RADIATIVE CORRECTIONS IN HIGH ENERGY COMPTON SCATTERING

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Radiative corrections (RC) to the differential and total Compton scattering cross sections are calculated. We consider the scattering of photon with a few GeV energy by the atomic electron. The reason is the unique possibility of precise measurement of respective cross sections in the PrimEx experiment (Jefferson Lab., CEBAF), where the high-precision value of $\pi^0 \to 2\gamma$ width was measured, and where this process used in normalization goals. RC include the contributions from one-loop diagrams as well the effects due to additional real photon emission. The FORTRAN CODE for calculation of elastic and inelastic cross sections which takes into account different restrictions on event selection is created. Some underlying semi-analytical formulas are given.

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1. INTRODUCTION

The PrimEx Collaboration at Jefferson Laboratory in the United States is performing a high precision test of a key prediction of the quantum chromodynamics, a theory that describes matter on a fundamental level. In the testing experiment the π^0-meson lifetime will be measured by means of the small-angle coherent photo-production of the π^0 in the Coulomb field of a nucleus, i. e., via the Primakoff effect.

This experiment, in which the decay width of the $\pi^0 \to 2\gamma$ will be determined at the 1.5% level, has the highest possible scientific rating at the laboratory (an A rating).

The peak value of the Primakoff cross section has a strong energy dependence, proportional to the four power of the incoming photon energy, and hence it is very critical to have a precise knowledge of the absolute energy of the photon beam in this experiment.

Currently, the absolute energy determination of the photon tagging facility, to be used for the PrimEx experiment, is known at the ~ 1% level. Developing an independent method to determine the energy of the tagged photon beam is very important. The scattering of photons on the atomic electrons (the Compton scattering) can provide an unique way to measure the energy value of the tagged photon beam with needed accuracy.

At present, there is the well-known formula of Nishina-Klein-Tamm for the Compton scattering cross-section on a free electron, which is supplemented with first-order radiative corrections (RC) [2]. These corrections include the one-loop contributions and the effect of soft-photon emission, which does not affect the two-body kinematics. As was shown in Ref. [3], the radiative tail caused by the double Compton scattering may give a considerable contribution on the level of the RC.

In this connection, our tasks for the Compton scattering calculations are to check the existing analytical results for the single and double Compton scattering, and to adopt them to the PrimEx kinematics.

2. BORN CROSS-SECTION AND RC

Cross section for Compton scattering in the Born approximation is represented by the formula of Nishina-Klein-Tamm

$$\frac{d\sigma_0}{d\cos\theta_1} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega_1}{\omega_0}\right)^2 U_0, \tag{1}$$

with

$$U_0 = \left(\frac{\omega_1}{\omega_0} + \frac{\omega_0}{\omega_1} - \sin\theta_1{}^2\right), \tag{2}$$

where ω_0, ω_1 are energies of the initial and scattered photons, θ_1 is the angle between momenta of the ingoing and scattered photons,

$$\omega_1 = \frac{\omega_0}{1 + \frac{\omega_0}{m}(1-\cos\theta_1)}. \tag{3}$$

The RC to the Born cross section (Eq. (1)) induced by one-loop diagrams have been evaluated by Brown and Feynman [1], according to whom the Compton scattering cross section accounting for this virtual corrections reads

$$d\sigma_0(1+\delta^c_{vir}), \tag{4}$$

where

$$\delta^c_{vir} = -\frac{\alpha}{\pi U_0} ReU^c_{vir}(\chi_1,\chi_2), \tag{5}$$

$$U^c_{vir}(\chi_1,\chi_2) = P(\chi_1,\chi_2) + P(\chi_2,\chi_1).$$

and in the laboratory system

$$\chi_1 = -\frac{2\omega_0}{m}, \quad \chi_2 = \frac{2\omega_1}{m}.$$

For the function $P(\chi_1,\chi_2)$ see APPENDIX.

In order to take into account the total first order RC it is necessary to include also the contribution caused by the radiation of the two photons. For the case in which the energy of one of the emitted photon is much smaller than the electron mass

$\omega_2 \le \omega_{2max} << m$,

the cross section $d\sigma_{real,soft}$ was obtained by Brown and Feynman and in laboratory system $d\sigma_{real,soft}$ is given by the Eq. (39) of Ref. [1]

$$d\sigma_{real,soft} = -\frac{\alpha}{\pi} d\sigma_0 \Big[4y \tanh(2y)^{-1}[h(2y)-1]$$

$$+2(1-2y\tanh(2y)^{-1}\left(\ln\frac{2\omega_{2max}}{\lambda}-\frac{1}{2}\right)\Big], \quad (6)$$

where

$$h(y) = \frac{1}{y}\int_0^y udu \tanh(u)^{-1},\ \ 2\sinh^2(y) = \frac{\omega_0-\omega_1}{m}.$$

The differential cross section for the case of double hard photon emitted with four-momenta k_1 and k_2 has been calculated by Mandle and Skyrme [4]

$$d\sigma_{real} = \alpha r_0^2 \frac{d\Omega_1 d\Omega_2}{(4\pi)^2} \frac{\omega_1\omega_2 d\omega_1}{m\omega_0} \frac{X}{T_1}, \quad (7)$$

with

$$T_1 = m + \omega_0(1-\cos\theta_2) - \omega_1(1-\cos\theta_{12}).$$

The quantity X is defined by Eqs. (8.3) and (8.4) of Ref.[4] as a function of invariant quantities

$$a_1 = -p_1k_1,\ a_2 = -p_1k_2,\ a_3 = p_1k_0,$$
$$b_1 = p_2k_1,\ b_2 = p_2k_2,\ b_3 = -p_2k_0.$$

The conservation law $p_1 + k_0 = p_2 + k_1 + k_2$ can be used to express $d\sigma_{real}$ in terms of variables $\omega_1, \theta_1, \theta_2$ and ϕ (see Fig. 1) in laboratory system.

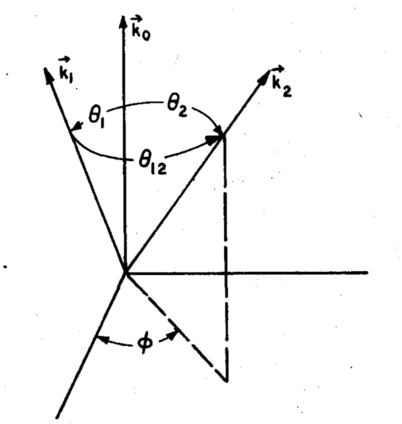

Fig. 1. *Relative directions in double Compton scattering*

In terms of this variables

$$\cos\theta_{12} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\phi$$

and

$$\omega_2 = \frac{L}{T_1},\quad L = m\omega_0 - \omega_1[m + \omega_0(1-\cos\theta_1)]. \quad (8)$$

In the sum the soft- and virtual-photon corrections, the infrared divergent term associated with the "photon mass" λ disappears, but instead the logarithmic dependence on the auxiliary parameter $\omega_{2\max}$ appears.

3. RADIATIVE TAIL

Consider now the Compton scattering in which one has to record photons emitted into the given angle θ_1. The energy of the emitted photon in the single Compton scattering process $\omega_1 = \omega_s$ is given by

$$\omega_1 \to \omega_s = \frac{\omega_0}{1+\frac{\omega_0}{m}(1-\cos\theta_1)}.$$

If the calorimeter has the energy resolution ΔE, then photons from both single- and double Compton scattering processes are selected and

$$\omega_s - \Delta E < \omega_1 < \omega_s.$$

If $\Delta E << m$, then one can use the soft photon approximation to compute the contribution due to double Compton scattering. In this case it needs simply to change $\omega_{2\max}$ by ΔE in the sum of soft and virtual corrections. Otherwise, we have to include to RC the double Compton cross section integrated over ω_1 from $\omega_s - \Delta E$ to upper limit defined by condition $\omega_2 > \omega_{2max}$.

$$d\sigma_{real,hard} = \int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\omega_s-\Delta E}^{\omega_{1max}} d\omega_1 \frac{d\sigma_{real}}{d\Omega_2 d\omega_1}, \quad (9)$$

where the inequality

$$\omega_2 = \frac{\omega_0(\omega_s-\omega_1)}{\omega_s[1+\frac{\omega_0}{m}(1-\cos\theta_2)-\frac{\omega_1}{m}(1-\cos\theta_{12})]} \ge \omega_{2max}$$

defines

$$\omega_{1max} = \omega_s - \omega_{2max} f_{12},\qquad f_{12} = \quad (10)$$

$$\frac{\left(\frac{\omega_0}{m}(1-\cos(\theta_1))+1\right)\left(\frac{\omega_0}{m}(1-\cos(\theta_2))+1\right)-\frac{\omega_0}{m}(1-\cos\theta_{12})}{\left(\frac{\omega_0}{m}(1-\cos(\theta_1))+1\right)\left(\frac{\omega_0}{m}(1-\cos(\theta_1))-\frac{\omega_{2max}}{m}(1-\cos\theta_{12})+1\right)}.$$

The differential cross section $d\sigma_{real}/d\Omega_2 d\omega_1$ in the Eq. (9) consists of the two parts. One of them has the logarithmic singularity, when $\omega_1 \to \omega_s$. So,

$$\frac{d\sigma_{real}}{d\Omega_2 d\omega_1} = f_1(\omega_1,\theta_1,\theta_2,\varphi) + \frac{f_2(\omega_1,\theta_1,\theta_2,\varphi)}{\omega_s-\omega_1}. \quad (11)$$

Let us rewrite Eq. (11) in the following form

$$\frac{d\sigma_{real}}{d\Omega_2 d\omega_1} = f_1(\omega_1,\theta_1,\theta_2,\varphi) +$$

$$\frac{f_2(\omega_1,\theta_1,\theta_2,\varphi)-f_2(\omega_s,\theta_1,\theta_2,\varphi)}{\omega_s-\omega_1} + \frac{f_2(\omega_s,\theta_1,\theta_2,\varphi)}{\omega_s-\omega_1}. \quad (12)$$

The integration of the last term in the Eq. (12) over ω_1 gives

$$\int_{\omega_s-\Delta E}^{\omega_{1max}} \frac{f_2(\omega_s,\theta_1,\theta_2,\varphi)}{\omega_s-\omega_1} d\omega_1 = f_2(\omega_s,\theta_1,\theta_2,\varphi)\ln\frac{\omega_{2max}f_{12}}{\Delta E} \quad (13)$$

The structure of function f_2 allows the simple analytical integration with respect to θ_2,φ, and after such integration the term with infrared parameter $\omega_{2\max}$ cancels the corresponding term in the soft-photon double Compton cross section. The rest is

$$d\sigma_{real,hard} = \int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\omega_s-\Delta E}^{\omega_s} d\omega_1 (f_1(\omega_1,\theta_1,\theta_2,\varphi)$$
$$+\frac{f_2(\omega_1,\theta_1,\theta_2,\varphi)-f_2(\omega_s,\theta_1,\theta_2,\varphi)}{\omega_s-\omega_1}) \quad (14)$$
$$+\int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi (f_2(\omega_s,\theta_1,\theta_2,\varphi)\ln\frac{f_{12}m}{\Delta E}).$$

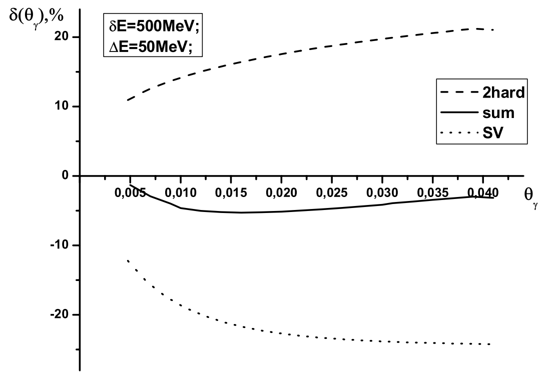

***Fig. 2**. $\theta_1(rad)$ -dependence of the* $\delta^{SV} = \frac{d\sigma^{SV}/d\theta_1}{d\sigma_0/d\theta_1}$, $\delta^{2hard} = \frac{d\sigma^{2hard}/d\theta_1}{d\sigma_0/d\theta_1}$, $\delta^{sum} = \frac{d\sigma^{sum}/d\theta_1}{d\sigma_0/d\theta_1}$ *at* $\omega_0 = 5$ GeV

The right-hand side of Eq. (14) does not contain any singularities and can be integrated numerically.

If in experiment the energy threshold of the recorded photon is δE, we need to change the limits of the integration in Eq. (14). Thus, for $0 \le \theta_1 \le \theta_{1max1}$ the limits of integration are

$$\int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\omega_s-\Delta E}^{\omega_s} d\omega_1$$

and for $\theta_{1max1} \le \theta_1 \le \theta_{1max2}$

$$\int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\delta E}^{\omega_s} d\omega_1,$$

where

$$\theta_{1max1} = \arccos\frac{(\Delta E+\delta E)(\omega_0+m)-\omega_0 m}{(\Delta E+\delta E)\omega_0};$$

$$\theta_{1max2} = \arccos(1-\frac{m}{\delta E}+\frac{m}{\omega_0}).$$

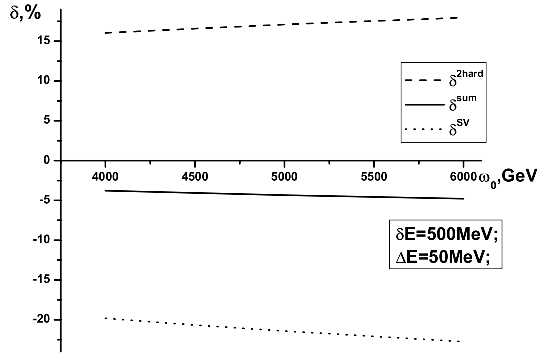

***Fig. 3**. Dependence the integrated over* θ_1 *values* δ^{SV}, δ^{2hard}, δ^{sum} *on ingoing photon energy* ω_0

To improve the accuracy of numerical integration we use variables t_1, t_2 instead of $\cos\theta_1, \cos\theta_2$, namely

$$\cos\theta_1 = \frac{1}{\beta}\tanh(\beta t_1),\ \cos\theta_2 = \frac{1}{\beta}\tanh(\beta t_2),$$

where $\beta = \sqrt{1+\frac{2m}{\omega_0}}$.

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$$\cos\theta_1 = \frac{1}{\beta}\tanh(\beta t_1),\ \cos\theta_2 = \frac{1}{\beta}\tanh(\beta t_2),$$

where $\beta = \sqrt{1+\frac{2m}{\omega_0}}$.

The results of the numerical calculations are shown in Figs. 2, 3. On these pictures σ^{SV} is the part of the cross-section caused by radiation of virtual and soft photons, σ^{2hard} — due to radiation of two hard photons, σ^{sum} is their sum.

In the real experiment the angular position of the recorded photon has the uncertainty $\Delta\theta$ that brings the additional uncertainty to its energy

$$\Delta E = \frac{d\omega_1}{d\theta_1}\Delta\theta = \frac{\omega_s^2 \sin\theta_1}{m}\Delta\theta. \quad (15)$$

In situation when there are both ΔE and $\Delta\theta$ we must replace

$$\Delta E \to \Delta E + \frac{\omega_s^2 \sin\theta_1}{m}\Delta\theta \quad (16)$$

in the respective above formulae.

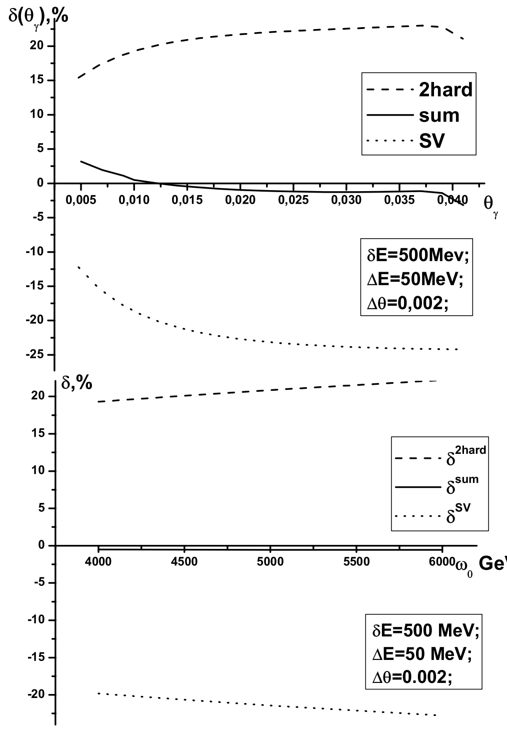

***Fig. 4**. The differential functions* δ *(upper picture) and integrated ones (low picture) if the energy (*ΔE*) and angular (*$\Delta\theta$*) uncertainties of detected photon are taken into account*

Results of numerical calculation in this case are shown in Fig. 4. If we detect the photon and electron energies in the single Compton scattering kinematics, both with accuracy ΔE, the energy of the second photon in the double Compton scattering must be restricted by

$$\omega_2 \le 2\Delta E. \quad (17)$$

The respective results are shown in Fig. 5.

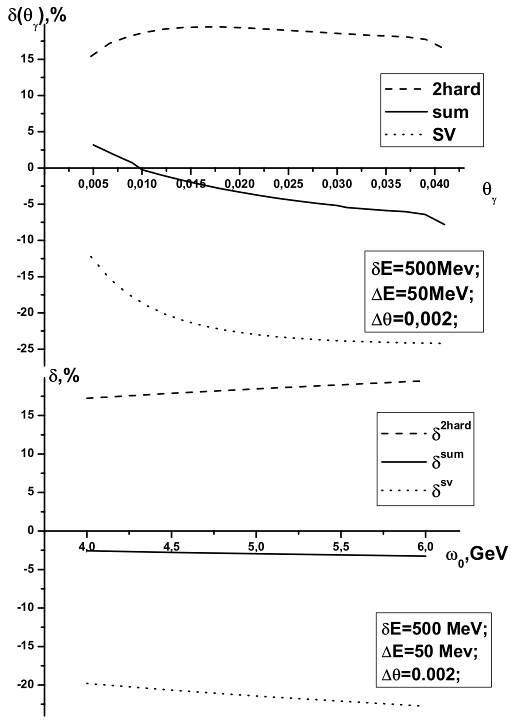

Fig. 5. The same as in Fig. 4, but with additional restriction due to inequality (17)

4. TOTAL CROSS SECTION FOR DOUBLE COMPTON EFFECT

In section 3 we calculated RC to the cross section at the single Compton scattering kinematics when some restrictions on event selection were applied. Here we will compute RC to the total cross section without any selection constraints.

In order to obtain the total cross for double Compton scattering in which both emitted photons are hard, we integrate the Eq. (7) over variables $\theta_1, \theta_2, \varphi, \omega_1$.

$$d\sigma^t_{real,hard} = \frac{1}{2}\alpha r_0^2 \frac{1}{8\pi}\int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\omega_{2max}}^{\omega_{1max}} d\omega_1 \frac{\omega_1\omega_2}{m\omega_0}\frac{X}{T_1}. \quad (18)$$

Before to perform the numerical integration in Eq. (18) one needs to separate the infrared singularities on both the low and upper limits of integration over variable ω_1. The respective terms are canceled with the infrared contribution from the soft photon emission. After such regularization we arrive at results shown in Fig. 6.

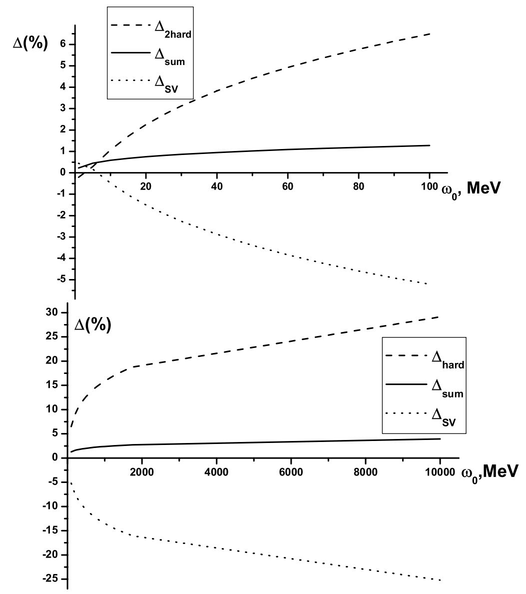

Fig. 6. *The dependence of functions Δ, $\Delta_{SV} = \sigma^{SV}/\sigma_0$, $\Delta_{hard} = \sigma^t_{real,hard}/\sigma_0$, $\Delta_{sum} = \sigma^{sum}/\sigma_0$, from ω_0 (here $\sigma^{sum} = \sigma^{SV} + \sigma^t_{real,hard}$)*

5. CONCLUSION

As one can see from the submitted results relative to the differential and integrated Compton scattering cross sections at the PrimEx kinematics, the accounting for the RC is very important and rather necessary for the description of respective events at the level of 1% accuracy. The main problem is the calculation of the positive contribution due to double hard photon emission, which essentially compensates the negative contribution caused by virtual and soft photon emission. It strongly depends on restrictions for the event selection. At the PrimEx kinematics values of the angles and energies the above mentioned contributions into RC are very large (up to 25% in absolute values) but opposite in sign. Thus, the total correction to the integrated cross section near the single Compton scattering kinematics (in this cases we use Eq. (9) for the double photon cross section) is negative whereas the corresponding correction to the total cross section (when any restriction on event selection is absent and Eq. (18) is applied) is positive. In both cases the absolute value of the total RC does not exceed 5% and slightly increases with the rise of the ingoing photon energy.

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APPENDIX

$$P(\chi_1,\chi_2)=(1-2y\coth(2y))\ln\left(\frac{\lambda}{m}\right)U_0-2U_0\coth(2y)(2h(y)-h(2y))y-4\left(\frac{1}{2}-\frac{1}{\chi_1}\right)\tanh(y)y$$

$$-\frac{4\left(\frac{2}{\chi_1}-\frac{3}{4}\frac{\chi_2^2}{\chi_1}-\frac{7}{4}\chi_1\right)y^2}{\chi_1+\chi_2}+4\left(\frac{1}{\chi_2}+\frac{1}{\chi_2}\right)^2+\frac{\frac{\chi_1}{\chi_2}+\frac{1}{2}}{\chi_1-1}+\frac{2\left(\frac{\chi_1^2}{\chi_2}+\frac{\chi_1}{\chi_2}+\chi_1+\frac{\chi_2}{2}-\frac{3}{\chi_2}-1+\frac{2}{\chi_1}+\frac{\chi_2}{\chi_1^2}\right)(F(\chi_1-1)-F(-1))}{\chi_1}$$

$$+\ln(|\chi_1|)(4y\coth(2y)(\frac{4\cosh^2(y)}{\chi_1\chi_2}+\frac{(\chi_1-6)\mathrm{sech}(2y)}{2\chi_2}+\frac{4}{\chi_1^2}-\frac{1}{\chi_1}-\frac{\chi_2}{2\chi_1}-\frac{\chi_1}{\chi_2}-1)+\frac{3\chi_2}{2\chi_1}+\frac{3\chi_2}{2\chi_1^2}+\frac{8}{\chi_1}-\frac{8}{\chi_1^2}+\frac{3}{\chi_2}+1$$

$$-\frac{2\chi_1^2+\chi_2}{2(\chi_1-1)^2\chi_2}+\frac{2\chi_1-\chi_2\chi_1^2-\chi_2^2}{2(\chi_1-1)\chi_1^2\chi_2}-\frac{7}{\chi_1\chi_2})+h(y)\left(2y\coth(y)-\frac{4y(2-\cosh(2y))\sinh(2y)}{\chi_1\chi_2}\right)-\frac{12}{\chi_1}-\frac{3\chi_1}{2\chi_2}-\frac{2\chi_1}{\chi_2^2},$$

where

$$F(x)=-Li_2(-x).$$

РАДИАЦИОННЫЕ ПОПРАВКИ В ВЫСОКОЭНЕРГЕТИЧЕСКОМ КОМПТОНОВСКОМ РАССЕЯНИИ

М.И. Кончатный, Н.П. Меренков

Вычислены радиационные поправки (РП) к дифференциальному и полному сечениям комптоновского рассеяния. Мы рассматриваем рассеяние фотонов, имеющих энергии порядка нескольких гигаэлектронвольт, на атомных электронах. Такое рассмотрение обусловлено возможностью прецизионного измерения сечения процесса комптоновского рассеяния в эксперименте PrimEx (Лаборатория Джефферсона, CEBAF), в котором выполняется высокоточное измерение ширины распада $\pi^0\to 2\gamma$, и в котором этот процесс используется для нормировочных целей. РП включают в себя как вклады от однопетлевых диаграмм, так и вклад, обусловленный излучением дополнительного реального фотона. Созданы программы на языке ФОРТРАН для вычисления упругого и неупругого сечений с учетом различных ограничений на отбор событий. Приведены некоторые основные полуаналитические формулы.

РАДІАЦІЙНІ ПОПРАВКИ В ВИСОКОЕНЕРГЕТИЧНОМУ КОМПТОНІВСЬКОМУ РОЗСІЯННІ

М.І. Кончатний, М.П. Меренков

Обчислені радіаційні поправки (РП) до диференціального і повного перерізів комптонівського розсіювання. Ми розглядаємо розсіювання фотонів, що мають енергії порядку декількох гигаелектронвольт, на атомних електронах. Такий розгляд обумовлений можливістю прецизійного вимірювання перерізу процесу комптонівського розсіювання в експерименті PrimEx (Лабораторія Джефферсона, CEBAF), в якому виконується високоточний вимір ширини розпаду $\pi^0\to 2\gamma$ і в котрому цей процес використовується для нормувальних цілей. РП включають в себе як вклади від однопетлевих діаграм, так і вклад обумовлений випромінюванням додаткового реального фотона. Написані програми на мові ФОРТРАН для обчислення пружного і непружного перерізів з врахуванням різних обмеженостей на відбір подій. Приведені деякі основні напіваналітичні формули.