EXCITATION OF HIGH NUMBERS HARMONICS BY FLOWS OF OSCILLATORS IN A PERIODIC POTENTIAL

V.A. Buts, V.I. Marekha, A.P. Tolstoluzhsky
NSC Kharkov Institute of Physics and Technology, Kharkov, Ukraine,
e-mail: vbuts@kipt.kharkov.ua

It is shown that the maximum of radiation spectrum of nonrelativistic oscillators, which move into a periodically inhomogeneous potential, can be in the region of high numbers harmonics. Spectrum of such oscillators radiation becomes similar to the radiation spectrum of relativistic oscillators. The equations, describing the non-linear self-consistent theory of excitations, of high numbers harmonics by ensemble of oscillators are formulated and its numerical analysis is conducted. The numerical analysis has confirmed the capability of radiation of high numbers of harmonics. Such peculiarity of radiation allows to expect of creation of nonrelativistic FEL.

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1. INTRODUCTION

Lately large attention is devoted to investigation of the mechanisms of short-wave radiation generating. One of the possible mechanisms of generating of short-wave radiation is the double Doppler effect realizable in lasers on free electrons. In addition the considerable frequency multiplication is reached while using relativistic electron beams $\nu \sim \gamma^2 \nu_0$. It is known also, that in vacuum the oscillator effectively radiate high numbers of harmonics only in the case when it has a large energy. So, for synchrotron emission the maximum of radiation falls on harmonics with number $\nu \sim \gamma^3$ [1]. It is possible one more mechanism of generating of short-wave radiation which not require to use high energy beams. This is the radiation of high numbers harmonics by nonrelativistic oscillators – by charged particles moving in external periodic in time electrical field, and in field of external periodic in space, potential.

The investigation of excitation of harmonics by ensembles of charged oscillators is carried out. These investigations were carried out by analytical and numerical methods. The analytical results are in the good agreement with numerical results.

2. RADIATION IN A PERIODIC POTENTIAL

Briefly we'll describe the mechanism of generating of high numbers harmonics by nonrelativistic oscillators. Let charged particle moves in the external periodic in time electrical field $E(t) = E\cdot\cos(\omega_0 \cdot t)$ and in the field of periodic potential $U(z) = U_0 + g\cdot\cos(\kappa\cdot z)$, $\kappa = 2\pi/d$. For simplicity we'll suppose, that the motion occurs only along Z-axis. In general the equations for electron motion in such field has a kind

$$\frac{dP}{dt} = e\vec{E} - e\vec{E}_U,$$

$$\frac{d\vec{r}}{dt} = \vec{V}, \quad \vec{V} = \vec{P}/\sqrt{1+P^2}, \qquad (1)$$

where $E_U = -\nabla U$.

Let's suppose that the intensity of these fields is small enough, so that it is possible to consider the particle motion in these fields as nonrelativistic. Besides we'll consider that $E >> g\cdot\kappa$.

As far as we first of all are interested by motion of particle along an axis z, rewriting the system (1) along an axis z, rewriting the system (1) in dimensionless variables we obtain:

$$\frac{dV_z}{d\tau} = \varepsilon\cos(\Omega\tau) + w\cdot\sin(\frac{\kappa}{k}\cdot z), \qquad \frac{dz}{d\tau} = V_z, \qquad (2)$$

where: $\varepsilon = eE/mc\omega$, $w = eg\kappa/mc\omega$, $\omega = kc$, $z = kz$, $V_z = V_z/(\omega/k)$, $\Omega = \omega_0/kc$, $\tau = kct$.

We'll solve the system (2) by a perturbation method. So we have the solution in zero approximation

$$\frac{dV_z}{d\tau} = \varepsilon\cos\Omega\tau \qquad (3)$$

It will be :

$$V_z = (\varepsilon/\Omega)\sin\Omega\tau, \qquad (4)$$

$$z = -(\varepsilon/\Omega^2)\cos\Omega\tau. \qquad (5)$$

Using expansion formula

$$\sin(x\cos\varphi) = 2J_1(x)\cos\varphi - 2J_3(x)\cos 3\varphi + 2J_5(x)\cos 5\varphi - \ldots \qquad (6)$$

in first approximation we can receive solution of the system (2)

$$V_z = \frac{\varepsilon}{\Omega}\sin(\Omega\tau) - 2w\sum_{n=0}^{\infty}(-1)^n \frac{J_{2n+1}(\mu)}{(2n+1)\Omega}\cdot\sin[(2n+1)\Omega\tau], \qquad (7)$$

$$z = -\frac{\varepsilon}{\Omega^2}\cos(\Omega\tau) - 2w\sum_{n=0}^{\infty}(-1)^n \frac{J_{2n+1}(\mu)}{[(2n+1)\Omega]^2}\cdot\cos[(2n+1)\Omega\tau], \qquad (8)$$

where $\mu = \frac{\kappa\varepsilon}{k\Omega^2}$, J_{2j+1} – Bessel functions of $(2j+1)$ – order.

For a harmonic oscillator of type

$$z = a\sin(\omega t) \qquad (9)$$

in [1] the formula for emission power for the first harmonic was find (dipole radiation, n = 1, radiation frequency ω)

$$\frac{\partial W}{\partial t} = \frac{e^2\omega^2}{4c}\int_0^{\pi}\sin(\theta)d\theta\,\sin^2(\theta)\beta^2\left(1-\frac{1}{4}\beta^2\cos^2(\theta)\right) = \frac{e^2\omega^2\beta^2}{3c}\left(1-\frac{1}{4}\beta^2\right) \approx \frac{e^2\omega^4 a^2}{3c^3}, \qquad (10)$$

$$\beta = a\omega/c.$$

In our case or excitation harmonics amplitude

$$\beta = 2wJ_{2n+1}(\mu)/[(2n+1)\Omega]^2$$

and so the formula (10) will be

$$\frac{\partial W}{\partial t} \approx \frac{4w^2e^2\omega^2}{3c}\left(\frac{J_{2n+1}(\mu)}{[(2n+1)\Omega]^2}\right)^2 . \qquad (11)$$

Thus, we can see, that conditions of radiation maximum $\mu = (2n+1) = 1/\Omega$ in this case completely coincides with condition of oscillator radiation in periodically inhomogeneous dielectric [2], i.e. in both cases the radiation maximum corresponds to the same frequency. When $eU_0/mc^2 > q\varepsilon^2$ the role of a periodic potential for radiation will be more significant.

3. QUANTUM CONSIDERATION

The considerable information about features of charged particles' radiation in a periodic potential one can obtain using the methods of quantum electrodynamics. Using the perturbation technique, it is easy to obtain following expression for a radiating power of charged particle

$$P = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int d(\hbar\omega)\cdot W_{f,i}\rho(\hbar\omega)\sin\theta . \qquad (12)$$

Here $W_{f,i} = \frac{2\pi}{\hbar}\,|H_{f,i}|^2\,\delta(\hbar\omega - \Delta E)$, $\rho = \frac{L^3\omega^2 n^3}{(2\pi c)^3\hbar}$,

$$H_{f,i} = \frac{dh}{m}\sqrt{\frac{2\pi h}{L^3 w_\lambda}}\cdot\int\sum|\Psi_{N+1}|^2\cdot\sqrt{N+1}\times$$

$$\times\left|(\vec{e}_\lambda \vec{k})(\Psi_f^*\Psi_i) + \vec{e}_\lambda(\Psi_f^*\hat{\vec{p}}\Psi_i)\right|\cdot exp\left(-i\Delta\vec{k}\cdot\vec{r}\right)d\vec{r} ,$$

$n^2 = \varepsilon$ - inductive capacity of medium, n – its refraction index $\Delta k = k_i - k_f - k_\lambda$.

If the particle moves in potential with a weak periodic inhomogeneity, its wave function can be represented as:

$$\Psi_i = \sum_m \Psi_{i,m}\exp\left[i\left(\vec{k}_i + m\cdot\vec{\kappa}\right)\cdot\vec{r}\right], \qquad (13)$$

where $\Psi_{i,m} = g^m \Psi_{i,0}$.

From (13) it is visible, that the wave function has addends, which can be identified with particles, which velocity exceeds the velocity of real particle. Such addends it is possible to identify with fast virtual particles. In themselves they do not exist. Here one can see the analogy to virtual waves in periodically inhomogeneous mediums. Only in the last case we were interested in the slow virtual waves. For a case of particles we'll be interested in the fast virtual particles. Substituting the wave function (13) into the formula (12), it is possible to obtain following expression for radiating power of charged particle, which one moves in periodic potential:

$$P = g^2\cdot\frac{q^2V}{c^2}(N+1)\int\omega\cdot d\omega\,(1-\frac{c^2}{v^2\varepsilon}), \qquad (14)$$

where $v = 2\cdot\omega/(\kappa\cdot e_i)$ at $v_i >> v_f$; $v = \omega/(\kappa\cdot e_i)$ at $v_i \sim v_f$; e_i - unit vector, directed along v_i.

If there are the oscillator moving in periodic potential, we obtain the formula, which coincides with formula (11).

4. RADIATION OF OSCILLATORS FLOW

While investigating the elementary mechanism of charged oscillator radiation, which moves in periodically inhomogeneous potential, the possibility of excitation of high numbers harmonics by nonrelativistic oscillators was shown. For the effectiveness of such radiation, it is necessary the fulfillment of following condition: $d \approx \beta\lambda$, $r_o \approx nd/2\pi$, here λ – wavelength of radiation, d – period of inhomogeneity, r_0 – amplitude of oscillator displacement from its equilibrium position, $\beta = v/c << 1$, v – oscillator velocity, n – number of radiated harmonic.

We'll consider the excitation of electromagnetic waves by ensemble of oscillators, which are in the field of periodically inhomogeneous potential $U(z) = U_0 + g\cdot\cos(\kappa\cdot z)$.

The fullest description of self-consistent process of interaction of charged particles with an exciting field implies the simultaneous solution of Maxwell equations for the electromagnetic field and equations of charged particles' motion in exited fields.

$$\frac{\partial B}{\partial t} = -c\,\mathrm{rot}\,\vec{E},\ \frac{\partial E}{\partial t} = c\,\mathrm{rot}\,\vec{H} - 4\pi\vec{j}$$

$$\frac{dp}{dt} = e\vec{E} + \frac{e}{c}\vec{v}\times\vec{B} + F_0\sin\omega_0 t - e\nabla U,\ \frac{dr}{dt} = \vec{v}, \qquad (15)$$

where ω_0 frequency of oscillation of the oscillator, F_0 - amplitude of the external force, which acts on oscillator (creates an oscillator). The oscillations of the oscillator will be considered to occur along axis Z.

During investigation of the elementary mechanism of radiation of the oscillator it was find out, that the directional diagram corresponds to dipole radiation, i.e. the radiation is directed in a transverse direction with respect to direction of the oscillator oscillation. Therefore we shall search for such solution for exited wave

$$E = Re\,A(t)exp(ikx) . \qquad (16)$$

We'll study time evolution of electromagnetic field, in which the only following components E_x, E_z, H_y are different from zero. Let's substitute expressions for fields (16) in the set of equations (15). Averaging the obtained equations on a space phase of perturbation, we'll obtain the following set of equations for finding fields and characteristics of oscillators:

$$\frac{dp_x}{d\tau} = \mathrm{Re}\,\varepsilon_x\exp(ix) - v_z\,\mathrm{Re}\,h_y\exp(ix),$$

$$\frac{dp_z}{d\tau} = \mathrm{Re}\,\varepsilon_z\exp(ikx) + v_x\,\mathrm{Re}\,h_y\exp(ix) +$$

$$+ f_0\cos\Omega\tau + w\sin(\kappa z)$$

$$\frac{dx}{d\tau} = v_x,\ \frac{dz}{d\tau} = v_z,\ \vec{v} = \vec{p}/\sqrt{1+p_x^2+p_z^2}, \qquad (17)$$

$$\frac{dh_y}{d\tau} = i\varepsilon_z,$$

$$\frac{d}{d\tau}\varepsilon_x = -\frac{2w_b^2}{2\pi}\int_0^{2\pi} v_x \exp(-ix)dx_o \,,$$

$$\frac{d}{d\tau}\varepsilon_z = -ih_y - \frac{2w_b^2}{2\pi}\int_0^{2\pi} v_z \exp(-ix)dx_o \,.$$

The integration in the right part of these equations for fields is over on initial values of oscillators coordinates. The set of equations (17) is written to in dimensionless variables:

$$kct \to \tau,\ k\vec{r} \to \vec{r},\ \vec{p}\big/mc = \vec{p},\ \vec{\kappa}\big/k = \vec{\kappa},\ \vec{\varepsilon} = e\vec{E}\big/mckc\ ,$$

$$\vec{h} = e\vec{H}\big/mckc,\ f_0 = F_0\big/mckc,\ \omega_b^2 = \frac{4\pi e^2 n_b}{4\pi e^2 n_b},\ \vec{v} = \vec{V}\big/\omega / k\ ,$$

where m, e - mass and charge of electrons, n_b - density of oscillators.

5. RESULTS OF THE NUMERICAL ANALYSIS

The numerical analysis of self-consistent set of equations (17) has confirmed the presence of instability in the considered system. The values of dimensionless parameters (frequency are standardized on kc, and wave vector on k) were following: ω_b =0.3, w=0.02, f_0 =0.02, n =5. In these conditions the excitation of 11-th harmonics by oscillators in periodic potential with $\kappa = 5k$ was observed. The results of simulation are represented on figures 1-4. In Figs. 1, 2 the dependencies of amplitude on time and spectrum of the field ε_z are shown. In Figs. 3, 4 one can see amplitude and spectrum of the field ε_x.

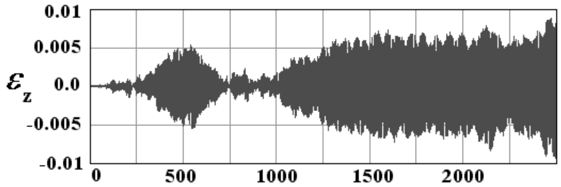

Fig.1. Dependence of amplitude ε_z on time

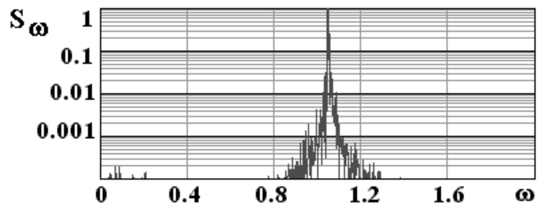

Fig.2. Power spectrum ε_z

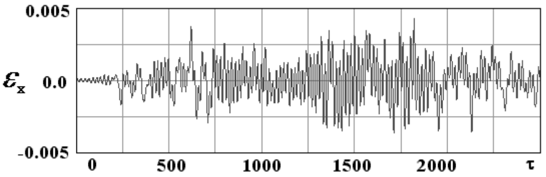

Fig.3. Dependence of amplitude ε_x on time

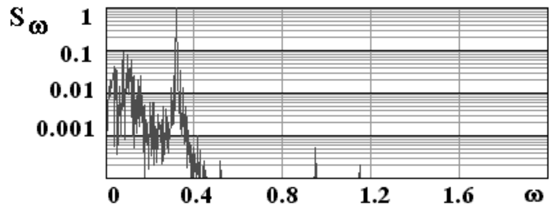

Fig.4. Power spectrum ε_x

The dark filling inside the envelope amplitude is high frequency oscillations. Spectrum of the field ε_z has a maximum on frequency, which equal 1, and that is according to 11 harmonic of oscillation frequency of oscillators. Spectrum of the field ε_x has a maximum on frequency, which equal ω_b. It is connected with, that field ε_x at obtaining equations (18) the resonance wasn't separated, and also with that the field ε_x represents quasi-longitudinal oscillations of an ensemble of oscillators.

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ВОЗБУЖДЕНИЕ ВЫСОКИХ НОМЕРОВ ГАРМОНИК ПОТОКАМИ ОСЦИЛЛЯТОРОВ В ПЕРИОДИЧЕСКОМ ПОТЕНЦИАЛЕ

В.А. Буц, В.И. Мареха, А.П. Толстолужский

Показано, что максимум спектра излучения нерелятивистских осцилляторов, которые движутся в периодически неоднородном потенциале, может находиться в области высоких номеров гармоник. Спектр излучения таких осцилляторов становится похожим на спектр излучения релятивистских осцилляторов. Сформулированы уравнения, описывающие самосогласованную нелинейную теорию возбуждения высоких номеров гармоник ансамблем осцилляторов, и проведен их численный анализ. Численный анализ подтвердил возможность излучения высоких номеров гармоник.

ЗБУДЖЕННЯ ВИСОКИХ НОМЕРІВ ГАРМОНІК ПОТОКАМИ ОСЦИЛЯТОРІВ В ПЕРІОДИЧНОМУ ПОТЕНЦІАЛІ

В.О. Буц, В.І. Мареха, О.П. Толстолужський

Показано, що максимум спектра випромінювання нерелятивістських осциляторів, які рухаються у періодично неоднорідному потенціалі, може знаходитись в області високих номерів гармонік. Спектр випромінювання таких осциляторів стає схожим на спектр випромінювання релятивістських осциляторів. Сформульовані рівняння, що описують самоузгоджену нелінійну теорію збудження високих номерів гармонік ансамблем осциляторів, і проведено їх чисельний аналіз. Чисельний аналіз підтвердив можливість випромінювання високих номерів гармонік.