RF FIELD PATTERN IN THE PLASMA CYLINDER OF FINITE LENGTH
FOR VARIOUS ANTENNAS

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The paper deals with numerical calculations of electromagnetic fields, which are launched in the plasma cylinder of finite length surrounded by metal vessel. The RF antenna is located at the end face of the cylinder and placed between two dielectric plates. Such configuration is typical for plasma sources used for technological purposes [1, 2]. The obtained outcomes can be used for improvement of the characteristics of similar systems.

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SETTING OF THE TASK

The scheme of device under consideration is represented in Fig. 1. In calculations the radius of the metal cylinder was \( a = 23 \text{ cm} \), the length was \( L = 5 \cdot 10^{-2} \text{ cm} \), the thickness of dielectric plates were (at the left on the right) \( g = 0.1 - 2.5 \text{ cm} \) and \( h - g = 0.1 - 0.5 \text{ cm} \). Three types of antennas were considered in calculations and are shown in Fig. 2.

![Fig. 1. The outline of plasma source and used in the calculations coordinate system](image)

![Fig. 2. The scheme of surveyed antennas: a – open ring, b – the combination of azimuth and radial currents, c – the spiral of Archimedes](image)

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TECHNIQUE OF SOLUTION

The hydrodynamic approximation was used for plasma dielectric permeability. The density of plasma in a device was considered as homogeneous. Then a plasma dielectric permeability is

\[
\varepsilon_p = \frac{\omega^2 - \frac{\omega_p^2}{\omega(\omega+i\nu)}}{\frac{\omega_p^2}{\omega(\omega+i\nu)}},
\]

where \( \omega = 2\pi f \), \( \omega_p = \sqrt{4\pi e^2 n_e/m_p} \) is plasma frequency of electrons, \( \nu \) is effective collision frequency. The collisions of electrons with neutral atoms are dominant for the range of pressures and electron temperature under consideration. In spite of the fact that \( \nu << \omega \), the collision frequency was taken into account in the vicinity of eigenfrequencies of the device. The kinetic effects were neglected in the consideration because \( \omega_p/\nu T << 1 \).

Here \( T \) is electron thermal velocity and \( 1/k \) is the RF field scale length. The electron mean free pass \( \lambda_e \) is defined by electron – neutral atom collisions. For the case under consideration \( \lambda_e \sim 0.5 \text{ cm} \) and \( \lambda_e << a \). So, plasma density profile is defined by RF field pattern.

Maxwell equations were written for three regions (two regions of dielectrics and one region of plasma) in a cylindrical coordinate system and were adjusted at boundaries. Set of the Maxwell's equations in each region is written as

\[
\begin{align*}
\text{rot} \vec{H}_k &= \frac{1}{c} \frac{\partial D_k}{\partial t} \\
\vec{D}_k &= \varepsilon_k \vec{E}_k \\
\text{rot} \vec{E}_k &= -\frac{1}{c} \frac{\partial \vec{B}_k}{\partial t} \\
\text{div} \vec{D}_k &= 0 \\
\text{div} \vec{B}_k &= 0
\end{align*}
\]

(1)

where \( k \) is index designating an appropriate region. In the set of equations (1) the absence of external charges in all three areas is taken into account. The currents and charges of an antenna are taken into account in boundary conditions.

Two independent modes, TE mode (\( E_z = 0 \)) and TM mode (\( H_z = 0 \)), represent the solutions of the system (1). They are connected through boundary conditions on the antenna. Then, the combination of two methods was used to solve the differential equations (1):
the finite-differences along coordinates $r$ and $z$ and Fourier series along coordinate $\varphi$. Application of Fourier expansion allowed us to avoid introduction of a three-dimensional grid and to reduce a set of Maxwell equations to sequential solution of the differential equations for two independent variables. Fifty harmonics of Fourier series were used in numerical calculations.

Finally, the integral-differential equations for Fourier harmonics were solved by the finite-differences method. We employed the non-uniform grid in $r$ and $z$ directions. It allowed us to get more accurate solutions in the regions of field inhomogeneity with smaller amount of nodes. Also, finite-differences method allows us to solve the problem for the case of arbitrary density distribution inside the device. In this specific case, the grid consisted of 26 points along $r$, and 28 points along $z$.

**BASIC RESULTS**

As the result of the numerical calculations, the values of the electromagnetic field components in grid points were obtained. For example, in the Fig. 3 the RF field pattern in plasma region is shown for density $n=10^8 \text{ cm}^{-3}$. As it is seen in this figure, the electric field iterates the geometry of the antenna. It vanishes while moving away the antenna. Only the forced oscillations are exited in the plasma at this density.

The dependencies of the modulus of the RF electric field in plasma on the device parameters are shown in Fig. 4 for $TM$ and $TE$ modes at plasma density $n=10^8 \text{ cm}^{-3}$. As it is seen in Fig. 4a and Fig. 4b, the change of the device length weakly influences the RF field value in plasma region. The increase of width of the first dielectric plate results in growth of $TE$ mode value (Fig. 4d), and increase of the width of second dielectric plate leads to decrease of $TM$ mode value (Fig. 4c). Therefore, for increase of RF electric field value in plasma region, it is necessary to increase width of the first dielectric plate and to reduce the width of second plate. When the values of RF fields are known, it is easy to find the energy electromagnetic field in plasma region. The $TM$ mode plays the dominant role on initial stage of discharge at plasma densities up to $n=10^9 \text{ cm}^{-3}$, as it is shown in Fig. 5. When the plasma density increases further, the contribution of the $TM$ mode decreases, and the contribution of the $TE$ modes grows. The number of RF energy peaks at densities up to $n=10^9 \text{ cm}^{-3}$ is due to resonant excitation of eigenoscillations of $TM$ mode.

![Fig. 3. Dependence $E_z=\frac{E_z(r, \varphi)}{I}$ at the point $z=0.5\text{cm}$ for the antenna type $a$, $I$ - current in the antenna](image)

![Fig. 4. The dependencies of the normalized modulus of electric field in plasma $4\pi^c_0 E \cdot I$ on parameters of the device](image)

![Fig. 5. Dependence of energy of RF electric field in plasma $c^2 \cdot W \cdot |E|^2/16\pi^2 \cdot I^2$ on plasma density for antenna of type $c$](image)

Also, the antenna design influences the total energy of RF electric field in plasma strongly. As it is shown in Fig. 6, the antenna in the form of the spiral of Archimedes has an advantage in comparison with antennas of other types at small density about $2 \cdot 2$
time, and at increase of density of plasma about 16
time. It happens, because it is longer than others. The
value of a charge is identical to all types of antennas,
however, current value on an inlet of an antenna goes up
with antenna length increase. At low densities, TM mode
plays the dominant role. It is stimulated only by charge
on an antenna. And at major densities TE mode starts to
play a dominant role. This mode is stimulated by current
on an antenna. Thus, the positive effects from an
elongation of an antenna and from increase of an entry
current add and give such major benefit.

\[
\begin{align*}
E^2 & \quad \omega^2 \quad v^2 \\
\int & \quad 8\pi \left( \frac{\omega^2 + v^2} {\omega^2} \right) dV \\
R & = \frac{1} {I^2} \left( \frac{\omega^2}{\omega^2 + v^2} \right) \frac{E^2} {v} \\
L & = 2c^2 W_e / I^2 \\
C & = e^2 / \left( 2W_e \right)
\end{align*}
\]

Fig. 6. Dependence of energy of an electric field in
plasma \( c^2 \cdot W_e, 1 / 16 \pi^2 \cdot I^2 \) on density for different
types of antennas

Having received values of energy of electrical and
magnetic fields in the device it is possible to
calculate such macro characteristic of antennas as
capacity \( C = e^2 / \left( 2W_e \right) \) and inductance
\( L = 2c^2 W_e / I^2 \). As main energy of an
electromagnetic field is concentrated in dielectric plates,
than capacity and inductance depend feebly on density of
plasma everywhere, except of regions of eigenmodes of
the device. Thus, for example, for an antenna of a type c
we have \( L = 3 \cdot 10^{-5} \text{ m} \) and
\( C = 3 \cdot 5 \cdot 10^{-4} \text{ mfd} \). Also, knowing effective
frequency of collision, it is possible to find energy, which
is delivered to plasma heating. Then, the ohmic resistance of an antenna loading equals

\[
R = \frac{1} {I^2} \int \frac{E^2} {v} \frac{\omega^2 \, v} {8\pi \left( \omega^2 + v^2 \right)} \, dV
\]

For the antenna of a
type c we have \( R = 0.01 \text{ Ohm} \).

CONCLUSION

The calculations of electromagnetic fields in the
device established the base for antenna optimization. The
no uniform grid of splitting was adopted using the finite-
differences method. It permits to get more precise
solutions in the RF field strong inhomogeneity while
using of a fewer number of points. Also, a finite-
differences method allows us to solve the equations in
case of arbitrary distribution of particles density in a
source's volume. It enables us to receive self-consistent
solution later. Fourier's decomposition has allowed to
reduce number of the finite-difference equations and to
increase speed of calculation considerably. The macro
characteristics (capacity, inductance and resistance) were
calculated for antennas. Also, the dependence of
integrated over source volume energy of RF electric field
on device parameters is obtained, that allows us to
estimate the influence of geometrical parameters change
on the source operation.

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РАСПРЕДЕЛЕНИЕ ВЧ ПОЛЕЙ В ПЛАЗМЕННОМ ЦИЛИНДРЕ ОГРАНИЧЕННОЙ ДЛИНЫ ДЛЯ РАЗЛИЧНЫХ АНТЕН

Д.Л. Греков, Д.В. Скляров

В работе проведен численный расчет электромагнитных полей, возбуждаемых в плазменном цилиндре
ограниченной длины, заключенном в металлический кожух. ВЧ поля возбуждаются антенной, расположенной
на торце цилиндра и помещенной между двумя диэлектрическими пластинами. Такая конфигурация характерна для
плазменных источников, используемых в технологических целях. Полученные результаты могут быть
использованы для улучшения характеристик подобных систем.

РОЗПОДІЛ ВЧ ПОЛЯ В ПЛАЗМОВОМУ ЦИЛІНДРІ ОБМЕЖЕНОЇ ДОВЖИНИ ДЛЯ РІЗНИХ АНТЕН

Д.Л. Греков, Д.В. Скляров

В роботі виконаний числовий розрахунок електромагнітних полів, які генеруються в обмеженому
металевими стінками плазмовому циліндрі. ВЧ поля генеруються антенною, розміщеною між двома
dіелектричними пластинами в торці циліндра. Така конфігурація характерна для плазмових джерел, які
виокремовуються в промислових цілях. Отримані результати можна використовувати для покращення
характеристик подібних систем.