_{1,2,3} \]  and \[ B_{1,2,3}(x) \] are known functions of \( x = \omega_e \tau_e \), \( \omega_e = \frac{e H}{m_e c} \), \( \tau_e = \frac{3 \sqrt{m_e} \left( k T \right)^{3/2}}{4 \sqrt{2} \pi \hat{e} e^2 Z^2 n} \).

In the case of axial flow symmetry \( \left( \partial / \partial \varphi = 0 \right) \) it is possible to introduce the vector potential \( A (H = rot A) \) so that \( d i \times H = 0 \) in the numerical simulations. The toroidal vector potential \( A_\varphi \) defines the poloidal components of the magnetic field \( H_z \) and \( H_r \).

As a result, we have 7 equations for variables \( \rho, T, V_z, V_r, V_\varphi, H_\varphi, \tilde{A}_\varphi \).

PARAMETERS, BOUNDARY CONDITIONS AND NUMERICAL METHODS

As the input dimensional units, we will take the characteristic values of the density and temperature \( \rho_o = m n_o \) and \( T_o = T_o^e = T_o^i \) at the channel inlet, the length \( L \) of the plasma accelerator channel and the strength \( H_o = H_o^o \) of the azimuthal magnetic field component, where \( J_p \) is the current in the system and \( R_o \) is the characteristic radius at the inlet. Using these quantities, we form the following units of velocity, pressure and time scales: the characteristic Alfvén velocity \( V_o = H_o \sqrt{4 \pi \rho_o} \), \( H_o / 4 \pi \) and \( L / V_o \), respectively. In this case the dimensionless parameters in numerical investigation have the form:

\[ \beta = \frac{8 \pi \rho_o}{H_o^2} \]  
\[ \tilde{\zeta} = \frac{c}{e L} \left( \frac{m_i}{4 \pi n_i} \right) \]  
\[ \nu = \frac{1}{\text{Re}_m} = \frac{c^2}{4 \pi L V_o \sigma} ; \]  
\[ \omega_e \tau_e = \frac{\tilde{\zeta} H}{\nu \rho} \]

The dimensionless conductivity \( \text{Re}_m = \sigma_o T^{3/2} \) (magnetic Reynolds number) contains \( \sigma_o \) which can be expressed in terms of the initial dimensional parameters and physical constant.

The boundary condition at the channel inlet corresponds to the subsonic plasma inflow with \( \rho(r) = f_1(r) \); \( T(r) = f_2(r) \); \( V_\varphi = 0 \); \( H_z(r) = H_z^o \), where \( f_1(r) \) and \( f_2(r) \) are known functions of \( r \). We will assume that the total electric current flowing through the system is supplied only through the electrodes and maintained constant. This generates the boundary condition at the inlet for azimuthal magnetic field \( r H_\varphi = r_0 = R_o / L \).

The accelerator plasma dynamics are investigated in different current transport regimes. In the electron current transport regime the streamlines of the ion plasma component lie on the impermeable electrode surfaces. In this case the electrodes are not equipotential. On the other hand, in the ion current transport regime the electrodes are equipotential surfaces. In this case they must be transparent for the plasma entering the channel across them. Most experiments [3-5] and models [6,8,10] are based on ion current transport.

In the present study we will consider cold plasma flow in the ion current transport regime. In this case we have \( E_t = 0 \) on the electrode surfaces: the cathode \( r_e(z) \) and the anode \( r_o(z) \). The shapes of the electrodes are given. In the presence of a longitudinal magnetic field it is necessary to determine the additional condition for magnetic field components \( H_z \) and \( H_r \). Within the framework of simple formulation of the problem it is natural to assume that \( H_o = 0 \) on the electrode surfaces. Besides we will suppose that \( \partial T / \partial n = 0 \). The anode flow enters the channel from the side of the anode. Therefore, on the anode surface we must assign two functions \( \rho(z) = f_3(z) \) and \( V_\varphi = f_4(z) \). The functions \( f_1(r), f_2(r), f_3(z), f_4(z) \) must be chosen depending on the particular details of the problem formulation. These functions can be determined from analytical model in order to compare the analytical and numerical solutions.

The channel geometry is specified by the electrode profiles, which correspond to the possibility of transonic flow so that at mid-channel the flow velocity passes through the local velocity of the fast magnetosonic wave. So at the accelerator outlet the boundary conditions correspond to a supersonic plasma flow.

The simulations are based on the adaptation of flux-corrected transport method for hyperbolic part of equations. The finite conductivity is taken into account in the parabolic equations for \( A_\varphi \) and \( H_\varphi \) fields by means of the implicit numerical scheme. A steady-state supersonic flow in the nozzle-type channel calculated by the relaxation method.

RESULTS OF COMPUTATION

The channel has the shape of a Laval nozzle of length equal to unity. The initial dimensional and dimensionless parameters are: \( n_o = 3.6 \cdot 10^{14} \text{ cm}^{-3} \); \( T_o = 2 eB \); \( L = 60 \text{ cm} \); \( J_p = 300 \text{ kA} \); \( \beta = 0.005 \); \( \tilde{\zeta} = 0.02 \); \( \sigma_o = 812.8 \). For hydrogen plasma we have \( \nu = 0.0012 \) and \( \omega_e \tau_e = 16.2 \) if \( T = 1 \) and
\[ \rho = 1. \] Within the framework of the model proposed the presence of a small insignificant longitudinal magnetic field \((H_z^0 = 0.1)\) makes it possible to realize transonic flow.

\[ (H_z, H_z) \Rightarrow (j_z, j_r) \Rightarrow r H_\varphi = \text{const} \]

\[ V_\varphi(z, r) \] computation theory

The calculation results correspond to the analytical model of rotatory plasma flow.

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REFERENCES

Представлена численная модель двумерных течений в коаксиальных каналах плазменных ускорителей при наличии продольного поля. В основу модели положены МГД-уравнения с учетом эффекта Холла, тензора проводимости среды. Проведено сопоставление аналитического решения и результатов расчетов установившегося течения плазмы в режиме ионного токопереноса.

МODELIROVANIE BRA’TSHAUCHHIHSHIY POTOKOV V KANALE PLOZMENNOGO USKORITELIYA PRI NAHLICHII PRODOLNOGO MAGNITNOGO POLYA

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MODELUVANIA OBERTOVIH POTOKIV U KANALI POZMOWOGO PRISKOYUCAH W PRISUTNOSTI POUZDOJNOGO MAGNITNOGO POLYA

A.M. Kozlov

Представлена чисельна модель двомірних течій в коаксіальних каналах плазмових прискорювачів в присутності повздовжнього поля. В основу моделі покладено МГД-рівняння з урахуванням ефекту Холла, тензора провідності середовища. Досліджено зіставлення аналітичного розв’язання і результатів розрахунків течії плазми у режимі іонного струмопереносу.