LANGMUIR WAVES EXCITATION BY ELECTRON BEAM WITH THE LIMITED CROSS-SECTION IN THE NEAR-EARTH ELECTRON FORE-SHOCK

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Model of the stripped monoenergetic electron beam with the sharp boundaries based on the results of CLUSTER measurements is proposed for the near-Earth foreshock vicinity. Dispersion equation is obtained and analyzed using numerical methods. Dependency of the equation roots corresponding to kinetic mechanism of beam-plasma instability on the model parameters is studied.
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1. INTRODUCTION

Multisatellite measurements that were performed in the international project CLUSTER fixed the electric field oscillations on the border between Earth magnetosphere and foreshock region [1]. Frequencies of these oscillations correspond to the Langmuir and electron-acoustic waves, respectively. Direct measurements of the electron velocity distribution function indicated the presence of electron beam that had been reflected from the shock front. This beam hypothetically causes excitation of the above-mentioned waves. Measurement results show that specified beam is not solid and can be considered as a system of separated radially restricted beams. Theoretical investigation of the waves' excitation by such a beam is a purpose of this work.

2. MODEL DESCRIPTION AND DISPERSION EQUATION

The simplest geometrical model is proposed where the stripped monoenergetic electron beam with the sharp boundaries pierces the warm plasma without magnetic field (Fig.1).

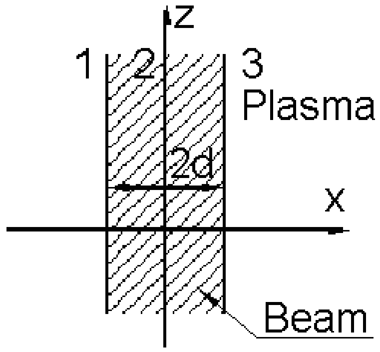

Fig.1. Model of the stripped beam piercing plasma

Considering all the physical magnitudes having the harmonic temporal dependence, *f(t)~exp(-iωt)*, dielectric permettivity for warm isotropic plasma without beam can be written as:

$$\varepsilon^l\left(\omega,\vec{k}\right)=1-\frac{\omega_{pe}^2}{\omega^2}-3k^2\lambda_D^2+\frac{i}{k^3\lambda_D^3}\sqrt{\frac{\pi}{2}}\exp\left(-\frac{1}{2k^2\lambda_D^2}\right). \quad (1)$$

Here ω_{pe} is an electron plasma frequency, and λ_D is a Debay radius. But taking into consideration the beam introduces the summand to expression (1):

$$\delta\varepsilon\approx\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dfrac{\omega_B^2}{\left(\omega-k_z v_0\right)^2}-\dfrac{\omega_B^2k_x^2}{\left(\omega-k_z v_0\right)^2k_z^2}\end{bmatrix}. \quad (2)$$

Here v_0 is a beam velocity, k_x and k_z are the transversal and longitudinal components of the wave vector, and ω_B is an electron plasma frequency of the beam.

Following dispersion equations can be obtained by solving Poisson expression (*div(ε∇φ)=0*) for infinite plasma and infinite beam:

$$k^2=k_{x2}^2+k_z^2=\frac{1}{3\lambda_D^2}\times\left[1-\frac{\omega_{pe}^2}{\omega^2}+\frac{i3\sqrt{\frac{3\pi}{2}}\omega^3}{\left(\omega^2-\omega_{pe}^2\right)^{3/2}}\exp\left(-\frac{3\omega^2}{2\left(\omega^2-\omega_{pe}^2\right)}\right)\right], \quad (3)$$

$$k^2=k_{x4}^2+k_z^2=\frac{1}{3\lambda_D^2}\times\left[1-\frac{\omega_{pe}^2}{\omega^2}-\frac{\omega_B^2}{\left(\omega-k_zv_0\right)^2}+\frac{i3\sqrt{\frac{3\pi}{2}}\omega^3}{\left(\omega^2-\omega_{pe}^2\right)^{3/2}}\exp\left(-\frac{3\omega^2}{2\left(\omega^2-\omega_{pe}^2\right)}\right)\right]. \quad (4)$$

Here k_{x2} is the Langmuir waves' transversal wavenumber for plasma without beam, k_{x4} is the transversal wavenumber for infinite beam in plasma, and $2d$ is a beam width.

In order to find dispersion equation for the beam in plasma boundary conditions for the potential on the beam-plasma border should be written. Potentials are specified as:

$$\varphi_1 = \left(\varphi_{m1} \exp\left(k_z x \right) + \varphi_{m2} \exp\left(-ik_{x2} x \right) \right) \exp\left(ik_z z \right);$$

$$\begin{aligned} \varphi_3 = (\varphi_{m7} \exp\left(-k_z x \right) + \\ +\varphi_{m8} \exp\left(ik_{x2} x \right)) \exp\left(ik_z z \right) \end{aligned}; \qquad (5)$$

$$\begin{aligned} \varphi_2 = (\varphi_{m3} \exp\left(k_z x \right) + \varphi_{m4} \exp\left(-k_z x \right) + \\ +\varphi_{m5} \exp\left(ik_{x4} x \right) + \varphi_{m6} \exp\left(-ik_{x4} x \right)) \exp\left(ik_z z \right). \end{aligned}$$

Here φ_1, φ_2 and φ_3 are the potentials in the areas 1, 2 and 3, respectively (see Fig.1). Boundary conditions that provide continuity of the potentials, their derivatives and Laplacians on the beam-plasma border have a form:

$$\frac{\partial}{\partial x}\left(\varphi_1 \right)\Big|_{x=-d} = 0, \; \frac{\partial}{\partial x}\left(\varphi_3 \right)\Big|_{x=d} = 0,$$

$$\frac{\partial}{\partial x}\left(\varphi_2 \right)\Big|_{x=\pm d} = 0, \qquad (6)$$

$$\varphi_1\big|_{x=-d} = \varphi_2\big|_{x=-d}, \; \varphi_2\big|_{x=d} = \varphi_3\big|_{x=d},$$

$$\Delta\varphi_1\big|_{x=-d} = \Delta\varphi_2\big|_{x=-d}, \; \Delta\varphi_2\big|_{x=d} = \Delta\varphi_3\big|_{x=d}.$$

As a result homogeneous equations' set is obtained. It has a non-trivial solution only in the case of its determinant is equal to zero. Desired dispersion equation found from this condition has a form (see, e.g., [2]):

$$\begin{aligned} F\left(\omega, \vec{k} \right) = [k_{x4}\left(k_z^2 + k_{x2}^2 \right) sh\left(k_z d \right) \cos\left(k_{x4} d \right) + \\ +\left(k_z + ik_{x2} \right)\left(k_z^2 + k_{x4}^2 \right) \sin\left(k_{x4} d \right) ch\left(k_z d \right)] \times \\ \times \left\{ \left[ik_{x2}\left(k_z^2 + k_{x4}^2 \right) + k_z\left(k_{x4}^2 - k_{x2}^2 \right) \right] sh\left(k_z d \right) \times \right. \\ \left. \times \cos\left(k_{x4} d \right) + k_{x4}\left(k_z^2 + k_{x2}^2 \right) \sin\left(k_{x4} d \right) ch(k_z d) \right\} = 0. \end{aligned} \qquad (7)$$

First co-factor in (7) corresponds to the antisymmetric modes and the second one – to the symmetric modes. Each co-factor was also obtained for symmetric and antisymmetric modes separately.

3. NUMERICAL SOLUTION OF THE DISPERSION EQUATION

Equation (7) was solved numerically. Dispersion function F was studied as a function of real and imaginary parts of frequency. Well defined maximums of the value ($-logF$) on the plane of complex frequency correspond to the roots of the dispersion equation. They become more acute and unreservedly grow if they are built more accurately. Identification of the waves' types was performed by investigation of extreme cases of the model.

The roots obtained correspond to Langmuir waves (Fig.2a) and beam-plasma modes (Fig.2b). Langmuir waves have symmetrical and antisymmetrical branches. Beam-plasma modes can be of stable and unstable type. The dispersion curve and increment (decrement) dependence on the wave number are plotted on Fig.3a-b, respectively.

In the presence of the beam the root corresponding to Langmuir wave in plasma is accompanied by a family of roots with stronger damping. The discreteness of the roots can be explained by the transversal restriction of the beam. In the direction normal to the beam motion standing waves occur.

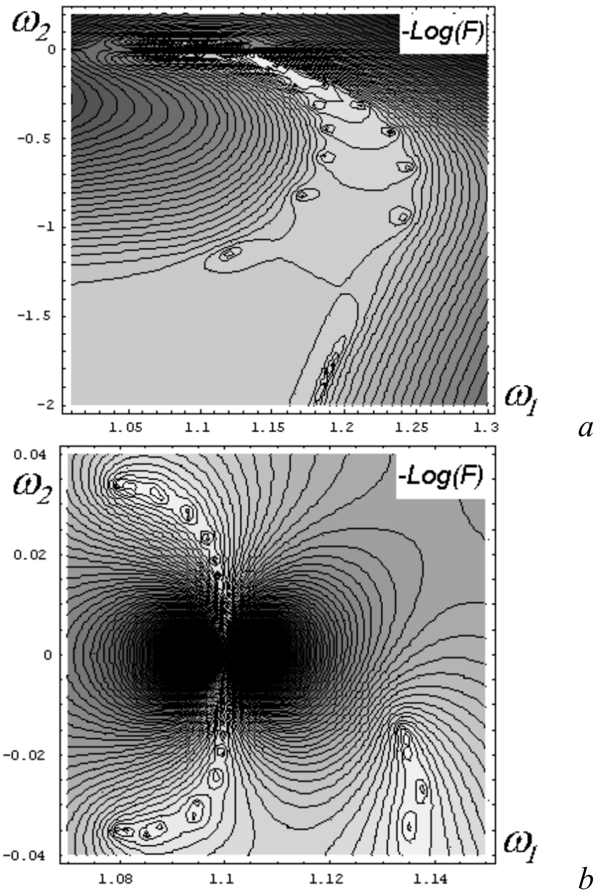

Fig.2. Family of roots corresponding to Langmuir waves (a) and beam-plasma modes (b): $\omega = \omega_1 + i\omega_2$, $\omega_b/\omega_p = 0.01$, $v_b/v_{Te} = 4.58$, $d/\lambda_D = 50$, $k_z\lambda_D = 0.24$

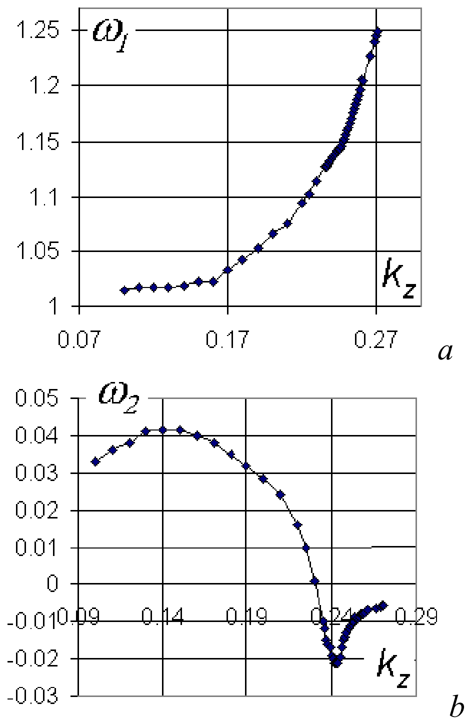

Fig.3. Dispersion dependency of the real (a) and imaginary (b) parts of the frequency for one of the roots near

the point of Cherenkov resonance ($\omega = \omega_1+i\omega_2$,, ω_b/ω_p=0.006, v_b/v_{Te}=4.58, d/λ_D=30)

Investigation shows that maximal increment does not depend on the beam width, but only on its density and velocity. It results from the fact that maximal increment corresponds to the purely longitudinal waves propagating along the beam motion.

Dependence of the real part of the squared transversal wavenumber in plasma is plotted at Fig.4.

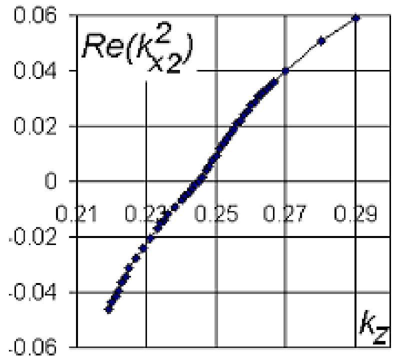

Fig.4. Dependence of the squared real part of the transversal wave number: $\omega = \omega_1+i\omega_2$, ω_b/ω_p=0.01, v_b/v_{Te}=4.58, d/λ_D=50, $k_z\lambda_D$=0.24

One can see that curve is lying in the area of the negative and positive values and passes zero value in the synchronism point. Magnitude $Re(k_{x2})^2$ defines en existence of the waves' excitation from the beam. So, in the positive values' area, on the right of the synchronism point, the Langmuir waves' emission from the beam takes place. On the left of the synchronism point there is an exponential reducing of the electric field out of the beam.

4. CONCLUSIONS

Dispersion equation for the stripped beam with sharp boundaries moving in the warm isotropic plasma was obtained. This equation analysis indicates the presence of two types of waves: Langmuir waves and beam-plasma modes. Langmuir waves have symmetrical and antisymmetrical families of the roots and beam-plasma modes are of dumping and growing types.

Large number of roots is a result of the presence of the preferential direction in the system and transversal limitation of the beam. Due to that in the system standing wave occurs.

Maximum increment corresponds to the purely longitudinal waves that is why it does not depend on the transversal dimension of the beam but on its density and velocity.

The real part of the squared transversal wave number can be of negative and positive value. In positive values area separate beams in periodical sequence of the beams can interact via the Langmuir waves excitation even if the distance between the beams is relatively large. Contrary, in the negative values area interaction between the beams is minimal and periodical sequence can be considered as a set of the independent beams if the distance between beams is more then specific length of the electric field reducing $|k_{x2}|^{-1}$.

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ИЗЛУЧЕНИЕ ЛЕНГМЮРОВСКИХ ВОЛН ЭЛЕКТРОННЫМ ПУЧКОМ ОГРАНИЧЕННОГО СЕЧЕНИЯ В ОБЛАСТИ ЕЛЕКТРОННОГО ФОРШОКА

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Предложена модель моноэнергетического ленточного пучка с резкими границами, которая базируется на результатах измерений эксперимента КЛАСТЕР в области форшока ударной волны Земли. Было получено дисперсионное уравнение и проанализировано числовыми методами. Изучены зависимости положения корней дисперсионного уравнения, соответствующих кинетическому механизму плазменно-пучковой неустойчивости, от параметров модели.

ВИПРОМІНЮВАННЯ ЛЕНГМЮРІВСЬКИХ ХВИЛЬ ЕЛЕКТРОННИМ ПУЧКОМ ОБМЕЖЕНОГО ПЕРЕРІЗУ В ОБЛАСТІ ЕЛЕКТРОННОГО ФОРШОКУ

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Запропоновано модель моноенергетичного стрічкоподібного пучка з різкими границями, яка базується на результатах вимірювань експерименту КЛАСТЕР в області форшоку ударної хвилі Землі. Було отримано дисперсійне співвідношення, яке проаналізовано числовими методами. Вивчені залежності положення коренів, що відповідають кінетичному механізму плазмово-пучкової нестійкості, від параметрів моделі.