ABOUT NEUTRON ACCELERATION IN UNIFORM ELECTROMAGNETIC FIELDS

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It has been shown some time ago that a neutron undergoes some acceleration in uniform electromagnetic fields. We found an exact solution of Dirac wave equation for a neutron imbedded in a uniform electromagnetic field and our consideration of the problem doesn't confirm the neutron acceleration. We provide the simple explanation why the statements about neutron acceleration are incorrect.

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1. INTRODUCTION

The derivation of neutron acceleration first obtained by J.Anandan [1, 2] is very simple and convincing. The acceleration is a consequence of the well-known Hamiltonian introduced by J. Schwinger in 1948,

$$H = \frac{\vec{p}^2}{2m} - \vec{\mu}\cdot(\vec{B} - \frac{1}{mc}[\vec{p}\vec{E}]), \qquad (1)$$

where p and m refer to the momentum and mass of the neutron. The magnetic moment is $\vec{\mu} = \mu\vec{\sigma}$ with $\vec{\sigma}$ being Pauli spin matrices. It is assumed that the fields $\vec{E}$ and $\vec{B}$ are uniform and time independent.

The Hamiltonian (1) may be regarded as generally valid in both the classical and quantum mechanical cases. Upon calculating the corresponding commutators (or Poisson brackets) Anandan and Hagen in [2] obtain

$$\vec{V} = \frac{d\vec{X}}{dt} = \frac{\vec{p}}{m} + \frac{1}{mc}[\vec{E}\vec{\mu}], \qquad \frac{d\vec{\mu}}{dt} = [\vec{\mu}(\vec{B} - \frac{1}{mc}[\vec{p}\vec{E}])]. \qquad (2)$$

In the original paper [3] J. Anandan states that acceleration (2) is due to relativistic effects. A key to understanding the new force is that for this dipole the kinetic momentum (classically mV) differs from the canonical momentum p. The canonical momentum is conserved if there is translational symmetry in space, as when the dipole interacts with a uniform electromagnetic field.

It follows from (2) that (if we put $d\vec{p}/dt = 0$)

$$\frac{d\vec{V}}{dt} = \frac{2|\vec{\mu}|}{m\hbar c}[\vec{E}[\vec{\mu}\vec{B}']] \qquad (3)$$

where, $\vec{B}' = \vec{B} - \frac{1}{c}\left[\vec{V}\vec{E}\right]$ is the magnetic field in the rest frame of the neutron that is moving with velocity $\vec{V}$ relative to the laboratory $(V << c)$.

2. ON INCORRECTNESS OF SOME EQUATIONS OF NEUTRON MOTIONS IN ELECTROMAGNETIC FIELDS

Unfortunately the conclusion (3) about acceleration of a neutron in uniform electromagnetic fields is incorrect.

Concerning the equation (3) if $\left(\vec{E}\vec{B}\right) = 0$ one can go in the frame of reference that is moving with velocity $\vec{V}_0$ relative to the laboratory so as (if $B << E$, then $V_0 << c$)

$$\vec{B}_0' = \vec{B} - \left[\vec{V}_0\vec{E}\right] = 0\,. \qquad (4)$$

Then according to (3) acceleration of a neutron is zero. We have absurd situation when the particle acceleration in (3) are nonzero in one system of reference and are zero in another one. So the acceleration of a neutron in uniform electromagnetic fields does not exist.

Let's consider now movement of a neutron in an electromagnetic field more carefully. As classical movement of a particle we shall understand such limiting case of quantum-mechanical movement in which it is possible to consider the certain trajectory of a particle.

Frequently classical movement of a particle with spin ½ is described by the equations of following type:

$$\frac{d\vec{V}}{dt} = \vec{F}(\vec{x},t,\vec{V},\vec{\mu}), \qquad \frac{d\vec{\mu}}{dt} = \vec{G}(\vec{x},t,\vec{V},\vec{\mu}), \qquad (5)$$

where x is the coordinate of a particle at the moment of time t, $\frac{d}{dt}\vec{V} = 0$. $\vec{F}$ and $\vec{G}$ are some functions. So, for example, no relativistic movement of a neutral particle with the abnormal magnetic moment of $\vec{\mu}$ in electric field $\vec{E}$ is described by the following equations [4]:

$$\frac{d\vec{V}}{dt} = \frac{1}{mc}\frac{\partial}{\partial\vec{x}}\left[\vec{\mu}\left[\vec{E}\vec{V}\right]\right], \qquad \frac{d\vec{\mu}}{dt} = \frac{2\mu}{c\hbar}\left[\vec{\mu}\left[\vec{E}\vec{V}\right]\right]. \qquad (6)$$

We shall discuss the reasons specifying that the equation of type (5) cannot be the equation describing quantum-mechanical movement of a particle in a limiting case of classical movement.

We shall consider movement of the particle described by the equations (6) in central field that is we shall put in (6)

$$\vec{E} = E(r)\frac{\vec{r}}{r}. \tag{7}$$

If the spin vector of a particle is perpendicular to a plane of scattering then according to the equations (6) direction of a vector will remain constant. In this case the movement of the particle will appear as it shown on Fig. 1.

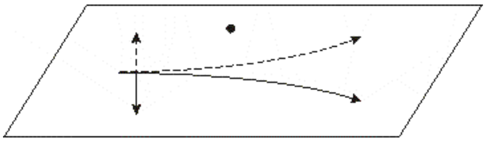

Fig. 1. *Movement of the particle with spin vector that is perpendicular to the plane of scattering*

If the spin vector of a particle locked in a plane of scattering, the vector of velocity of a particle will remain constant, and a spin vector will rotate, remaining in a plane of scattering. In this case the movement of a particle will appear as it shown on Fig. 2.

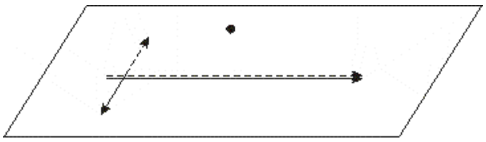

Fig. 2. *Movement of the particle with spin vector that is parallel to the plane of scattering*

Let's consider, further, scattering in the central field of no polarized beam of particles. If no polarized bunch to imagine as a mix of particles with opposite directed spins, orthogonal to a plane of scattering such beam will be split on two various beams (see Fig. 1). If no polarized beam to imagine as a mix of particles with opposite directed spins lying in a plane of scattering such beam will not be split (see Fig. 2). Thus, by scattering no polarized beam, we could establish what mix of completely polarized particles represents this beam. As it is impossible, is clear, that the equations (6) do not describe classical movement of a particle with spin ½, possessing the abnormal magnetic moment.

This difficulty is connected with the fact that trajectory of a particle described by the equations (6) depends on a direction of a vector a spin. It is clear therefore that classical movement of a particle with spin ½ cannot be described by the equations of type (5), except for that case when function $\vec{F}$ does not depend on a direction of a spin vector.

3. DIRAC EQUATION AND CLASSICAL MOTION OF NEUTRON

The force on a magnetic dipole in an electromagnetic field has been studied for more than 100 years. The expression found in textbooks depends on the spatial gradient of the applied magnetic field, $\vec{F} = \frac{\partial}{\partial \vec{x}}(\vec{\mu}\vec{B})$. If we imagine the magnetic dipole to be an infinitesimal current loop, then $\vec{\mu}$ is proportional to the angular momentum or the spin of the circulating current. Then $\vec{F}$ may be understood as the net force resulting from the different Lorentz forces acting on different parts of the current loop due to the variation of the magnetic field over the loop.

The acceleration of magnetic dipole obtained by J.Anandan was surprising because it exists even when the fields do not vary in space. It cannot therefore be obtained from the intuitive picture. It was surprising also because of its dependence on the electric field: as the current loop representing the dipole may have no net electric charge it may be expected not to couple to an electric field.

The classical limit of a neutron motion following from Dirac theory was considered for example in [5, 6]. Further we shall obtain classical equation of a neutron motion following from Dirac equation. We shall examine the classical movement of the neutron described by the equation (we put now c=1)

$$(\hbar\gamma_\mu \frac{\partial}{\partial x_\mu} - \mu F_{\mu\nu}\sigma_{\mu\nu} + m)\Psi(x) = 0. \tag{8}$$

Under name “neutron” we shall understand a neutral particle with the abnormal magnetic moment.

We shall present the solution of the equation (8) in the form:

$$\Psi(x) = e^{i\frac{S(x)}{\hbar}}\psi(x). \tag{9}$$

Then

$$\left[i\frac{\partial S}{\partial x_\mu} - \mu F_{\mu\nu}\sigma_{\mu\nu} + m\right]\psi + \hbar\gamma_\mu \frac{\partial \psi}{\partial x_\mu} = 0. \tag{10}$$

Neglecting in (10) a member $\hbar\gamma_\mu \frac{\partial \psi}{\partial x_\mu}$, we shall receive:

$$\left[i\frac{\partial S}{\partial x_\mu} - \mu F_{\mu\nu}\sigma_{\mu\nu} + m\right]\psi = 0. \tag{11}$$

From (11) follows:

$$\det\left[i\frac{\partial S}{\partial x_\mu} - \mu F_{\mu\nu}\sigma_{\mu\nu} + m\right] = 0. \tag{12}$$

Calculating a determinant (12), we obtain analogue of Hamilton-Jacobi equation for the neutron [5, 6]

$$\left[\left(\frac{\partial S}{\partial x_\rho}\right)^2 + m^2 - \frac{1}{2}\mu^2 F_{\rho\sigma}F_{\rho\sigma}\right]^2 + \left(\frac{1}{2}\mu^2 \tilde{F}_{\rho\sigma}F_{\rho\sigma}\right)^2 = 4\mu^2\left(\tilde{F}_{\rho\sigma}\frac{\partial S}{\partial x_\sigma}\right)^2, \tag{13}$$

$$\tilde{F}_{\rho\sigma} = \frac{1}{2i}\varepsilon_{\rho\sigma\mu\nu}F_{\mu\nu}. \tag{14}$$

Let's examine, further, movement of the neutron in case of: $\tilde{F}_{\rho\sigma}F_{\rho\sigma} = 0$ that is, $(\vec{E}\vec{B}) = 0$ and the equation (13) becomes:

$$\left[\left(\frac{\partial S}{\partial x_\rho}\right)^2 + m^2 - \frac{1}{2}\mu^2 F_{\rho\sigma} F_{\rho\sigma}\right]^2$$

$$= 4\mu^2\left(\tilde{F}_{\rho\sigma}\frac{\partial S}{\partial x_\sigma}\right)^2 . \qquad (15)$$

The equation (15) can be satisfied with action have been expressed in the form of following integral:

$$S(x) = \int_{x_0}^{x} P_\rho \, dx_\rho$$

$$= \int_{x_0}^{x}\left(mu_\rho - \mu\frac{\tilde{F}_{\rho\sigma} a_\sigma}{a}\right) dx_\rho , \qquad (16)$$

$$u_\rho = \frac{dx_\rho}{ds} \equiv \dot{x}_\rho ,$$

s is the proper time of a particle

$$a_\rho = \mu\tilde{F}_{\rho\sigma} u_\sigma ,$$

$$a = \pm\sqrt{a^2} = \pm\sqrt{\left(\mu\tilde{F}_{\rho\sigma}\right)^2} . \qquad (17)$$

It is easy to see that calculated according (16) canonical momentum

$$\frac{\partial S}{\partial x_\rho} = P_\rho = mu_\rho - \mu\frac{\tilde{F}_{\rho\sigma} a_\sigma}{a} \qquad (18)$$

really satisfies to the equation (17). According (18),

$$S = \int L \, ds , \qquad (19)$$

where the Lagrange function

$$L = -m \pm a = -m \pm \sqrt{\left(\mu\tilde{F}_{\rho\sigma} u_\sigma\right)^2} . \qquad (20)$$

Varying action (19), we shall receive the following equations of Lagrange describing movement of the neutron in an electromagnetic field (in the case of when tensor an electromagnetic field $F_{\rho\sigma}$ satisfies to a ratio $\tilde{F}_{\rho\sigma} F_{\rho\sigma}$ =0):

$$\frac{d}{ds}\left(mu_\rho - \mu\frac{\tilde{F}_{\rho\sigma} a_\sigma}{a}\right) = \frac{\partial}{\partial x_\rho} a . \qquad (21)$$

CONCLUSIONS

We have considered classical limit of Dirac equation and have obtained the classical action and classical equation of a neutron motions. According to these equations neutron acceleration in uniform electromagnetic fields is equal to zero.

We live in the relativistic world and, in the case when $\left(\vec{E}\vec{B}\right) = 0$, we can always pass in the system of reference in which $\vec{E}$, or $\vec{B}$ are equal zero. In this system of reference Anandan acceleration $\frac{d}{dt}\vec{V} = 0$. We have absurd situation when the particle acceleration is nonzero in one system of reference and is zero in another system. So the acceleration of a neutron in uniform electromagnetic fields does not exist.

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ОБ УСКОРЕНИИ НЕЙТРОНА В ОДНОРОДНОМ ЭЛЕКТРОМАГНИТНОМ ПОЛЕ

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Некоторое время назад было показано, что нейтрон ускоряется в однородных электромагнитных полях. Найдено точное решение волнового уравнения Дирака для нейтрона, помещенного в однородное электромагнитное поле, и анализ вопроса не подтверждает ускорение нейтрона. Приведено простое объяснение, почему утверждения об ускорении нейтрона неверны.

ЩОДО ПРИСКОРЕННЯ НЕЙТРОНА В ОДНОРІДНОМУ ЕЛЕКТРОМАГНІТНОМУ ПОЛІ

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Деякий час тому було показано, що нейтрон прискорюється в однорідних електромагнітних полях. Знайдено точне рішення хвильового рівняння Дірака для нейтрона, який вміщено в однорідне електромагнітне поле, і наш аналіз питання не підтверджує прискорення нейтрона. Дано просте пояснення, чому твердження про прискорення нейтрона є хибними.