STABILIZATION OF BEAM INSTABILITY IN WAVE-WAVE INTERACTION

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The new mechanism of stabilization of beam instability is proposed. The basis of this mechanism is the process of dynamical chaos at weak nonlinear interaction of waves. It is shown that mechanisms of stochastic decay are responsible for the stabilization. In this process the part of the energy is removed by eigen waves of plasma which take part in decay process. Besides the fields with random amplitudes and phases effectively transfer their energy to plasma particles. It is shown, that efficiency of excitation (reinforcing) of waves with random characteristics is sufficiently lower than efficiency of excitation of regular waves.

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1. INTRODUCTION

In the majority of actual conditions of the experiment the stabilization of beam instabilities occurs as the result of entrapment of beam particles by the field of exited wave. In some rarer cases, the amplitudes of exited waves can be so high, that the requirements of development of local instability of particle motion of the beam are fulfilled. These requirements mostly appear in the systems with high density of the beam. Thus the chaotic character of particle motion of the beam also leads to restriction of the level of oscillations exited by the beam; this level can be defined by the conditions of the development of stochastic instability. In the literature (see [1]) one more mechanism of stabilization of instabilities (including beam instabilities) is considered, where the exited wave decays into other waves. And, at least, one of these waves or rapidly damps, or rapidly removes from the area of interaction. Thus the channel of dissipation appears which can cause the stabilization of instability.

The other opportunity of appearance of the dissipation channel, as result of stabilization of instability is considered in the present report. The essence of mechanism of stabilization can be explained by the following way. Let the conditions of local instability of a wave-wave process are fulfilled [2] in the investigated system. At the same time dynamics of fields becomes chaotic. Efficiency of interaction of a charged beam with fluctuating field is much lower than with fields of regular waves. In addition fluctuating fields rapidly convey their energy to heating of plasma particles. Thus a new channel of rapid dissipation of energy of exited waves appears. As a result stabilization of instability or even failure of process of excitation of waves takes place. The results of some experiments at which, apparently, the described mechanism of failure of plasma-beam instability is realized, are described in this report.

2. STOCHASTIC INSTABILITY OF DYNAMICS OF WEAK NONLINEAR INTERACTION OF WAVES

At rather high amplitudes of the waves exited in plasma it is possible that effective nonlinear interactions of these waves with other eigen waves of plasma electrodynamical structure take place. The dynamics of this interaction can be both regular and chaotic. We are interested in chaotic regimes. Such regimes appear in different schemes of nonlinear wave interaction. The most prime are modified decay and also the case of three-wave interaction, when during interaction the fourth wave can participate. By its performances this wave is close, for example, to a low-frequency wave participating in the interaction. The last case we shall term the quasi-four-wave. The stochastic instability develops only when amplitude of a decaying wave (pump wave) exceeds some threshold value. Let us consider these two cases in details.

2.1 QUASI-FOUR-WAVE INTERACTION

Let the wave with amplitude \( a_1 \) wave number \( k_1 \) and frequency \( \omega_1 \) decay into two waves \( a_2, k_2, \omega_2 \) and \( a_3, k_3, \omega_3 \). Besides that let us assume, that there is one more wave with the following parameters \( a_4, k_4, \omega_4 \); \( k_4 = k_3, \quad \omega_2 - \omega_4 << \omega_1 \). Let us consider that the fourth wave does not influence the process of decay. Then the modification of amplitudes of three interacting waves in time can be described according to [2,3]:

\[
\dot{a}_1 = iV_1 a_2 a_3, \quad \dot{a}_2 = iV_1 a_2 a_3, \quad \dot{a}_3 = iV_1 a_3 a_2^*,
\]

where \( V_1 = |V_1| \exp(i\Phi_0) \) is the matrix element of interaction, \( a_j = |a_j| \exp(i\Phi_j) \). On the linear stage \((|a_1| = \text{const}, \Phi_1 = \text{const})\) of decay the amplitudes \(|a_1| \) and \(|a_2| \) growth exponentially with increment \( G = |a_1||V_1| \). The phase change \( \Phi = 2(\Phi_1 - \Phi_2 - \Phi_3 + \Phi_0) \) obeys equation of mathematical pendulum:

\[
\ddot{\Phi} + (2|a_1||V_1|)^2 \sin \Phi = 0.
\]

It is seen from Eq.(2) that the half width of nonlinear resonance equals \( 4G \). If we replace the third wave by forth wave we obtain the following set of equations:

\[
\begin{align*}
\dot{a}_1 &= iV_2 a_3 a_4 \exp(-i\delta \tau), \\
\dot{a}_2 &= iV_2 a_4 a_3 \exp(i\delta \tau), \\
\dot{a}_3 &= iV_2 a_3 a_4^* \exp(i\delta \tau), \\
\dot{a}_4 &= iV_2 a_1 a_2^* \exp(i\delta \tau)
\end{align*}
\]

where \( \delta = \omega_1 - \omega_2 - \omega_4 \).

On the linear stage phase \( \Psi = 2(\Phi_1 - \Phi_2 - \Phi_3 + \Phi_0 + \delta \tau) \) satisfies Eq.(2), where it...
is necessary to change \( V_1 \) on \( V_2 \). Half-width of this nonlinear resonance is equal to \( 4G_2 \), where \( G_2 = |a_1 ||V_2| \). This means that the distance between nonlinear resonances is equal to \( 2\delta \). Assuming the width of nonlinear resonance for the forth wave is small \((G >> G_2)\) we obtain the condition of the nonlinear resonance overlapping and, correspondingly, the criterion of stochastic instability:

\[
K = 2G / \delta > 1. \tag{4}
\]

Fig.1-2 show the dynamics of amplitudes of interacting waves, which depends on time for regular case \( K < 1 \) (Fig.1) and for chaotic case \( K > 1 \) (Fig. 2). It is possible to show that the spectra of interacting waves are wide and correlation functions fall down quickly. Fig.1 and Fig.2 describe the dynamics of decay of transverse electromagnetic wave \( \varepsilon_t \) into transverse \( \varepsilon_s \) and Langmuir wave \( \rho \). In this case the function of the fourth wave plays return Langmuir wave. It is necessary to notice that the similar dynamics is typical for decay with participation of other waves.

\[
\frac{d\varepsilon}{d\xi} = \mu \left[ \cos \Phi + \cos \left( \Phi + \frac{2\xi}{V_{ph}} \right) \right], \quad \frac{d\Phi}{d\xi} = \frac{1}{\sqrt{\varepsilon}} - \frac{1}{V_{ph}}, \tag{5}
\]

where \( \varepsilon = V^2 / V_0^2 \) — dimensionless energy of particle, \( V, V_0 \) — current and initial velocity of particle, \( V_{ph} = v_{ph}/V_0 \), \( v_{ph} \) — phase velocity of wave, \( \mu = (2E)/(moV_0) \ll 1 \) — velocity of wave, \( \Phi = cot - k_2 \) — phase of wave, \( L \) —length of a system, \( \xi = z\omega / V_0 \) — normalized coordinate of a particle, \( \Delta \Phi \) — random phase of wave. If \( \Delta \Phi = 0 \), then the system of equations (5) coincides with system which were investigated in [5]. System (5) can be solved by step-by-step method:

\[
\varepsilon = 1 + \mu \varepsilon^{(1)} + \mu^2 \varepsilon^{(2)} + \mu^3 \varepsilon^{(3)} + ... \]

We will assume that distribution \( \Delta \Phi \) is uniform with central tendency \( \langle \Delta \Phi(\xi) \rangle = 0 \), and its maximal value is equal \( \Delta \Phi_m \). Then from (5) for central tendency \( \langle \varepsilon^{(1)} \rangle \) we can obtain:

\[
\langle \varepsilon^{(1)} \rangle = \frac{\varepsilon^{(1)}}{\delta} \sin \Delta \Phi_m / \Delta \Phi_m. \tag{6}
\]

In the case when \( \delta \) - correlated fluctuations, i.e. at realization of a requirement \( \langle \Delta \Phi(\xi) \Delta \Phi(\xi') \rangle = N\delta(\xi - \xi') \) for average value of the correction of the second degree to dimensionless energy we shall receive:

\[
\langle \varepsilon^{(2)} \rangle = \frac{\varepsilon^{(2)}}{\delta} \sin^2 \Delta \Phi_m / \Delta \Phi_m^2. \tag{7}
\]

From expressions (6) and (7) follows that the efficiency of interaction of the electron beam with fluctuating electromagnetic fields reduces significantly.

4. HEATING OF PLASMA PARTICLES BY A FIELD OF NOISE WAVES

Let us estimate the efficiency of energy transmission from random field to the particles. For this purpose we shall choose the most prime model. Let us consider that the charged particles move in random field where there are no correlations, i.e. \( \langle E(t_1) \cdot E(t_2) \rangle = A^2 \cdot \delta(t_1 - t_2) \).

From the common equation of charged particle motion in such electromagnetic fields it is easy to find the following expression for average square of energy changing:

\[
\langle (\Delta y)^2 \rangle = \langle (y(t) - y(0)) \rangle = v^2 \cdot A^2 \cdot \tau. \tag{8}
\]

As it is seen from (8) the energy of plasma electrons can vary from several \( eV \) (electronvolt) up to \( K eV \) (kilelectronvolt) in a time about hundreds of periods of high-frequency field.

5. EXPERIMENTAL RESULTS

Below we shall shortly describe the results of the experiment, in which above described mechanism of stabilization of level of waves exited by beam was probably observed.

The electron beam with current 1-10 A and energy 10-40 keV was injected in the interacting region. Density of plasma varied from \( 5 \cdot 10^{11} \) up to \( 1 \cdot 10^{13} \) e/cm\(^3\). All
The qualitative situation is simple enough. Let us give its brief description. At the beginning the beam excites one of eigen surface waves. When its level is high enough, eigen wave disintegrates into transverse electromagnetic wave and on low-hybrid one. As the transverse wave is not eigen, it easily leaves plasma. At this moment the effective radiation from the plasma cylinder is observed. This part of the process is in good qualitative agreement with the experiment. At the moment of radiation of transverse waves from plasma two channels of sink of energy appear. The first channel is outlet of energy together with improper electromagnetic waves. The second one is effective heating of particles of a beam and plasma by fields of fluctuating waves. These two channels cause depression of level of fields in plasma and as a result failure of three-wave decay. The radiation from plasma stops. The process of plasma-beam instability can be resumed. This situation coincides well with a situation, which is observed in experiment.

REFERENCES