POLARIZATION PHENOMENA IN THE PROCESS $e^+ + e^- \leftrightarrow N + \overline{N}$ IN PRESENCE OF TWO-PHOTON EXCHANGE

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General properties of the differential cross section and of various polarization observables are derived for the $e^+ + e^- \to N + \overline{N}$ reaction, in presence of two-photon exchange. Polarization effects are investigated for longitudinally polarized electron beam and polarized antinucleon and/or polarized nucleon in the final state.

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1. INTRODUCTION

The measurement of the nucleon electromagnetic form factors (FFs) in the space-like region of momentum transfer squared q^2 has a long history. The electric and magnetic FFs were traditionally determined using the Rosenbluth separation. More recently, the polarization transfer method [1-2], which requires longitudinally polarized electron beam and the measurement of the recoil proton polarization, could be applied. It turned out that the ratio of the proton electric and magnetic FFs determined by the two methods lead to appreciably different results [3]. One possible explanation for this discrepancy is the presence of a two-photon exchange (TPE) contribution to the elastic electron-nucleon (*eN)* scattering [4-5]. Model independent properties of TPE in elastic *ep*-scattering have been derived in [6].

Experimental investigation of the nucleon FFs in the time-like (TL) region could shed new light on this problem and bring additional valuable information on the internal structure of the proton and neutron.

In TL region the nucleon FFs (which are complex functions) can be studied through the annihilation reactions $e^+ + e^- \to N + \overline{N}$, $N + \overline{N} \to e^+ + e^-$, which are the crossing channel reactions of elastic electron-nucleon (electron-proton and electron-neutron) scattering. From crossing symmetry one expects that the reaction mechanisms are common for the scattering and the annihilation channels.

Here we consider the reaction

$$e^+ + e^- \to N + \overline{N} \tag{1}$$

and derive the expressions for the differential cross section and the polarization observables for the case when the matrix element contains TPE contribution.

2. MATRIX ELEMENT AND OBSERVABLES

The matrix element of this reaction can be obtained by analytic continuation of the matrix element for the elastic electron-nucleon scattering [6], parameterizing the TPE contribution in axial form:

$$M = -\frac{e^2}{q^2}\{\overline{u}(-k_2)\gamma_\mu u(k_1)\overline{u}(p_2)[F_{1N}(q^2,t)\gamma_\mu - \\ -\frac{1}{2m}F_{2N}(q^2,t)\sigma_{\mu\nu}q_\nu]u(-p_1) + \overline{u}(-k_2)\gamma_\mu\gamma_5 u(k_1) \\ \times\overline{u}(p_2)\gamma_\mu\gamma_5 u(-p_1)A_N(q^2,t)\}, \tag{2}$$

where $q = k_1 + k_2 = p_1 + p_2$ and k_1 (k_2) and p_1 (p_2) are the four-momenta of the electron (positron) and antinucleon (nucleon), respectively; m is the nucleon mass.

The complex amplitudes, $F_{1N,2N}$, A_N, which are functions of two variables, q^2 and $t = (k_1 - p_1)^2$, fully describe the spin structure of the matrix element – for any number of exchanged virtual photons. In the Born approximation (one-photon-exchange mechanism) these amplitudes reduce to:

$$F_{1N}^{Born}(q^2,t) = F_{1N}(q^2),\ F_{2N}^{Born}(q^2,t) = F_{2N}(q^2), \\ A_N^{Born}(q^2,t) = 0, \tag{3}$$

where $F_{1N}(q^2)$ and $F_{2N}(q^2)$ are the nucleon electromagnetic FFs.

In the following we use the standard FFs**:** magnetic FF $G_{MN}(q^2)$ and charge FF $G_{EN}(q^2)$. We introduce some combinations of the $F_{1N,2N}(q^2,t)$ amplitudes [6], which in the Born approximation correspond to the Sachs FFs and single out the dominant contribution (to separate the effects of the Born and TPE contributions):

$$\widetilde{G}_{MN}(q^2,t) = F_{1N}(q^2,t) + F_{2N}(q^2,t) = \\ G_{MN}(q^2) + \Delta G_{MN}(q^2,t), \\ \widetilde{G}_{EN}(q^2,t) = F_{1N}(q^2,t) + \tau\, F_{2N}(q^2,t) = \\ G_{EN}(q^2) + \Delta G_{EN}(q^2,t), \tag{4}$$

where $\tau = q^2/4m^2$.

The functions ΔG_{MN}, ΔG_{EN}, and A_N are of the order of α $(\alpha = e^2/4\pi \cong 1/137)$, while FFs are of the order of α^0. Since the TPE terms are small in comparison with the dominant ones, we neglect in the following their bilinear combinations.

The unpolarized differential cross section of the reaction (1) has the form

$$\frac{d\sigma_{un}}{d\Omega}=\frac{\alpha^2\sqrt{1-4m^2/q^2}}{4q^2}D,$$

$$D=(1+\cos^2\theta)(|G_{MN}|^2+2\,\mathrm{Re}\,G_{MN}\Delta G_{MN}^*)+\frac{1}{\tau}\sin^2\theta$$
$$\times(|G_{EN}|^2+2\,\mathrm{Re}\,G_{EN}\Delta G_{EN}^*)-4z\cos\theta\,\mathrm{Re}\,G_{MN}A_N^*,\quad(5)$$

where $z=\sqrt{(\tau-1)/\tau}$ and θ is the angle between the momenta of the electron and the antinucleon, in the reaction CMS.

The TPE contribution brings three new terms.

As it was shown in Refs. [7-8], for the scattering channel, symmetry properties of the amplitudes with respect to the $\cos\theta\to-\cos\theta$ transformation can be expressed in the annihilation channel as

$$\Delta G_{MN,EN}(\cos\theta)=-\Delta G_{MN,EN}(-\cos\theta),$$
$$A_N(\cos\theta)=A_N(-\cos\theta).\quad(6)$$

Let us consider the situation when the experimental apparatus does not distinguish the nucleon from the antinucleon. Then the following sum of the differential cross sections is measured

$$\frac{d\sigma_+}{d\Omega}=\frac{d\sigma}{d\Omega}(\cos\theta)+\frac{d\sigma}{d\Omega}(-\cos\theta).$$

We can stress, using the properties (6), that this quantity does not depend on the TPE contributions. This statement agrees with the conclusion of the paper [9]: for the processes of the type $e^++e^-\to a^++a^-$, if the apparatus which detects the final particles does not distinguish the particle from the antiparticle, then the interference term between the matrix elements corresponding to the one-photon and two-photon exchange diagrams does not contribute to the differential cross section.

Note also that the TPE contributions does not contribute to the total cross section of the reaction (1), which can be written as

$$\sigma_t(q^2)=\frac{4\pi}{3}\frac{\alpha^2\beta}{q^2}[|G_{MN}(q^2)|^2+\frac{1}{2\tau}|G_{EN}(q^2)|^2].$$

Let us define a coordinate frame in the reaction CMS: the z axis is directed along the momentum of the antinucleon $(\vec{p})$, the y axis is orthogonal to the reaction plane and directed along the vector $\vec{k}\times\vec{p}$, ($\vec{k}$ is the electron momentum), and the x axis forms a left-handed coordinate system.

The polarization of the outgoing antinucleon (all particles in the initial state of the reaction (1) are nonpolarized) is

$$P_y=\frac{2\sin\theta}{D\sqrt{\tau}}\{\cos\theta[\mathrm{Im}\,G_{MN}G_{EN}^*+\mathrm{Im}(G_{MN}\Delta G_{EN}^*$$
$$-G_{EN}\Delta G_{MN}^*)]+z\,\mathrm{Im}\,G_{EN}A_N^*\}.\quad(7)$$

From this expression one can see that

– The polarization of the outgoing antinucleon in this case is determined by the polarization component which is perpendicular to the reaction plane.

– The polarization, being a T-odd quantity, does not vanish even in the one-photon-exchange approximation due to the complexity of the nucleon FFs in the TL region (to say more exactly, due to the non-zero difference of the phases of these FFs). This is the principal difference with the elastic electron-nucleon scattering.

– In the Born approximation this polarization becomes equal to zero for the scattering angle $\theta=90^0$ (as well for $\theta=0^0$ and $\theta=180^0$). The presence of the TPE terms leads to non-zero value of the polarization at this angle.

Using the properties (6), we can remove the contribution of the TPE effects by constructing the difference $P_y(q^2,\theta)-P_y(q^2,\pi-\theta)$. The magnitude of the TPE contribution to the polarization P_y can be seen in the sum $P_y(q^2,\theta)+P_y(q^2,\pi-\theta)$.

If one of the colliding beams is longitudinally polarized, then the antinucleon acquires x- and z-components of the polarization, which lie in the reaction plane. These components are

$$P_x=-\frac{2\sin\theta}{D\sqrt{\tau}}[\mathrm{Re}(G_{MN}G_{EN}^*+G_{MN}\Delta G_{EN}^*$$
$$+G_{EN}\Delta G_{MN}^*)-z\cos\theta\,\mathrm{Re}\,G_{EN}A_N^*];\quad(8)$$
$$P_z=\frac{2}{D}[\cos\theta(|G_{MN}|^2+2\,\mathrm{Re}\,G_{MN}\Delta G_{MN}^*)$$
$$+z(1+\cos^2\theta)\,\mathrm{Re}\,G_{MN}A_N^*].$$

These polarization components are T-even observables and they are non-zero also for the elastic electron-nucleon scattering in the Born approximation.
The polarization component along the z axis vanishes when the proton is emitted at an angle $\theta=90^0$ in the Born approximation. But the presence of the TPE term A_N in the electromagnetic hadron current may lead to non-zero value of this quantity if the TPE amplitude $A_N(\theta=90^0)$ is not zero, since the value of this component is determined by the term $\mathrm{Re}\,G_{MN}A_N^*$.

The components of the polarization correlation tensor $P_{ik},(i,k=x,y,z)$ of the nucleon and the antinucleon in presence of the TPE mechanisms, are

$$P_{xx}=\frac{\sin^2\theta}{\tau D}[\tau(|G_{MN}|^2+2\,\mathrm{Re}\,G_{MN}\Delta G_{MN}^*)+$$
$$|G_{EN}|^2+2\,\mathrm{Re}\,G_{EN}\Delta G_{EN}^*];$$
$$P_{yy}=\frac{\sin^2\theta}{\tau D}[|G_{EN}|^2+2\,\mathrm{Re}\,G_{EN}\Delta G_{EN}^*-$$
$$\tau(|G_{MN}|^2+2\,\mathrm{Re}\,G_{MN}\Delta G_{MN}^*)];$$

$$P_{zz} = \frac{1}{\tau D}[\tau(1+\cos^2\theta)(|G_{MN}|^2 + 2\,\mathrm{Re}\,G_{MN}\Delta G_{MN}^*)$$
$$-\sin^2\theta(|G_{EN}|^2 + 2\,\mathrm{Re}\,G_{EN}\Delta G_{EN}^*)$$
$$-4z\tau\cos\theta\,\mathrm{Re}\,G_{MN}A_N^*];$$
$$P_{xz} = P_{zx} = -2\frac{\sin\theta}{D\sqrt{\tau}}[\cos\theta\,\mathrm{Re}(G_{MN}G_{EN}^* + G_{MN}\Delta G_{EN}^*$$
$$+G_{EN}\Delta G_{MN}^*) - z\,\mathrm{Re}\,G_{EN}A_N^*].$$

For the completeness we give also the non-zero coefficients in the case of a longitudinally polarized electron beam

$$P_{xy} = P_{yx} = -\frac{1}{D}\sqrt{\frac{\tau-1}{\tau}}\sin^2\theta\,\mathrm{Im}\,G_{MN}A_N^*;$$
$$P_{zy} = P_{yz} = \frac{\sin\theta}{D\sqrt{\tau}}\mathrm{Im}[G_{MN}G_{EN}^* + G_{MN}\Delta G_{EN}^*$$
$$-G_{EN}\Delta G_{MN}^* + \sqrt{\frac{\tau-1}{\tau}}\cos\theta\,G_{EN}A_N^*].$$

One can easily verify that the following relation holds:

$$P_{xx} + P_{yy} + P_{zz} = 1.$$

Let us note the following properties for these coefficients.

▪ The components of the tensor describing the polarization correlations P_{xx}, P_{yy}, P_{zz}, P_{xz}, and P_{zx} are T-even observables, whereas the components P_{xy}, P_{yx}, P_{yz}, and P_{zy} are T-odd ones.

▪ The relative contribution of the interference amplitudes (between one- and two-photon-exchange terms) in these observables will increase as the value of the momentum transfer squared becomes larger since it is expected that the TPE amplitudes decrease more slowly with the momentum transfer squared compared with the nucleon form factors.

At the reaction threshold, the polarization correlation tensor components have some specific properties**:**

▪ All correlation coefficients (both T-odd and T-even) do not depend on the function A_N.

▪ In the Born approximation the P_{yy} observable is zero, but the presence of the TPE term leads to a non-zero value, which is determined by the following quantity $\mathrm{Re}\,G_N(\Delta G_{EN} - \Delta G_{MN})^*$ (at the reaction threshold the relation $G_N = G_{EN} = G_{MN}$ holds).

▪ At the scattering angle $\theta = 90^0$ we have the relation $P_{yy} + P_{zz} = 0$.

3. CONCLUSIONS

Precise measurements of the elastic electron-hadron scattering arose the question of the importance of the TPE mechanism. This problem enters also in the determination of the nucleon FFs in the TL region investigated with the reaction (1), since this process is the crossed channel of the elastic electron-nucleon scattering.

Using the properties of these amplitudes with respect to the $\cos\theta \to -\cos\theta$ transformation, it was shown how to remove (or enhance) the TPE contribution from various observables.

We showed that the real part of the TPE term can be accessed through the difference between the differential cross sections measured at angles θ and $\pi-\theta$, whereas for the imaginary part of the TPE term one needs the measurement of the nucleon or antinucleon polarization when all the other particles are unpolarized.

The polarization of the produced proton in the electron-positron annihilation may be more difficult to measure than in the elastic electron-nucleon scattering, due to the luminosity. On the other hand, angular distribution is easier to achieve as it is necessary to make measurements at different angles, with fixed beam energy, whereas in scattering reactions measurements should be done at fixed momentum transfer squared, which requires to change simultaneously beam energy and scattering angle.

Such model independent derivation of polarization observables has been completed in the elastic electron-proton scattering and in the crossing channels, electron-positron annihilation to the nucleon and antinucleon and inverse reaction.

These results will be especially useful in view of the ongoing programs at Novosibirsk and Bejing as well as at the future facilities at Frascati and Darmstadt.

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ПОЛЯРИЗАЦИОННЫЕ ЯВЛЕНИЯ В ПРОЦЕССЕ $e^{+} + e^{-} \leftrightarrow N + \overline{N}$ ПРИ НАЛИЧИИ ДВУХФОТОННОГО ОБМЕНА

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Получены общие свойства дифференциального сечения и различных поляризационных наблюдаемых для реакции $e^{+} + e^{-} \to N + \overline{N}$ при наличии двухфотонного обмена. Исследованы поляризационные эффекты, когда начальный пучок электронов имеет продольную поляризацию, а в конечном состоянии поляризованы антинуклон и/или нуклон.

ПОЛЯРИЗАЦІЙНІ ЯВИЩА В ПРОЦЕСІ $e^{+} + e^{-} \leftrightarrow N + \overline{N}$ ПРИ НАЯВНОСТІ ДВУХФОТОННОГО ОБМІНУ

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Отримані загальні властивості диференційного перерізу та різних поляризаційних спостережуваних для реакції $e^{+} + e^{-} \to N + \overline{N}$ при наявності двухфотонного обміну. Досліджені поляризаційні ефекти, коли початковий пучок електронів має повздовжню поляризацію, а в кінцевому стані поляризовані антинуклон і/або нуклон.