FEEDBACK EFFECTS BETWEEN RESONANCE SURFACES AND SPACE HARMONICS OF EXTERNAL PERTURBATIONS IN TOKAMAK

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Resonant magnetic surfaces in a tokamak can amplify the spatial harmonics of external perturbations, which may come from other resonant surfaces, from error fields, or from a feedback system. The behavior of this active resonant media can be roughly approximated with a system of coupled Van der Pole oscillators. The effect of frequency injection locking (or spatial harmonics injection locking in the plasma frame) is typical for these nonlinear systems. It happens when the amplitude of one modes increases and this mode becomes a dominant mode. Transition into synchronized condition can occur in a time scale of ~ 50 -100 μsec. For a tokamak it means that the stability of a large scale MHD perturbation can change jumpily, because frequency (phase) lock may create a positive feedback between resonant surfaces (or between resonant surfaces and the external feedback system). This effect probably determines the explosive dynamic of the disruptive instability.
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We use the following definitions:

- *Active mode* is the tearing or kink mode which has positive or close to 0 positive growth. The behavior of this mode depends on main plasma parameters such as shear, pressure profile, etc
- *Passive mode* is the mode which has negative growth or close to 0 negative growth. The behavior of this mode depends on amplitude and phase of the external perturbations.
- *Dominant mode* is the mode, which determines the global phase synchronization of the secondary active or passive modes.

Perturbations in a torus are coupled so that the development of an active mode at a resonance surface may excite a cascade of secondary passive (slave) modes at other appropriate m/n resonance surfaces q(r)=(m/n), where r- m is the poloidal, and n is the toroidal number of the helical perturbation (Fig.1a). Real excitation of these modes depends on the spatial structure of the driving perturbation (which comes from the active mode or from an external error field) and depends on conditions of the mode excitation, such as the geometry of the plasma, the current and pressure density profile, the values of toroidal magnetic field and plasma current, electromagnetic conditions of the vacuum wall and also depends on the relative velocity of the resonance surfaces and external perturbations. In the laboratory frame these phenomena can be described in frequency terms as resonance, frequency capture or mode locking. Destruction of resonances between different resonance surfaces and between resonance surfaces and error fields may greatly improve high-performance tokamak operation.

Experiments show the number of different m/n resonant perturbations, measured in different discharges at q≤5, do not exceed 16. This particular case is shown in Fig. 1b.

Usually, total amount of active and passive modes, which are developing in one shot simultaneously do not exceed 4-6 (for the case Fig. 1a). In Fig. 1b the circus show possible modes excitation in the case, for example, when the active primary mode m/n=1 excites the set of passive secondary modes 2/1, 3/1, 4/1, 5/1.

a

b

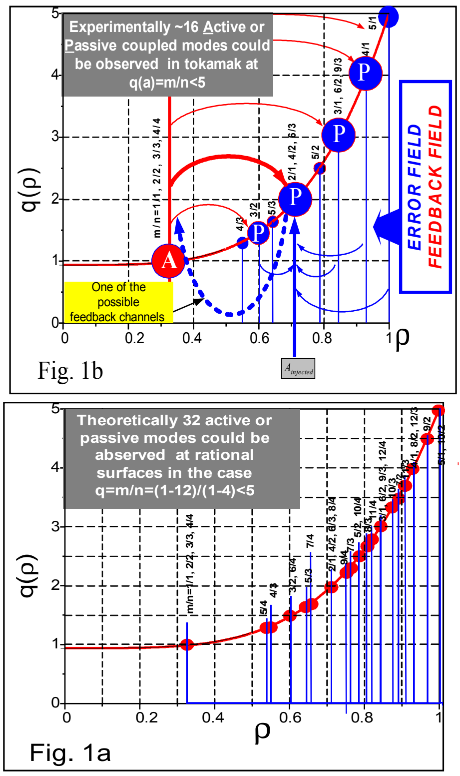


Fig.1

The secondary modes interact with each other on the tertiary basis and interact with space harmonics of error fields - A_{error}^{sum} and feedback systems - $A_{feedback}^{sum}$. In such a way the complicated system of resonant interactions is realized [1, 2, 3]. At each resonance surface the sum of these external perturbations are forming the resultant perturbation $A_{injected}^{sum}$, which determines the dynamics of this magnetic surface (for example 2/1 at Fig. 1b).

Experimentally it is possible to observe 3 forms of the mode excitation: First, the passive eigenmode excitation by corresponding harmonic of the external perturbation. Second, the triggering of the active eigenmode (this excitation is similar to hard excitation of the generator by external perturbation or, for example, an excitation of pendulum clock). In this case at the resonant surface, in laboratory frame, it is possible to observe two identical m/n modes with different frequencies. For example at q=2 (Fig. 1b), we can measure the free running 2/1eigenmode and injected 2/1 mode with the frequency equals to 1/1 at q=1. Third, it is possible to observe all modes synchronization or frequency locking under the influence of dominant mode. It happens when amplitude one of the modes reaches the level of ~ 5 Gauss (data were corrected according [4]). Synchronization (mode-locked mode) is accompanied by sharp increase in amplitude. Mode-locked mode, as a rule is observing directly before disruptive instability. To all appearance, the sharp modes amplitude growth is possible to explain by onset of positive feedback between active modes. One possible feedback channels is shown by dashed curve in Fig. 1b.Previously described MHD-perturbation dynamics in coupled system of resonant magnetic surfaces is similar to dynamics of coupled nonlinear oscillators. For the first time it was mentioned by Huygens [5] for the coupled pendulum system. This phenomenon is well known in modern engineering and describes by system of coupled quadrupoles (see Fig. 2) based on Van der Pole generators [6, 7, 8].

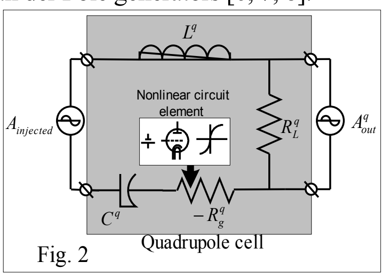


Fig.2

Van der Pole equations set for coupled magnetic surfaces is possible to write as:

$$\ddot{A}_{out}^q - \alpha(1-\beta {A_{out}^q}^2)\dot{A}_{out}^q + \omega_0^2 A_{out}^q = A_{injected}^{sum}$$

where

$$A_{injected}^{sum} = \sum_{k\neq q} \mu^k A_{out}^k Cos(\Omega^k t + \Phi^k(t)) + A_{error}^{sum} + A_{feedback}^{sum}$$

– the sum amplitude of the injected perturbations at resonant surface q;

$\omega_0^q = \sqrt{\dfrac{1}{L^q C^q}}$ - free running frequency at c

$q=k=m/n$; Ω^k, Φ^k, μ^k - injected frequency, phase and coupling coefficient between other resonant surfaces at $q\neq k=m/n$;

α, β, R_g^q, R_L^q - Van der Pole generator parameters

As is known, the locking amplitude $A_{injected}^{sum} = A_{injected}^{locking}$ describes for this system by Adler ratio[9]:

$$\frac{d\Phi^k(t)}{dt} = \omega_0^q (1/2\text{Q}) \text{Im}(A_{injected}^{sum} / A_{out}^q)$$

where Q is the resonant quality of the system.

Application of the Adler ratio to resonant magnetic surfaces is the subject of the next article. Note if the coupled system has high Q and small frequency difference, the locking maybe happen under very small $A_{injected}^{locking}$ amplitude.

The fact of modes slippage and sharp transformation into condition when dominant mode guides all other modes is very important for feedback control studies because the perturbation from feedback system in some cases could play the role of dominant mode and excite local positive feedback between internal modes, which could lead to disruptive instability. Analytical model of resonant field amplification in tokamaks is considered by V. Pustovitov [10] and A. Boozer [11].

Simulation of the frequency locking dynamics as a function of injected perturbation amplitude $A_{injected}^{sum}$ was carried out by Dr. Yu. Mitrishkin. Fig. 3 shows the sharp suppression of the free running perturbation and transition into locking condition for the case of high frequency difference $\omega_{injected} = 2\omega_0$.

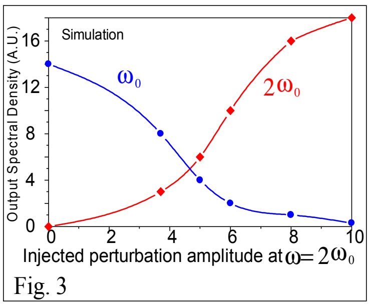
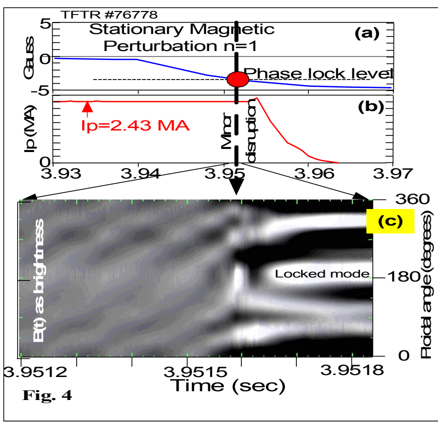


Fig.3

Note that the external error field can play an important role as dominant mode with ω=0. Fig. 4 shows that case (TFTR). As amplitude of stationary perturbation n=1 increases (Fig. 4a) the abrupt changes in mode structure are happened (Fig. 4c).



a

b

Fig.4

In the moment (t=3.9516 s) the stationary dominant mode m=3/n=1 changes the rotational dominant mode m=4/n=1 or “rotational Locked mode” m=4/n=1 transforms into stationary locked mode m=3/n=1. Transition is accompanied by minor disruption and lasts 150-200 μs. (Toroidal rotation of the plasma column apparently lasts out.). The dynamics of minor disruption was described in details in [12] but it was not clear the reason of stationary Locked mode. Above discussed mechanism of phase velocities alignment (similarly to Adler ratio) by dominant mode fills this gap.

Note, in the case of a stationary error field it is difficult to observe the amplification of error field directly using magnetic probes which produces a false impression of precursor absence in some disruptions. Amplification of a pulsed error field is easily observed in RWM stabilization experiments [13]. The modes synchronization with externally applied active perturbation and amplification of this perturbation was observed in T-10 [14], [15].

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ЭФФЕКТЫ ОБРАТНОЙ СВЯЗИ МЕЖДУ РЕЗОНАНСНЫМИ ПОВЕРХНОСТЯМИ И ПРОСТРАНСТВЕННЫМИ ГАРМОНИКАМИ ВНЕШНИХ ВОЗМУЩЕНИЙ В ТОКАМАКЕ

И.Б. Семенов, С.В. Мирнов, Е.Д. Фредриксон, В.А. Вознесенский

Резонансная магнитная поверхность в токамаке может усиливать пространственные гармоники возмущений других резонансных поверхностей, возмущения пространственных гармоник обмоток полоидального и тороидального полей (Error field) или обмоток обратных связей (Feedback field). Поведение этой активной резонансной среды грубо можно аппроксимировать системой связанных генераторов Ван дер Поля. Эффект захвата частоты (или захвата пространственных гармоник возмущений в системе координат, связанной с плазмой), является типичным для подобных нелинейных систем. Он происходит в том случае, когда амплитуда одной из мод увеличивается и эта мода становится доминантной модой. Переход в состояние захвата (синхронизации) частоты происходит за времена ~ 50 -100 μsec. В этот момент устойчивое состояние крупномасштабных МГД-возмущений может скачком стать неустойчивым вследствие появления положительной обратной связи между резонансными поверхностями (или между резонансными поверхностями и системой обратных связей). Этот эффект возможно определяет взрывной характер развития неустойчивости срыва.

ЕФЕКТИ ЗВОРОТНОГО ЗВ'ЯЗКУ МІЖ РЕЗОНАНСНИМИ ПОВЕРХНЯМИ І ПРОСТОРОВИМИ ГАРМОНІКАМИ ЗОВНІШНІХ ЗБУРЮВАНЬ У ТОКАМАЦІ

І.Б. Семенов, С.В. Мирнов, Е.Д. Фредріксон, В.А. Вознесенський

Резонансна магнітна поверхня в токамаці може підсилювати просторові гармоніки збурювань інших резонансних поверхонь, збурювання просторових гармонік обмоток полоідального і тороідального полів (Error field) або обмоток зворотних зв'язків (Feedback field). Поводження цього активного резонансного середовища грубо можна апроксимувати системою зв'язаних генераторів Ван дер Поля. Ефект захоплення частоти (або захоплення просторових гармонік збурювань у системі координат, зв'язаної з плазмою), є типовим для подібних нелінійних

систем. Він відбувається в тому випадку, коли амплітуда однієї з мод збільшується і ця мода стає домінантною модою. Перехід у стан захоплення (синхронізації) частоти відбувається за часи ~ 50 -100 μsec. У цей момент стійкий стан великомасштабних МГД-збурювань може стрибком стати нестійким унаслідок появи позитивного зворотного зв'язку між резонансними поверхнями (або між резонансними поверхнями і системою зворотних зв'язків). Цей ефект можливо визначає вибуховий характер розвитку нестійкості зриву.