QUANTUM THEORY OF THE VAVILOV-CHERENKOV RADIATION FOR A PARTICLE WITH ARBITRARY SPIN

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We consider the Vavilov-Cherenkov radiation by a particle with arbitrary spin using the relativistic quantum treatment of the one-photon Cherenkov emission. The probability of the one-photon emission by relativistic particle is calculated in the framework of quantum electrodynamics using the covariant parameterization of electromagnetic current for particle with arbitrary spin.

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1. INTRODUCTION

The theory for the Vavilov-Cherenkov radiation (VCR) of electrical and magnetic multipoles has a long history. The VCR radiation of a magnetic dipole and of electric and magnetic dipoles was first considered by Ginzburg (1940) and Frank (1942) [1,2]. In 1952 appeared two other publications by Ginzburg and Frank on the same subject [3,4], and they were followed on by their publications [5,6] in 1984. In detail this subject was considered in the books by Ginzburg and Frank [7,8] (Ginzburg regarded electric and magnetic dipoles only, Frank considered VCR of arbitrary electrical and magnetic multipoles). Note also that the exact electromagnetic fields and the VCR of electric, magnetic and toroidal dipoles moving uniformly in unbounded non-dispersive medium was considered in [9,10].

To calculate the probability of VCR for particle of arbitrary spin S we take a relativistic expression for electromagnetic current through "physical" form factors $Q_l(q^2)$ and $M_l(q^2)$ [11-13]. The expression for electromagnetic current mentioned above is based essentially on using Chebyshev polynomials of a discrete variable [14,15].

2. THE COVARIANT PARAMETERIZATION OF ELECTROMAGNETIC CURRENT FOR PARTICLE WITH ARBITRARY SPIN

The electrical and magnetic multipole momenta are proportional to completely symmetrical traceless "multipole" tensors $S_{i_1 i_2 \ldots i_l}$ ($l \le 2S$):

$$Q_{i_1 i_2 \ldots i_l} = Q_l \frac{(2l)!(2S-l)!}{(2S)!(l!)^2} S_{i_1 i_2 \ldots i_l}; \tag{1}$$

$$M_{i_1 i_2 \ldots i_l} = M_l \frac{(2l)!(2S-l)!}{(2S)!(l!)^2} S_{i_1 i_2 \ldots i_l}, \tag{2}$$

where Q_l and M_l are equal to averaged quantities of $Q_{z \ldots z}$ and $M_{z \ldots z}$ in state with spin S projection on z axis. Tensors $S_{i_1 i_2 \ldots i_l}$ may be unambiguously expressed through spin operators S_i

$$S_i S_k - S_k S_i = i\varepsilon_{ikl} S_l, \; S_i S_i = S(S+1)I \tag{3}$$

up to factor, that we fix by the next condition: the construction $S_{i_1 i_2 \ldots i_l} q_{i_1} q_{i_2} \ldots q_{i_l}$ represents i-power polynomial of $(\vec{S}\vec{q})$ with the unit coefficient by $(\vec{S}\vec{q})^l$, $\vec{q}$ is arbitrary vector.

It is known [13], that

$$\frac{(2l)!}{l!} S_{i_1 i_2 \ldots i_l} a_{i_1} a_{i_2} \ldots a_{i_l} = \varphi_l(\vec{S}\vec{a}), \tag{4}$$

where a_i is unit vector, $\varphi_l(x)$ are Chebyshev polynomials.

Acting by the polynomial $\varphi_l(\vec{S}\vec{a})$ on the wave function of a particle with spin S and its projection m on direction $\vec{a}$, for eigenvalue we obtain Chebyshev polynomial $\varphi_l(m)$ of discrete variable m:

$$\varphi_l(\vec{S}\vec{a}) v(S,m) = \varphi_l(m) v(S,m). \tag{5}$$

Polynomials are different for different values of spin S, i.e. by $\varphi_l(m)$ we always mean $\varphi_l(m,S)$.

Note, that Chebyshev polynomials $\varphi_l(m)$ are not so popular as famous Chebyshev polynomials $T_n(x)$ and $U_n(x)$. In view of it let's consider some properties of polynomials $\varphi_l(m)$.

A notation $\varphi_l(m)$ was introduced by Chebyshev [14], but in modern mathematical literature [15] instead of them a little different polynomials are used:

$$t_l(S+m) = \tfrac{1}{l!} \varphi_l(m). \tag{6}$$

For some purposes polynomials $p_l(m)$ are more convenient,

$$p_l(S+m) = \frac{1}{l!} \sqrt{\frac{(2S-l)!(2l+1)}{(2S+l+1)!}} \varphi_l(m). \tag{7}$$

Recurrence expressions for Chebyshev polynomials are:

$$\varphi_{l+1}(m) = (2l+1)2m\varphi_l(m) - l^2\left[(2S+1)^2 - l^2\right]\varphi_{l-1}(m).$$

Orthogonality relations are:

$$\sum_{n=-S}^{S} p_l(m)\, p_{l'}(m) = \delta_{ll'};$$

$$\sum_{l=0}^{2S} p_l(m)\, p_l(m') = \delta_{mm'}.$$

The trace of two Chebyshev polynomials product is

$$Sp\left[p_l(\vec{S}\vec{a})\, p_{l'}(\vec{S}\vec{b})\right] = \delta_{ll'} P_l(\vec{a}\vec{b}), \tag{8}$$

where

$$P_l(x) = \frac{1}{2^l\, l!}\frac{d^l}{dx^l}\left(x^2-1\right)^l$$

is a Legendre polynomial. The relation with Klebsch-Gordan coefficients is

$$p_l(S,m) = (-1)^{S-m}\langle SSm-m\,|\,l0\rangle, \tag{9}$$

where

$$\langle j_2 j_1 m_2 m_1\,|\,jm\rangle \equiv \langle j_2 j_1 m_2 m_1\,|\,j_2 j_1 jm\rangle \tag{10}$$

are Klebsch-Gordan coefficients, that defined so as, for example, in [16].

First six Chebyshev polynomials have next explicit form:

$$\varphi_0(m) = 1;$$

$$\varphi_1(m) = 2m;$$

$$\varphi_2(m) = 12m^2 - \left[(2S+1)^2 - 1\right];$$

$$\varphi_3(m) = 120m^3 - 6\left[3(2S+1)^2 - 7\right]m;$$

$$\varphi_4(m) = 1680m^4 - 120\left[3(2S+1)^2 - 13\right]m^2 + 9\left[(2S+1)^2 - 1\right]\left[(2S+1)^2 - 9\right];$$

$$\varphi_5(m) = 30240m^5 - 8400\left[(2S+1)^2 - 7\right]m^3 + 30\left[15(2S+1)^4 - 230(2S+1)^2 + 407\right]m.$$

Now we can write an expression for matrix elements of electromagnetic current in the case when a particle is described by Bargman-Wigner equations [13]:

$$(2\pi)^3\sqrt{4E_1E_2}\langle p_2|j_\mu|p_1\rangle = \frac{1}{[-(p_2+p_1)]^S} \times \sum_l (i)^l \frac{(2S-l)!}{(2S)!(2l)!l!} 2^l q^l (\bar{u}_2(p_2)\{Q_l(q^2)(p_2+p_1)_\mu$$
$$\times \varphi_l\left(\frac{S_\rho q_\rho}{q}\right) + \frac{i}{l}\frac{\sqrt{-(p_2+p_1)^2}}{2m} M_l(q^2)\varepsilon_{\mu\nu\rho\sigma}(p_2+p_1)_\nu \times q_\rho \frac{\partial}{\partial q_\sigma}\varphi_l\left(\frac{S_\rho q_\rho}{q}\right)\}(u_1(p_1)), \tag{11}$$

where $q = p_2 - p_1$,

$$S_\rho = \frac{1}{2i}\varepsilon_{\rho\mu\nu\sigma}S_{\mu\nu}\frac{(p_2+p_1)_\sigma}{\sqrt{-(p_2+p_1)^2}}; \tag{12}$$

$$S_{\mu\nu} = \sigma_{\mu\nu}^{(1)} + \sigma_{\mu\nu}^{(2)} + \ldots + \sigma_{\mu\nu}^{(2S)}; \tag{13}$$

$$\sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{4i} \tag{14}$$

overhead index shows on which index of spin-tensor $u_1(p_1)$ $(u_{1\alpha_1\alpha_2\ldots\alpha_{2S}}(p_1))$ or conjugate spin-tensor $\bar{u}_2(p_2)$ $(\bar{u}_2^{\alpha_1\alpha_2\ldots\alpha_{2S}}(p_2))$ acts matrix $\sigma_{\mu\nu}$. $u_1(p_1)$ and $\bar{u}_2(p_2)$ are normalized by condition:

$$(\bar{u}(p)u(p)) \equiv \bar{u}^{\alpha_1\alpha_2\ldots\alpha_{2S}}(p)u_{\alpha_1\alpha_2\ldots\alpha_{2S}}(p) = (2m)^{2S}.$$

Spin-tensor $u(p)$ satisfies Bargman-Wigner equations:

$$(ip_\mu\gamma_\mu + m)^{\alpha_i} u_{\alpha_1\alpha_2\ldots\alpha_i\ldots\alpha_{2S}} = 0. \tag{15}$$

$Q_l(q^2)$ and $M_l(q^2)$ in (11) are "physical" form factors, that are electric and magnetic multipole moments of the particle in the Breit system of reference [11,12].

3. VAVILOV-CHERENKOV RADIATION FOR PARTICLE WITH ARBITRARY SPIN

The calculated probability of the photon radiation by particle with arbitrary spin per unit of length l and per frequency unit ω is:

$$\frac{d^2w}{dl\, d\omega} = \frac{e^2}{\hbar c^2}\{[E(q^2) + M(q^2)]\sin^2\theta + \frac{c^4}{V_1^2E_1^2}(4m^2c^2 + q^2)M(q^2)\}; \tag{16}$$

$$q^2 = (n^2-1)\left(\frac{\hbar\omega}{c}\right)^2,$$

where V_1 and E_1 are the initial particle velocity and energy, θ is the angle of photon emission relative to direction of motion of the initial particle, n is the medium refractive index,

$$e^2E(q^2) = \sum_{l=0}^{2S} A_l q^{2l} Q_l^2(q^2);$$
$$e^2M(q^2) = \sum_{l=1}^{2S}\frac{l+1}{l}A_l q^{2l} M_l^2(q^2); \tag{17}$$

$$A_l = \frac{2^{2l}}{[(2S)!(2l)!]^2}\frac{(2S+l+1)!(2S-l)!}{(2S+1)(2l+1)}. \tag{18}$$

4. DISCUSSION

The theory developed by Frank [6] is the theory for the Vavilov-Cherenkov radiation of arbitrary *classical* multipoles. We investigated the Vavilov-Cherenkov radiation of arbitrary *quantum* multipoles. The quantum multipoles are the particles with arbitrary spin S and with electrical and magnetic multipole momenta that are described by some form factors. This paper is intrinsically connected with our papers [17,18], where the probability of the one-photon emission by relativistic electron was calculated in the framework of quantum electrodynamics and the behavior of the radiation intensities for large energy–momentum transfer was analyzed. It is known that "physical" form factors for electron are [11]

$$Q_0(q^2) = M_1(q^2) = e\left[1 + \frac{q^2}{4m^2}\right]^{-\frac{1}{2}}.$$

Putting these form factors in Eq. (16) gives the result obtained and analyzed in [17].

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КВАНТОВАЯ ТЕОРИЯ ИЗЛУЧЕНИЯ ВАВИЛОВА-ЧЕРЕНКОВА ЧАСТИЦЕЙ С ПРОИЗВОЛЬНЫМ СПИНОМ

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Рассмотрено излучение Вавилова-Черенкова частицей с произвольным спином. Вероятность излучения одного фотона релятивистской частицей рассчитана в рамках квантовой электродинамики с использованием ковариантной параметризации матричных элементов электромагнитного тока частицы с произвольным спином.

КВАНТОВА ТЕОРІЯ ВИПРОМІНЮВАННЯ ВАВІЛОВА-ЧЕРЕНКОВА ЧАСТИНКОЮ З ДОВІЛЬНИМ СПІНОМ

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Розглянуте випромінювання Вавілова-Черенкова частинкою з довільним спіном. Імовірність випромінювання одного фотона релятивістською частинкою обчислена в рамках квантової електродинаміки з використанням коваріантної параметризації матричних елементів електромагнітного струму частинки з довільним спіном.