PARITY NONCONSERVATION IN TRINUCLEON BOUND STATES

V. Kotlyar[1] and A. Nogga[2]

[1]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
e-mail: kotlyarv@kipt.kharkov.ua;
[2]Institut für Kernphysik, Forschungszentrum Jülich, Jülich, Germany;
e-mail: a.nogga@fz-juelich.de

Tensor and operator representations for wave function (WF) of three–nucleon bound states are discussed. It is supposed that space parity is not conserved in interaction between nucleons. The WFs of ^{3}He and ^{3}H are expressed in terms of 16 scalar functions, which depend on the magnitudes of the relative momenta and the angle between them. These functions are real in the case when nuclear forces do not violate time reversal symmetry. Characteristic features appropriate to WFs of deuteron and 3N nuclei in operator form (OF) are compared.

PACS: 21.45.+V, 27.10.+h, 11.30.Er

1. INTRODUCTION

Properties of the three– and four–nucleon bound states were investigated [1] by E. Gerjuoy and J. Schwinger under the assumption that the nuclear forces are invariant with respect to the space inversion. Three–nucleon system of even parity and with the total angular momentum $I = 1/2$ was considered in [1].

Tritium WF can be obtained [1] from the 3N spin state that has zero spin in two–nucleon subsystem with the help of a set of spin–angular operators $g_\lambda(\hat{\vec{\rho}}, \hat{\vec{\eta}})$ multiplied by scalar functions $f_\lambda(|\vec{\rho}|, |\vec{\eta}|, \vec{\rho}\cdot\vec{\eta})$. The operators g_λ are constructed from unit vectors $\hat{\vec{\rho}}, \hat{\vec{\eta}}$ directed along the Jacobi coordinates $\vec{\rho}, \vec{\eta}$ and Pauli matrices of nucleons. All three-nucleon states $^{2\Sigma+1}\Lambda_I = {}^2S_{1/2}$, $^2P_{1/2}$, $^4P_{1/2}$, and $^4D_{1/2}$ with total orbital momentum $\Lambda = 0, 1, 2$ and total spin $\Sigma = 1/2, 3/2$, that may appear in 3N bound state with $I^P = 1/2^+$, are generated by operators [1].

Relationship between the OF and the partial–wave decompositions [2] of the WF was investigated in [3]. Eight scalar functions, needed to build up the WF, were calculated for modern realistic models of nuclear forces. Results of Ref. [3] can be used to compute WFs of ^{3}He and ^{3}H nuclei in momentum space[1] .

An OF for the WF was also derived in [4] transforming WF [5] in tensor representation (TR). Different sets of the operators are employed in [1,3] and [4]. Operators and corresponding scalar functions [4] are linear combinations of ones defined in [3]. WF in the TR [5] was used in [4] to calculate and analyze structure of spin–dependent momentum distributions of nucleons and proton–deuteron clusters in 3N nuclei.

Parity–conserving nuclear forces and correspondingly WFs of ^{3}He and ^{3}H nuclei with positive parity were treated in Refs. [1,3-5]. Aim of this report is to incorporate into TR and OF for the WF contributions due to parity violating interaction between nucleons.

2. TENSOR FORM OF THE 3N BOUND – STATE WAVE FUNCTION

WF of the three-nucleon bound states $|\Psi; I m'\rangle$ with the total angular momentum $I = 1/2$ and its projection m' in TR reads [5]

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q}) = {}_{23,1}\langle \vec{p},\vec{q}; SM, 1/2\, m|\Psi; 1/2\, m'\rangle. \qquad (1)$$

Tensor (1) has 16 complex components.

Eq. (1) is written in the center of mass system of the nucleus. The Jacobi momenta are

$$\vec{p} = (\vec{k}_2 - \vec{k}_3)/2,\ \vec{q} = (2\vec{k}_1 - \vec{k}_2 - \vec{k}_3)/3,$$

where $\vec{k}_i$ is the momentum of nucleon with number $i\ (i = 1, 2, 3)$. The total spin S of the nucleons 2 and 3 takes on values 0 and 1. The spin states are defined as

$$|SM\rangle_{23} = \sum_{m_2\, m_3} C^{SM}_{1/2\, m_2\, 1/2\, m_3} |1/2\, m_2\rangle_2 |1/2\, m_3\rangle_3, \qquad (2)$$

where $C^{a\alpha}_{b\beta\, c\lambda}$ is Clebsch–Gordan coefficient [6]. Vector $|1/2\, m_i\rangle_i$ describes nucleon i with the projection of spin m_i.

The isospin formalism is not employed in the present paper. The nucleon labeled by 1 is chosen to be neutron (proton) for ^{3}He (^{3}H) nucleus. In turn, nucleons 2 and 3 are protons (neutrons).

WF (1) transforms under the permutation (2,3) of identical nucleons 2 and 3 according to

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q}) = (-1)^S \Psi^{SMm}_{m'}(-\vec{p},\vec{q}). \qquad (3)$$

As far as we assume that interaction between nucleons does not conserve space parity, nuclear states are superpositions of terms with opposite parity:

$$|\Psi; 1/2m'\rangle = \sum_{N=0,1} |\Psi; 1/2m'; N\rangle,$$

so $P|\Psi; 1/2m'; N\rangle = (-1)^N |\Psi; 1/2m'; N\rangle$, where P is the space inversion operator. The components

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q}; N) = \langle \vec{p},\vec{q}; SM, 1/2\, m|\Psi; 1/2\, m'; N\rangle \qquad (4)$$

of WF (1), that have definite parity $(-1)^N$, obey

[1] The functions can be downloaded from http://www.phy.ohiou.edu/~elster/h3wave/index.html

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q};N) = (-1)^N \Psi^{SMm}_{m'}(-\vec{p},-\vec{q};N). \quad (5)$$

A consequence of (3) and (5) is

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q};N) = (-1)^{S+N} \Psi^{SMm}_{m'}(\vec{p},-\vec{q};N). \quad (6)$$

Eqs. (3), (5), (6) contain the WF values for the same quantum numbers S, M, m, m' at different points in the space of Jacobi momenta. These relations can be used to control numerical calculations of the WF in TR (1).

In the case of time reversal invariant nuclear forces the WF fulfills

$$\Psi^{SMm}_{m'}(\vec{p},\vec{q}) = (-1)^{S+M+m-m'} \times \left(\Psi^{S,-M,-m}_{-m'}(-\vec{p},-\vec{q})\right)^{*}. \quad (7)$$

Eqs. (5) and (7) impose constraints

$$\Psi^{SMm}_{-m'}(\vec{p},\vec{q};N) = (-1)^{S+M+m+m'+N} \times \left(\Psi^{S,-M,-m}_{m'}(\vec{p},\vec{q};N)\right)^{*}. \quad (8)$$

Since in both sides of Eq. (8) the WF values are taken at the same point of $\vec{p},\vec{q}$–space, the components of tensor (4) with $m' = -1/2$ can be obtained from ones having $m' = 1/2$. However, the total number of independent functions, needed to built tensor (1), when time reversal invariance holds, and space inversion symmetry is violated, is not reduced by Eq. (8). In the next section WF (1) will be constructed using minimal amount of real scalar functions.

3. DECOMPOSITION OF THE WAVE FUNCTION OVER POLARIZATION OPERATORS

WF (1) is the set of four 2×2 matrices $\Phi^{SM}(\vec{p},\vec{q})$, defined as $\langle 1/2\, m | \Phi^{SM}(\vec{p},\vec{q}) |\, 1/2\, m'\rangle = \Psi^{SMm}_{m'}(\vec{p},\vec{q})$. The matrices can be decomposed over the complete system of polarization operators (POs) $T^{KM}(s)$ for spin $s = 1/2$

$$\Phi^{SM}(\vec{p},\vec{q}) = \sqrt{2}\, \Sigma_{K=0,1;M'} \overline{\Psi}^{SM}_{KM'}(\vec{p},\vec{q})\, T^{KM'}(1/2). \quad (9)$$

Properties of the POs are discussed in [6-8]. In the present report we follow conventions of Ref. [6]. The contravariant POs are given by $T^{KM}(s) = (-1)^M T_{K,-M}(s)$. The matrices of the covariant POs are

$$\langle s\, m | T_{KM}(s) |\, s\, m'\rangle = (2K+1)^{1/2} (2s+1)^{-1/2} C^{sm}_{sm'\,KM}.$$

Tensor $\overline{\Psi}^{SM}_{KM'}(\vec{p},\vec{q})$ is to be constructed from the Jacobi momenta $\vec{p},\vec{q}$. The component of the tensor with $S = K = 0$ is a scalar function

$$\overline{\Psi}^{00}{}_{00}(\vec{p},\vec{q}) = \psi_1(p,q,\xi), \quad (\xi = \hat{\vec{p}}\cdot\hat{\vec{q}}), \quad (10)$$

where a unit vector $\hat{\vec{A}} = \vec{A}/|\vec{A}|$. Below we denote $\psi_\lambda = \psi_\lambda(p,q,\xi)$, where λ=1, 2,….

Tensors $\overline{\Psi}^{SM}_{KM'}(\vec{p},\vec{q})$ with S=0, K=1 or S=1, K=0, which are equivalent to three-vectors, can be written as

$$\overline{\Psi}^{00}{}_{1M}(\vec{p},\vec{q}) = \sqrt{2}\, v'_M\, \psi_2 + \hat{p}_M \psi_9 + \hat{q}_M \psi_{10}; \quad (11)$$

$$\overline{\Psi}^{1M}{}_{00}(\vec{p},\vec{q}) = \sqrt{2}\, v'^M\, \psi_3 + \hat{p}^M \psi_{11} + \hat{q}^M \psi_{12}, \quad (12)$$

where $v'_M = \{\hat{\vec{p}}\otimes\hat{\vec{q}}\}_{1M}$.

Irreducible tensor product that has rank J of two irreducible tensors $A_{J'M'}$ and $B_{J''M''}$ is [6]

$$\{A_{J'}\otimes B_{J''}\}_{JM} = \Sigma_{M'M''} C^{JM}_{J'M'\,J''M''} A_{J'M'} B_{J''M''}. \quad (13)$$

The contravariant and covariant components of (13) are related by

$$\{A^{J'}\otimes B^{J''}\}^{JM} = (-1)^M \{A_{J'}\otimes B_{J''}\}_{J,-M}.$$

Structure of the reducible tensor $\overline{\Psi}^{S=1,M}_{K=1,M'}(\vec{p},\vec{q})$ can be determined from decomposition

$$\overline{\Psi}^{1M}_{1M'}(\vec{p},\vec{q}) = \sum_{k=0,1,2;\kappa} w_{k\kappa}(\vec{p},\vec{q}) \times \langle 1M\, | T^{k\kappa}(1) |\, 1M'\rangle, \quad (14)$$

where, similarly to Eqs. (10)–(12), one has

$$k = 0: \quad w_{00}(\vec{p},\vec{q}) = \sqrt{3}\, \psi'_4; \quad (15)$$

$$k = 1: \quad w_{1M}(\vec{p},\vec{q}) = 2\, v'_M\, \psi'_5 - \sqrt{2}\, \hat{p}_M \psi_{13} - \sqrt{2}\, \hat{q}_M \psi_{14}. \quad (16)$$

Term $w_{2\kappa}(\vec{p},\vec{q})$ in (14) receives contributions from irreducible tensor products of rank two, which are quadratic and cubic in relative moment,

$$w_{2\kappa}(\vec{p},\vec{q}) = -\{\hat{\vec{p}}\otimes\hat{\vec{p}}\}_{2\kappa}\psi_6 - \{\hat{\vec{q}}\otimes\hat{\vec{q}}\}_{2\kappa}\psi_7 - \{\hat{\vec{p}}\otimes\hat{\vec{q}}\}_{2\kappa}\psi'_8 - 2\sqrt{2}\, \{\hat{\vec{p}}\otimes v'\}_{2\kappa}\psi_{15} - 2\sqrt{2}\, \{\hat{\vec{q}}\otimes v'\}_{2\kappa}\psi_{16}. \quad (17)$$

The matrices $\Phi^{SM}(\vec{p},\vec{q})$, in the same way as the 3N state $|\Psi; 1/2m'\rangle$, split into parts with definite parity

$$\Phi^{SM}(\vec{p},\vec{q}) = \Sigma_{N=0,1} \Phi^{SM}(\vec{p},\vec{q};N). \quad (18)$$

Eqs. (9)–(17) yield for the terms with positive parity

$$\Phi(\vec{p},\vec{q};N=0) = \psi_1 + i\, \vec{\sigma}\cdot\vec{v}\, \psi_2; \quad (19)$$

$$\vec{\Phi}(\vec{p},\vec{q};N=0) = i\, \vec{v}\, \psi_3 + \vec{\sigma}\, \psi_4 + \hat{\vec{p}}\, \vec{\sigma}\cdot\hat{\vec{q}}\, \psi_5 + \hat{\vec{p}}\, \vec{\sigma}\cdot\hat{\vec{p}}\, \psi_6 + \hat{\vec{q}}\, \vec{\sigma}\cdot\hat{\vec{q}}\, \psi_7 + \hat{\vec{q}}\, \vec{\sigma}\cdot\hat{\vec{p}}\, \psi_8, \quad (20)$$

where it is denoted $\Phi(\vec{p},\vec{q};N) = \Phi^{S=M=0}(\vec{p},\vec{q};N)$, and $\vec{v} = [\hat{\vec{p}}\times\hat{\vec{q}}]$. The contravariant cyclic components of the three-vector $\vec{\Phi}(\vec{p},\vec{q};N)$ are $\Phi^{S=1,M}(\vec{p},\vec{q};N)$.

WF $\Psi^{SMm}_{m'}(\vec{p},\vec{q};N{=}0)$ with positive parity depends on functions $\psi_\lambda(p,q,\xi)$ with $\lambda = 1,\ldots,8$ where

$$\psi_4 = \psi'_4 - (\psi_6 + \psi_7 + \xi\psi'_8)/3; \quad \psi_5 = \psi'_5 + \psi'_8/2, \quad \psi_8 = -\psi'_5 + \psi'_8/2. \quad (21)$$

Other eight functions ψ_λ with $\lambda = 9,\ldots,16$ appear in the negative parity contributions to the WF. Component $\Psi^{SMm}_{m'}(\vec{p},\vec{q};N{=}1)$ with zero spin S reads

$$\Phi(\vec{p},\vec{q};N=1) = \vec{\sigma}\cdot\hat{\vec{p}}\, \psi_9 + \vec{\sigma}\cdot\hat{\vec{q}}\, \psi_{10}. \quad (22)$$

The part of the WF with S=1 is

$$\vec{\Phi}(\vec{p},\vec{q};N=1) = \hat{\vec{p}}\, \psi_{11} + \hat{\vec{q}}\, \psi_{12} + i\, [\vec{\sigma}\times\hat{\vec{p}}]\, \psi_{13} + i\, [\vec{\sigma}\times\hat{\vec{q}}]\, \psi_{14} + i\, (\hat{\vec{p}}\, \vec{\sigma}\cdot\vec{v} + \vec{v}\, \vec{\sigma}\cdot\hat{\vec{p}})\, \psi_{15} + i\, (\hat{\vec{q}}\, \vec{\sigma}\cdot\vec{v} + \vec{v}\, \vec{\sigma}\cdot\hat{\vec{q}})\, \psi_{16}. \quad (23)$$

Scalar functions $\psi_\lambda(p,q,\xi)$ with $\lambda = 1,\ldots,16$ are real, when nuclear forces are time–reversal invariant.

4. OPERATOR FORM OF THE WAVE FUNCTION

Relation between TR and OFs for the WF can be obtained from decomposition

$$\left|\Psi; 1/2\, m'\right\rangle = \sum_{SMm} \int d^3p\, d^3q\;\; \Psi_{m'}^{SMm}(\vec{p},\vec{q}) \times \left|\vec{p},\vec{q}; SM, 1/2\, m\right\rangle_{23,1}. \qquad (24)$$

Spin states with S=1 in the r.h.s. of (24) can be generated by the operator $\vec{\sigma}(23) = 1/2\,(\vec{\sigma}(2) - \vec{\sigma}(3))$ from one with S=0. Operator $\vec{\sigma}(23)$ was introduced in [1]. With the help of identity $\left|1M\right\rangle_{23} = \sigma_M(23)\left|00\right\rangle_{23}$, Eq. (24) can be cast into the form

$$\left|\Psi; 1/2\, m'\right\rangle = \int d^3p\, d^3q\, \hat{\Psi}(\vec{p},\vec{q}) \left|S{=}M{=}0\right\rangle_{23} \left|1/2\, m'\right\rangle_1. \qquad (25)$$

The covariant cyclic components of the vector $\vec{\sigma}(23)$ are denoted by $\sigma_M(23)$.

Operator $\hat{\Psi}(\vec{p},\vec{q})$ acts in the spin space and can be expressed through the matrices $\Phi^{SM}(\vec{p},\vec{q};N)$, viz.,

$$\hat{\Psi}(\vec{p},\vec{q}) = \sum_{N=0,1} \left(\Phi(\vec{p},\vec{q};N) + \vec{\Phi}(\vec{p},\vec{q};N)\cdot\vec{\sigma}(23)\right). \qquad (26)$$

In Eq. (26) $\Phi^{SM}(\vec{p},\vec{q};N)$ are given by (19), (20), (22), and (23) where $\vec{\sigma}$ is substituted by $\vec{\sigma}(1)$.

Representation (26) can be also derived, using decomposition of $\hat{\Psi}(\vec{p},\vec{q})$ in terms of the cartesian components of the operators $\vec{\sigma}(1)$ and $\vec{\sigma}(23)$,

$$\begin{aligned} \hat{\Psi}(\vec{p},\vec{q}) &= \Psi(\vec{p},\vec{q};1) + \\ &+ \sum_n (\Psi_n(\vec{p},\vec{q};2)\,\sigma_n(1) + \Psi_n(\vec{p},\vec{q};3)\,\sigma_n(23)) + \\ &+ \sum_{nn'} \Psi_{nn'}(\vec{p},\vec{q})\,\sigma_n(1)\,\sigma_{n'}(23), \qquad (n,n' = x,y,z). \end{aligned} \qquad (27)$$

Scalar $\Psi(\vec{p},\vec{q};\lambda=1)$, vectors $\Psi_n(\vec{p},\vec{q};\lambda=2,3)$ and tensor $\Psi_{nn'}(\vec{p},\vec{q})$ are to be build in terms of the relative momenta $\vec{p},\vec{q}$. Operator (27) obtained in this way agrees with (19), (20), (22), (23), and (26).

Representations of the WF in terms of the POs as well as with the use of cartesian tensors allow one to relate functions $\psi_\lambda(p,q,\xi)$ and components of the WF with definite values of total orbital angular momenta. As seen from Eqs. (19) and (20), the S-state $^2S_{1/2}$ originates not merely from $\psi_{\lambda=1}$ and $\psi_{\lambda=4}$. As far as in Eq. (20) a symmetric traceless tensor $\Psi^s_{nn'}(\vec{p},\vec{q})$ is not singled out, $\psi_{\lambda=6,7}$ and $\psi'_{\lambda=8}$ contribute to the S–state, and through tensor $\Psi^s_{nn'}(\vec{p},\vec{q})$ these functions are related to D–wave $^4D_{1/2}$. Pseudovector $\vec{v}$ in (19) and (20) together with antisymmetric part of (20) determine P–waves. Component $^2P_{1/2}$ depends on $\psi_{\lambda=2}$, while $^4P_{1/2}$ state is due to both $\psi_{\lambda=3}$ and $\psi'_{\lambda=5}$.

Parity–odd components $\Phi^{SM}(\vec{p},\vec{q};N=1)$ with spin values S=0 and 1, which contain ψ_λ with $\lambda = 9, 10$ and $\lambda = 11,\ldots,14$, produce negative–parity $^2P_{1/2}$ and $^4P_{1/2}$ states, respectively. The irreducible pseudotensors $\{\hat{\vec{p}}\otimes\{\hat{\vec{p}}\otimes\hat{\vec{q}}\}_1\}_{2\kappa}$ and $\{\hat{\vec{q}}\otimes\{\hat{\vec{p}}\otimes\hat{\vec{q}}\}_1\}_{2\kappa}$, which have rank two and involve three relative momenta, together with functions $\psi_{\lambda=15,16}$ (see Eq. (17)) generate P-odd $^4D_{1/2}$ components of the WF.

Eqs. (26) and (27) can be written as

$$\hat{\Psi}(\vec{p},\vec{q}) = \sum_{\lambda=1,\ldots,16} \psi_\lambda(p,q,\xi)\, u_\lambda(\hat{\vec{p}},\hat{\vec{q}}), \qquad (28)$$

where spin–angular operators $u_\lambda(\hat{\vec{p}},\hat{\vec{q}})$ corresponding to the scalar functions are separated out. The operators and functions in (28) are superpositions of ones in Gerjuoy–Schwinger representation [1,3].

Another way to get an OF for the WF is to transform decomposition (24) eliminating explicit contribution of the spin state $\chi_0 = \left|S=0, M\right\rangle_{23}$. Identity

$$\sigma^M(23)\,\chi_{M'} = \delta_{MM'}\,\chi_0, \qquad (29)$$

where $\chi_M = \left|S=1, M\right\rangle_{23}$, can be used with this end.

Instead of Eq. (25) one has

$$\left|\Psi; 1/2\, m\right\rangle = \int d^3p\, d^3q\, \hat{\vec{\mathrm{X}}}(\vec{p},\vec{q})\cdot\vec{\chi}\,\chi_{1/2\,m},$$

where covariant cyclic components of $\vec{\chi}$ are χ_M, and $\chi_{1/2\,m} = \left|1/2\, m\right\rangle_1$. Expression for the operator $\hat{\vec{\mathrm{X}}}(\vec{p},\vec{q})$ generating the 3N–state involves 2×2 matrix $\vec{\Gamma}(\vec{p},\vec{q};N)$, which comes from the term $\Psi_{m'}^{S=M=0,m}(\vec{p},\vec{q})\,\chi_0$ in (24),

$$\vec{\mathrm{X}}(\vec{p},\vec{q}) = \sum_{N=0,1} (\vec{\Gamma}(\vec{p},\vec{q};N) + \vec{\Phi}(\vec{p},\vec{q};N)).$$

Parity–even component of $\vec{\Gamma}$ is given by

$$\begin{aligned} \vec{\Gamma}(\vec{p},\vec{q};N=0) &= (3a_1 + a_2 + a_3)^{-1}\left(a_1\,\vec{\sigma}(23)\right. \\ &+ a_2\,\vec{\sigma}(23)\cdot\hat{\vec{p}}\,\hat{\vec{p}} + a_3\,\vec{\sigma}(23)\cdot\hat{\vec{q}}\,\hat{\vec{q}}\left.\right)\psi_1 \\ &+ i/2\,(\vec{\sigma}(1)\cdot\vec{\sigma}(23)\,\vec{v} + \vec{\sigma}(23)\cdot\vec{v}\,\vec{\sigma}(1))\,\psi_2. \end{aligned}$$

The representation for $\vec{\Gamma}(\vec{p},\vec{q};N=0)$ is not unique. Choice of coefficients a_i, $(i=1,2,3)$ depends on the purpose of the transformation. Parity-odd $^2P_{1/2}$ state

$$\begin{aligned} \vec{\Gamma}(\vec{p},\vec{q};N{=}1) &= (\vec{\sigma}(1)\cdot\vec{\sigma}(23)\,\hat{\vec{p}} + \vec{\sigma}(23)\cdot\hat{\vec{p}}\,\vec{\sigma}(1))\,\psi_9/2 \\ &+ (\vec{\sigma}(1)\cdot\vec{\sigma}(23)\,\hat{\vec{q}} + \vec{\sigma}(23)\cdot\hat{\vec{q}}\,\vec{\sigma}(1))\,\psi_{10}/2 \end{aligned}$$

originates from (22), (23), (26) and contains no ambiguities.

5. COMMON FEATURES OF TENSOR AND OPERATOR REPRESENTATIONS FOR DEUTERON AND ^{3}He WAVE FUNCTIONS

In calculations of reaction amplitudes, e.g. two-body photodisintegration of ^{3}He or the momentum distributions of proton–deuteron clusters in ^{3}He, both WFs of 2N and 3N nuclei are constructed making use of partial-wave components. The deuteron WF is known [9-12] to contain $^{2S+1}L_J = {}^3S_1,\ {}^1P_1,\ {}^3P_1,\ {}^3D_1$ states when nucleon–nucleon potential includes both P–even and P–odd contributions. In the case of a three-body bound state partial-wave series involve infinite number of terms [2].

WFs of deuteron and ^{3}He in the TR consist of 9 and 16 complex components regardless of whether nuclear forces conserve parity. The components of WF in the TR are not independent.

In the center of mass system WF of deuteron with the total angular momentum $J=1$ and its projection M' reads

$$\phi^{SM}_{1M'}(\vec{p}) = {}_{23}\langle \vec{p};\, SM \,|\, \phi;\, J=1, M'\rangle, \quad (S=0,1). \quad (30)$$

Nucleons in deuteron have number labels 2 and 3.

Component of the WF (30) with S=0 can be written as

$$\phi^{S=M=0}_{1M}(\vec{p}) = \hat{p}_M\ \phi({}^1P_1), \quad (31)$$

where notation $\phi({}^{2S+1}L_J) = \phi(p;\, {}^{2S+1}L_J)$ is used.

Tensor $\phi^{S=1,M}_{1M'}(\vec{p})$ is a 3×3 matrix, which can be decomposed similarly to $\overline{\Psi}{}^{1M}_{1M'}(\vec{p},\vec{q})$ over the POs with spin $s=1$ (see Eq. (14)). The coefficients in the decomposition, that are denoted by $\phi_{k\kappa}(\vec{p})$, (k=0,1,2), can be constructed as follows

$$\begin{aligned}&\phi_{00}(\vec{p}) = \sqrt{3}\ \phi'({}^3S_1), \quad \phi_{1\kappa}(\vec{p}) = \sqrt{2}\ \hat{p}_\kappa\ \phi({}^3P_1),\\ &\phi_{2\kappa}(\vec{p}) = -\{\hat{p}\otimes\hat{p}\}_{2\kappa}\ \phi({}^3D_1).\end{aligned} \quad (32)$$

Time reversal invariance of NN interaction implies that the functions $\phi(p;\, {}^{2S+1}L_J)$ are real.

Part of the deuteron WF with S=1 is

$$\begin{aligned}&\textstyle\sum_{MM'} A_M B^{M'}\ \phi^{1M}_{1M'}(\vec{p}) = \vec{A}\cdot\vec{B}\ \phi({}^3S_1) +\\ &+ i\hat{\vec{p}}\cdot[\vec{A}\times\vec{B}]\ \phi({}^3P_1) + \vec{A}\cdot\hat{\vec{p}}\ \vec{B}\cdot\hat{\vec{p}}\ \phi({}^3D_1),\end{aligned} \quad (33)$$

where $\phi({}^3S_1) = \phi'({}^3S_1) - \phi({}^3D_1)/3$, and auxiliary vectors $\vec{A}, \vec{B}$ are introduced for convenience.

Eq. (33) can be written in terms of cartesian components $\phi_{nn'}(\vec{p}) = \sum_{N=0,1} \phi_{nn'}(\vec{p};N)$ of the tensor $\phi^{1M}_{1M}(\vec{p})$. The P-even part of $\phi_{nn'}(\vec{p})$

$$\phi_{nn'}(\vec{p}; N=0) = \delta_{nn'}\ \phi({}^3S_1) + \hat{p}_n\ \hat{p}_{n'}\ \phi({}^3D_1) \quad (34)$$

is symmetric in indices nn' and real. The P–odd contribution to the WF originating from the $^3P_1$ wave is antisymmetric and imaginary

$$\phi_{nn'}(\vec{p}; N=1) = i\varepsilon_{nn'l}\ \hat{p}_l\ \phi({}^3P_1), \quad (S=1), \quad (35)$$

whereas one coming from $^1P_1$ wave (31) is real

$$\phi_n(\vec{p}) = \hat{p}_n\ \phi({}^1P_1), \quad (S=0). \quad (36)$$

Eqs. (31), (32) and (34)–(36) correspond to partial–wave decompositions [9–12] of WF (30). Compact representation (34)–(36) for the deuteron WF prompts us to search for corresponding constructions in the case of 3N and 4N systems.

OFs for the deuteron WF can be derived from expressions (34)–(36). Vector $|\vec{\phi}\rangle$ with cyclic components that coincide with the deuteron state $|\phi;\, J=1, M\rangle$ is produced according to

$$|\vec{\phi}\rangle = \int d^3p\ \vec{\phi}(\vec{p})\,|\vec{p}\rangle\,|S=M=0\rangle_{23} \quad (37)$$

by operator $\vec{\phi}(\vec{p})$ acting in the spin space. Axial- and polar-vector parts of

$$\begin{aligned}&\vec{\phi}(\vec{p}) = \vec{\sigma}(23)\ \phi({}^3S_1) + \hat{\vec{p}}\ \phi({}^1P_1)\\ &+ i\,[\vec{\sigma}(23)\times\hat{\vec{p}}]\ \phi({}^3P_1) + \vec{\sigma}(23)\cdot\hat{\vec{p}}\ \hat{\vec{p}}\ \phi({}^3D_1)\end{aligned}$$

spring from parity conserving and violating nucleon–nucleon interaction.

The operator $\vec{\phi}(\vec{p})$ is applied in (37) at the two-nucleon state $|S=M=0\rangle_{23}$ with zero total spin. In this respect, Eq. (37) is similar to decomposition (25) in the case of 3N nuclei. While transformations of (37) under rotations are provided by the operator $\vec{\phi}(\vec{p})$, in the case of 3N bound state, vector $|1/2\, m'\rangle_1$ is responsible for the correct properties of $|\Psi;\, 1/2\, m'\rangle$, that is generated by the scalar operator $\hat{\Psi}(\vec{p},\vec{q})$.

One can get from (31) and (32) other representation

$$|\vec{\phi}\rangle = \int d^3p\ \vec{\chi}(\vec{p})\,|\vec{p}\rangle, \quad (38)$$

that involves explicitly only the 2N spin states $\vec{\chi}$ with S=1. Vector in the spin space $\vec{\chi}(\vec{p})$ is given by

$$\begin{aligned}&\vec{\chi}(\vec{p}) = \vec{\chi}\ \phi({}^3S_1) + (\vec{\sigma}(23)\cdot\hat{\vec{p}}\ \vec{\chi} + \vec{\sigma}(23)\ \hat{\vec{p}}\cdot\vec{\chi})\ \phi({}^1P_1)\\ &+ i[\vec{\chi}\times\hat{\vec{p}}]\ \phi({}^3P_1) + \vec{\chi}\cdot\hat{\vec{p}}\ \hat{\vec{p}}\ \phi({}^3D_1).\end{aligned} \quad (39)$$

Identity (29) has been employed to obtain contribution of $^1P_1$ state to (39). Derivation of Eqs. (38) and (39) does not suffer from ambiguities in contrast to the transformation of the S=0 contribution to the 3N WF that yields the matrix $\Gamma(\vec{p},\vec{q};N=0)$. Really, angular dependence of $^1P_1$ component (31) is determined by the vector $\hat{\vec{p}}$ and application of the identity is straightforward.

6. SUMMARY AND OUTLOOK

WF of 3N bound state with the total angular momentum $I=1/2$ in TR (1) consists of 16 complex components, which are not independent. Decompositions over POs are used to study structure of the reducible tensor $\Psi^{SMm}_{m'}(\vec{p},\vec{q})$. Under the assumption that nuclear forces are time reversal invariant, the WF is expressed in terms of 16 real scalar functions $\psi_\lambda(|\vec{p}|,|\vec{q}|,\hat{\vec{p}}\cdot\hat{\vec{q}})$. First eight of them produce parity–even components of the WF. Ones with $\lambda = 9,\ldots,16$ appear when nuclear forces are parity non–conserving. In the obtained representation for the WF the contributions with definite values of total angular orbital momenta are singled out.

The TR for the WF is transformed into the OFs. In one of the OFs the spin–angular structure of the WF is generated by operators $u_\lambda(\hat{\vec{p}},\hat{\vec{q}})$, $(\lambda = 1,\ldots,16)$, applied at the spin state $|S = M = 0\rangle_{23}\,|1/2\,m'\rangle_1$ with zero total spin S in the pair of identical nucleons. The unit vectors $\hat{\vec{p}},\hat{\vec{q}}$ and the nucleon spin operators are used to build u_λ, that are scalars with respect to rotations in space of the Jacobi momenta. The operators u_λ and functions ψ_λ are superpositions of the respective quantities in the Gerjuoy–Schwinger representation [1, 3] for the WF.

The WF can be also written as the operator $\hat{\vec{\mathrm{X}}}(\vec{p},\vec{q})$ acting on the state $|S{=}1;\,M\rangle_{23}\,|1/2\,m'\rangle_1$. Under the rotations operator $\hat{\vec{\mathrm{X}}}$ transforms like a vector. Non–uniqueness of this alternative OF is related to existence of several ways to get contributions from the term

$$\psi_{\lambda=1}\,|S = M = 0\rangle_{23}\,|1/2\,m'\rangle_1$$

to ${}^{2\Sigma+1}\Lambda_I = {}^2S_{1/2}$ component of the WF.

The deuteron state vector, which has total angular momentum $J = 1$, can be obtained from the NN spin states $|S\,M\rangle_{23}$ with total spin either S=0 or S=1. The OFs for the deuteron WF are equivalent to partial–wave decompositions [9–12]. The derived OFs differ from Rarita–Schwinger representation [13,14].

In papers [1,3] and the present report the isospin formalism is not employed. TR (1) is convenient [4] to derive an OF for the WF taking into account isospin degrees of freedom. Explicitly antisymmetric operator representation for the WF within the isospin formalism can be constructed from decomposition $|\Psi\rangle=(1-(1,2)-(1,3))|\Psi^{(1)}\rangle$, where an OF is used for component $|\Psi^{(1)}\rangle$. Transposition of nucleon quantum numbers in momentum, spin and isospin space is denoted by (i,j). Being represented in one of the above discussed OF, vector $|\Psi^{(1)}\rangle$ meets the requirement $(2,3)|\Psi^{(1)}\rangle = -|\Psi^{(1)}\rangle$, which provides antisymmetrization of $|\Psi\rangle$.

Other approach to construct evidently antisymmetric WF is widely known [15–20]. The WF can be built in terms of functions $\Psi^{[\nu]}(\vec{p},\vec{q})$, ($\nu$ = symmetric, antisymmetric, and mixed), spin, and isospin states that belong to irreducible representations of the symmetric group S_3. The corresponding expressions for parity-conserving nuclear forces remain unaltered when effects of parity violation are incorporated, while functions $\Psi^{[\nu]}$ are to be modified with the aim to include P-odd contributions.

Both TR and OFs for the WFs are convenient for analysis of quantitative features of the matrix elements involving the WF. Influence of parity–violating nuclear forces on the spin–dependent momentum distributions of nucleons in polarized 3N nuclei will be discussed in a forthcoming paper. Within the approaches elaborated, TR and OFs can be derived for ^{4}He WF and related to the partial-wave decompositions [21] used in solution of the Faddeev-Yakubovsky equations.

REFERENCES

1. E. Gerjuoy, J. Schwinger. On tensor forces and the theory of light nuclei //*Phys. Rev.* 1942, v. 61, p. 138-146.
2. W. Glöckle. *The Quantum Mechanical Few-Body Problem.* Berlin-Heidelberg-New York: “Springer”, 1983, 283 p.
3. I. Fachruddin, W. Glöckle, Ch. Elster, A. Nogga. Operator form of 3H (3He) and its spin structure //*Phys. Rev. C.* 2004, v. 69, 064002(1-16).
4. V.V. Kotlyar, J. Jourdan. Spin structure of three-nucleon bound states //*Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations"* (45). 2005, N6, p. 24-29.
5. V.V. Kotlyar, A.V. Shebeko. Nucleon-nucleon interaction and meson exchange current effects in 3He two-body breakup by polarized photons //*Zeitschrift für Physik A.* 1987, v. 327, p. 301-309.
6. D.A. Varshalovich, A.N. Moskalev, V.K. Khersonskii. *Quantum Theory of Angular Momentum*. Singapore: “World Scientific”, 1989, 439 p.
7. G.G. Ohlsen. Polarization transfer and spin correlation experiments in nuclear physics //*Rep. Mod. Phys.* 1972, v. 35, p. 717-801.
8. O.F. Nemets, A.M. Yasnogorodsky. *Polarization researches in nuclear physics.* Kiev: “Naukova dumka”, 1980, 352 p. (in Russian).
9. H. Arenhövel. Electroweak processes in few-nucleon systems //*Few Body Syst.* 1999, v. 26, p. 43-98.
10. C.-P. Liu, G. Prezeau, M.J. Ramsey-Musolf. Hadronic parity violation and inelastic electron-deuteron scattering //*Phys. Rev. C.* 2003, v. 67, 035501(1-13).
11. C.P. Liu, C.H. Hyun, B. Desplanques. Deuteron anapole moment with heavy mesons //*Phys. Rev. C.* 2003, v. 68, 045501(1-12).
12. R. Schiavilla, J. Carlson, M.W. Paris. Parity violating interaction effects in the np system //*Phys. Rev. C.* 2004, v. 70, 044007(1-25).
13. W. Rarita, J. Schwinger. On the neutron-proton interaction //*Phys. Rev.* 1941, v. 59, p. 436-452.
14. I. Fachruddin, Ch. Elster, W. Glöckle. New forms of deuteron equations and wave function re–presentations //*Phys. Rev. C.* 2001, v. 63, 054003 (1-14).
15. M. Verde. On the elastic scattering of neutrons by deuterons //*Helv. Phys. Acta.* 1949, v. XXII, p. 339-360.
16. G. Derrick, J.M. Blatt. Classification of triton wave functions //*Nucl. Phys.* 1958. v. 8, p. 310-324.
17. V.F. Kharchenko. On the problem of the scattering of a nucleon by the bound state of two others //*Ukrainian Journal of Physics*. 1962, v. 7, p. 573-581 (in Ukrainian).

18. A.G. Sitenko, V.F. Kharchenko. On the binding and scattering of the three-nucleon system //*Nucl. Phys*. 1963, v. 49, p. 15-28.
19. A.G. Sitenko, V.F. Kharchenko. Bound states and scattering in a system of three particles //*Sov. Phys. Uspekhi*. 1971, v. 14, p. 125-153.
20. B. Blankleider, R.M. Woloshyn. Quasielastic scattering of polarized electrons on polarized 3He //*Phys. Rev. C.* 1984, v. 29, p. 538-552.
21. A. Nogga, H. Kamada, W. Glöckle, B.R. Barrett. The α particle based on modern nuclear forces //*Phys. Rev. C.* 2002, v. 65, 054003(1-18).

НАРУШЕНИЕ ПРОСТРАНСТВЕННОЙ ЧЕТНОСТИ В СВЯЗАННЫХ СОСТОЯНИЯХ СИСТЕМЫ ТРЕХ НУКЛОНОВ

В. Котляр, А. Ногга

Исследуются тензорное и операторное представления для волновой функции связанного состояния трех нуклонов. Предполагается, что пространственная четность не сохраняется во взаимодействии между нуклонами. Волновые функции ядер ^{3}He и ^{3}H выражены через 16 скалярных функций, которые зависят от величин относительных моментов и угла между ними. Функции вещественные, когда ядерные силы являются инвариантными относительно обращения времени. Проведено сравнение операторных форм для волновых функций дейтрона и 3N-ядер.

ПОРУШЕННЯ ПРОСТОРОВОЇ ПАРНОСТІ В ЗВ'ЯЗАНИХ СТАНАХ СИСТЕМИ ТРЬОХ НУКЛОНІВ

В. Котляр, А. Ногга

Досліджуються тензорне та операторне представлення для хвильової функції зв'язаного стану трьох нуклонів. Вважається, що просторова парність не зберігається при взаємодії між нуклонами. Хвильові функції ядер ^{3}He і ^{3}H виражаються через 16 скалярних функцій, які залежать від величин відносних моментів та кута поміж ними. Функції є дійсними, коли ядерні сили інваріантні відносно обернення часу. Проведено порівняння операторних форм для хвильових функцій дейтрона та 3N-ядер.