ON E(11) OF M-THEORY: 1. HIDDEN SYMMETRIES OF MAXIMAL SUPERGRAVITIES AND LEGO OF DYNKIN DIAGRAMS

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We review a graphical way of classifying hidden symmetry algebras and groups of D=11, 10 maximal supergravities in terms of Dynkin diagrams, the shapes of which are determined by the bosonic field content of supergravities supermultiplets. The approach we follow is tightly related to the West's conjecture on a hidden symmetry of M-theory, and we discuss benefits of the approach in compare to other ways of searching for hidden symmetries of String Theory.

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1. INTRODUCTION

The subject under the focus of these notes is the hidden symmetry structure of M-theory. Its study is of considerable interest, since the action of full M-theory with non-perturbative degrees of freedom is still lacking. The encountered situation is a remnant of 60^{th} in physics of strong interactions, when the study of hidden symmetries was very efficient in classifying various resonances. It resulted in the QCD foundation in the end, with the well-defined action on the ground of SU(3) symmetry. Hence, unmasking the symmetry structure of M-theory is extremely important in searching for the dynamical underlying principle.

To introduce the reader into the field of study, we review, in the following section, dualities of String Theory and its hidden symmetries. We recall then a way of identifying the hidden symmetries in the supergravity approximation, and demonstrate after that an effective technique of recovering the results in terms of Dynkin diagrams. Our conclusions are collected in the last section. No attempt has been made of giving a careful set of references, rather most of details of what is discussed here in a sketched manner can be read off textbooks [1-3] and review papers [4-6].

2. STRING THEORY DUALITIES AND HIDDEN SYMMETRIES

We begin with two pictures that characterize the past and the presence of Sting Theory. The first picture, Fig. 1 below, indicates the state of affairs in the "old String Theory" [1], which mainly deals with perturbative degrees of freedom of the theory.

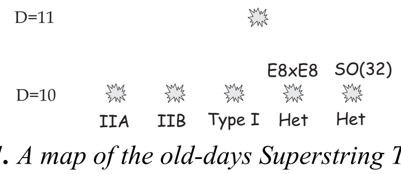

Fig. 1. *A map of the old-days Superstring Theory*

When non-perturbative effects are taken into account, it drastically changes the picture to the following map [2] which is a pictorial representation of five different types of Superstrings (which stay in the corners of the star in Fig. 2) in frames of an eleven-dimensional non-perturbative unifying theory, called M-theory.

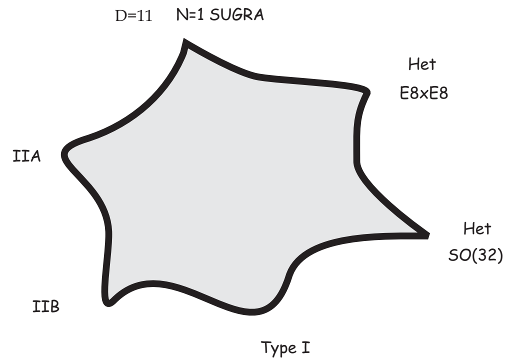

Fig. 2. *M-theory map*

Since M-theory lives in D=11, while we observe just four-dimensional world, it is an apparent problem for the M-theory setting (as well as for the "old String Theory") to figure out a way of correspondence between D=11 and D=4 physics. A bridge between high and low dimensions is provided by compactification of additional coordinates, which form an internal (very small and invisible for us and our experimental tools) space. Though an internal space is hidden, its properties are very important for connecting high- and low-energy physics, and different internal spaces lead to different four-dimensional effective theories.

Compactifying M-theory, one arrives at M(oduli space)-theory, which depends on moduli, i.e. some parameters of an effective theory arising upon the compactification. The moduli, but rather transformations of the moduli under (hidden) symmetry groups, form the moduli space, different points of which (points A and B in Fig. 3) correspond to different effective coupling regimes.

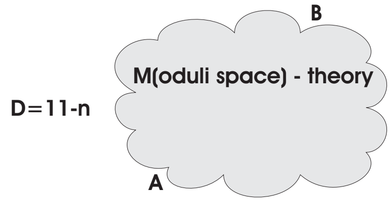

Fig. 3. *A low-dimensional effective theory after compactifying on a n-dimensional internal manifold*

The effective coupling, say in the A-point, may become weak, so one can study the effective theory perturbatively there. But A and B points of the Moduli space are related to each other via Duality, and it makes possible to predict the behavior of theory in the strong coupling point B by studying the theory in the weak coupling point A.

There are three types of Dualities which connect points in the Moduli space. S-duality, the duality between strong and weak coupling regimes of the same or different type theories, applies for analysis of non-perturbative effects due to Dp-branes. Recall that Dp-branes are massive BPS states of type I and type II superstring theories with coupling constant g_s. They become light states in the dual theory with coupling constant $1/g_s$. It turns out that type IIB superstring theory is invariant under S-duality. Together with the invariance of type IIB theory under constant shifts of RR fields it is realized in SL(2,Z) symmetry of type IIB theory. On the type IIA side S-duality has a different realization: A stack of n D0-branes with masses $M \sim n/g$ gets transformed into a smooth spectrum of massless particles in the strong coupling constant limit $g \to \infty$. Such a process may be interpreted as a decompactification of type IIA D=10 string theory into a D=11 theory in the strong coupling limit. Then, the spectrum of type IIA n D0-branes naturally arises upon the compactification of D=11 theory on the circle of radius $R \sim g$. Therefore, the strong coupling limit of type IIA theory is a theory in D=11, referred to as M-theory.

Target-space duality, or T-duality for short, arises when a string is embedded into a target space of the configuration $M_D = M_{D-n} \times T_n$, where T_n is a n-dimensional internal torus under which a string is wrapped m times. The energy of a string wrapped on T_n is described by the following Hamiltonian

$$H_{T_n} \sim \sum_n \frac{p_n^2}{r_n^2} + w_n^2 r_n^2. \tag{1}$$

Eq. (1) indicates the symmetry under exchanging the KK momenta p_n with winding modes w_n, supplemented with exchanging the radii of the torus r_n with their inverse, i.e.

$$p_n \leftrightarrow w_n, \quad r_n \leftrightarrow \frac{1}{r_n} \quad . \tag{2}$$

Winding modes are solitonic states, so T-duality is the invariance of String Theory under exchanging the solitonic and standard modes. But not at all, since the invariance under the radii exchanging allows one to treat more carefully different singularities, which can not be in principle resolved in frameworks of usual QFT.

The group structure of T-duality is easy to understand. A flat metric on a n-dimensional torus has the same number of degrees of freedom as the following coset space

$$\frac{SL(n)}{SO(n)} \times R^+, \tag{3}$$

where we have selected the torus volume parameter R^+. In String Theory we also have a two-form gauge field B_2, whose contribution into degrees of freedom on the torus is

$$\Lambda^2 R^n, \ dim[\Lambda^2 R^n] = \frac{n(n-1)}{2}. \tag{4}$$

We finally get

$$\frac{SL(n)}{SO(n)} \times R^+ \times \Lambda^2 R^n \simeq \frac{O(n,n)}{O(n) \times O(n)}, \tag{5}$$

that is the contribution of gauge fields leads to the enhancement of the original global symmetry of the torus $SL(n)$.

Lastly, there is U-duality, which unifies the dualities mentioned in the above. To establish the U-duality group, one should study both type IIA/IIB theories in different coupling regimes and in $M_D = M_{D-n} \times T_n$ space-times. In type IIA picture we have $SL(n,R)$ to $O(n,n)$ enhancement due to T-duality, and $SL(n,R)$ to $SL(n+1,R)$ enlargement through the M-theory interpretation (S-duality). Together, these symmetries generate the larger U-duality group. A convenient way to realize the group is to play with Dynkin diagrams of algebras in the above [3]. $SL(n,R)$ algebra corresponds to A_{n-1} diagram with $n-1$ nodes (see Fig. 4)

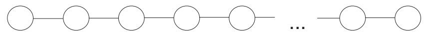

Fig. 4. *A(n-1) Dynkin diagram*

The enlargement to $O(n,n)$ corresponds to D_n Dynkin diagram with n nodes as in Fig. 5

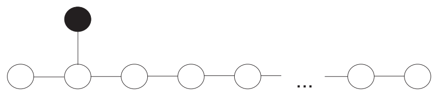

Fig. 5. *D(n) Dynkin diagram*

while the enlargement to $SL(n+1,R)$ algebra gets A_n diagram (with n nodes)

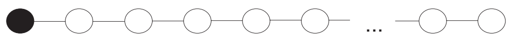

Fig. 6. *A(n) Dynkin diagram*

An entanglement of two diagrams, Fig. 5 and Fig. 6, is realized in E_{n+1} diagram (with $n+1$ nodes, Fig. 7)

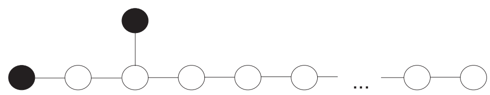

Fig. 7. *E(n+1) Dynkin diagram*

The latter group is the hidden symmetry global group of String theory in $M_D = M_{D-n} \times T_n$ target-space.

A symmetry group of String theory should also incorporate the symmetry groups of the low-energy effective actions, viz. supergravities. Since $M_D = M_{D-n} \times T_n$ target space configuration is viewed as the toroidal reduction, (a subgroup of) E_{n+1} should appear in the toroidally reduced maximal supergravities. An interpretation of E_{n+1} for $n < 2$ is subtle (as well as for $n > 8$, since E_8 is the end of E_n sequence of classical algebras), rather it is a unifying notation for global symmetry groups of the moduli space in the reduced theories. Taking into account the relation of D=11 N=1 supergravity to type IIA supergravity via the reduction on a one-torus, the E_n sequence of hidden symmetries can be assigned to D=11 supergravity, and should be incorporated into M-theory. The moduli space of the reduced M-theory in the low-energy approximation is as follows (Fig. 8)

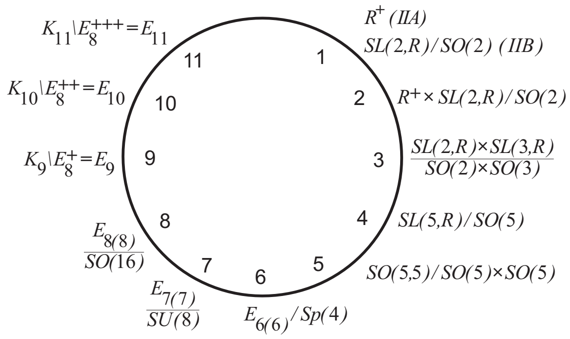

Fig. 8. *M(oduli)-theory clock*

An amazing fact one can read off Fig. 7, that then the reduction goes over three-dimensional space-time down to two, one and zero, the E_n sequence of global symmetry algebras still continues. When $n > 8$, the global symmetry algebras become Kac-Moody-type infinite-dimensional algebras. The local algebras of cosets E_n / K_n, n >8 also become infinite-dimensonal. The end of the sequence is E_{11}, and this point corresponds to M-theory finite in all directions. It is absolutely unclear why E_n "conspiracy" arises, unless it has presented in the unreduced theory. Following this way, we arrive at West's conjecture on E_{11} as a hidden symmetry of M-theory.

This conjecture has several non-trivial corollaries. For instance, M-theory constructed in such a way contains infinitely many massless fields. Some of them may be auxiliary, so a problem is to relate the fields to the perturbative string spectrum (after figuring out a mechanism of mass generation). Next, the realization of E_{11} algebra requires not only the generators for gauge fields and their duals, but a special generator, which corresponds to the field dual to graviton, Hence, M-theory based on E_{11} is dual to gravitational theory. Moreover, it has to be a theory of higher-spin interacting fields, that rises a long-standing problem of the dynamical realization of such a theory.

Leaving apart many details of the conjecture, let us just accept it and try to realize Fig. 8 within its framework. As we have mentioned in the above, this picture (but up to the point 10) comes from reduced supergravities, and the way it appears is as follows.

3. DYNAMICAL ORIGIN OF HIDDEN SYMMETRIES

To establish the hidden symmetry group of dimensionally reduced supergravities one should start with a higher-dimensional supergravity action and recover the following structure upon the reduction to $d = (D-n)$ space-time dimensions [7, 8]

$$S = \int d^d x \sqrt{-g}\, R$$
$$+\int \frac{1}{2} d\vec{\varphi} * d\vec{\varphi} + \frac{1}{2} \sum e^{\vec{\alpha}\cdot\vec{\varphi}}\, d\chi_{\vec{\alpha}} * d\chi_{\vec{\alpha}} + ... \qquad (6)$$

Here $\vec{\varphi}$ is a dilaton vector which includes the original supergravity dilaton and those appeared in the reduction. Constant vectors $\vec{\alpha}$ label axionic scalar fields $\chi_{\vec{\alpha}}$. The scalar part of the action is that of a G/H sigma model if one can identify $\vec{\alpha}$ as positive roots of the algebra corresponding to G. The local group H is the maximum compact subgroup of G. It is worth mentioning that to identify G to that of Fig. 8, the dualisation has to be taken into account (see [7, 8]).

To make the identification of groups more clear and visible, it is used a graphical representation of the group algebras, the Dynkin diagrams. Any symmetry group (more precisely, its algebra) is uniquely determined by the Cartan matrix constructed out of simple roots α_i forming a basis of the root vectors. The Cartan matrix is defined by

$$A_{ij} = \frac{2\alpha_i \alpha_j}{\alpha_i \alpha_i} \qquad (7)$$

and it is in one-to-one correspondence to the appropriate Dynkin diagram [3]. The machinery of calculating the Cartan matrix for different supergravities is very well established (see e.g. [9,10]), with results marked in Fig. 8, up to the reduction on a ten-torus.

4. DYNKIN LEGO FOR M-THEORY

Let's now turn to the following question: Does the conjectured E_{11} so natural as a symmetry of M-theory? And if so, could it be possible, without doing the reduction routine, to realize Fig. 8 in a graphical way?

We note to this end that dealing with Dynkin diagrams is in common the same as dealing with the representation theory. From the point of view of the latter, to study representations (reps.) of an infinite-dimensional algebra it is convenient to choose its maximal regular finite-dimensional subalgebra.

The horizontal line of E_{11} Dynkin diagram (cf. Fig. 7 with 11 nodes) is A_{10} diagram (Fig. 9)

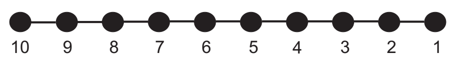

Fig. 9. *A_{10} diagram (gravity line)*

and the diagram corresponds to the finite classical algebra $SU(11) \sim SL(11)$. Non-negative numbers on the diagram are reserved for simple roots coming from the gravitational degrees of freedom. The finite-dimensional algebra $SL(11)$ is the maximal regular subalgebra, called the gravity line. From the point of view of the representation theory, we can decompose the field content of our infinite-dimensional algebra into reps. of the subalgebra $SL(11)$. An advantage of such a decomposition is that the resulting fields of the low-level reps. can be recognized in terms of the fields one is familiar with. For instance, if we enumerate the nodes of the gravity line like in Fig. 9, the additional node needed to form E_{11} will be exactly connected to the node ‘3’ that tells us this additional simple root is originated from a 3rd rank gauge field A_3. This field enters D=11 supergravity multiplet.

Such an observation suggests a naïve ‘mnemonic’ rule to construct a diagram which corresponds to hidden symmetries of higher-dimensional supergravities: We have to draw the ‘gravity line’ – ten nodes for D=11 supergravity, nine nodes for D=10 theory – and to look at the gauge fields content of the supergravity multiplet. As a next step, we shall connect an additional n’th rank tensor field node to the n’th node of the gravity line and look at the so obtained diagram.

For D=11, 10 maximal supergravities it results in (Figs. 10-12)

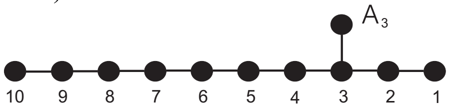

Fig. 10. *Naive diagram of D=11 supergravity*

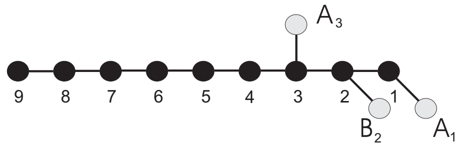

Fig. 11. *Naive diagram of type IIA supergravity*

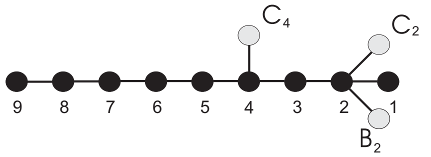

Fig. 12. *Naive diagram of type IIB supergravity*

As in Fig. 9, we have denoted the gravity line with numbers from one to nine, while other nodes are decorated with antisymmetric tensor fields of the rank from one to four which enter the supergravity multiplet.

The last three diagrams are incorrect and there is a very simple objection for them: They do not correspond to the ‘very-extension’ (i.e. to the extension of the diagram with three additional nodes) of ‘classical’ algebras. For the type IIA naïve diagram there are two ways to relate it to the ‘very-extended’ diagrams: To omit the A_3 node that leads to a E_{11}-like diagram (Fig. 13)

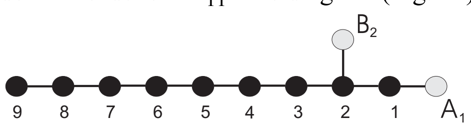

Fig. 13. E_{11} *diagram of type IIA supergravity*

or to delete the B_2 node together with the gravity line ninth node that results in a E_7^{+++}-like diagram (Fig. 14)

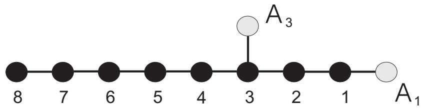

Fig. 14. E_7^{+++} *diagram of type IIA supergravity*

The E_{11}-type diagram Fig. 13 tells us that the hidden symmetry of type IIA supergravity is the same as that of D=11 supergravity. The meaning of Fig. 14 will become clear in a minute.

For type IIB supergravity we arrive at E_7^{+++}-like diagram (Fig. 15) after deleting the B_2 and C_2 nodes in Fig. 12

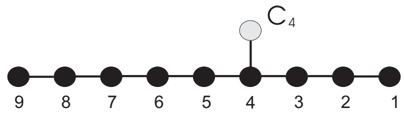

Fig. 15. E_7^{+++} *diagram of type IIB supergravity*

This diagram corresponds to the subsector of type IIB supergravity that includes gravity and C_4 antisymmetric tensor field. When the complete bosonic subsector of the type IIB supermultiplet is taken into account, it results in Fig. 16.

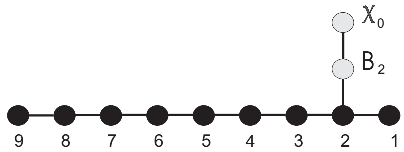

Fig. 16. E_{11} *diagram of type IIB supergravity*

χ_0 here is the type IIB supergravity axion.

If we now compare two E_{11} diagrams of type IIA/IIB theories (Fig. 17)

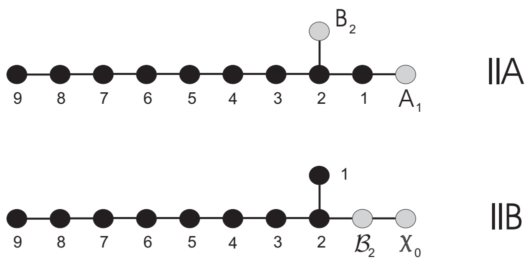

Fig. 17. E_{11} *diagrams of IIA/B supergravities*

we find what is usually called T-duality rules

$$i_z A_1^{IIA} \sim \chi_0^{IIB}\ ,\quad i_Z B_2^{IIB} \sim g_{zm}^{IIA}\ ,$$

$$i_z B_2^{IIA} \sim g_{Zm}^{IIB}\ . \tag{8}$$

The rest of T-duality rules can be read off comparing Fig. 15 to Fig. 16.

We have found the hidden symmetries of unreduced maximal supergravities. The space-time reduction goes on with deleting the gravity line nodes, starting from the ninth. Then, the following chain of algebras occurs in the process

$$E_{10} \to E_9 \to E_8 \to E_7 \to E_6 \to$$

$$\to D_5 \to A_4 \to A_2 \times A_1 \to ...$$

which are precisely the global symmetry algebras one encounters in Fig. 8 when comes anticlockwise from 10 to 3. As in the standard approach resulted in Fig. 8, the interpretation of E_{n+1} for $n < 2$ is subtle.

5. SUMMARY

The hidden symmetry groups of the toroidally reduced supergravities can be found via studying the reduced actions and involved calculations of the field transformations. It is an art to find the correct transformations and to identify the group. The method of calculations of the axionic label vectors which correspond to the simple roots of the hidden symmetry group is more transparent, simple and elegant. However, it also requires the reduction of supergravities action.

Dynkin diagrams, constructing out of a LEGO-like system elements such as nodes and lines, give us a way of identifying the hidden symmetries without reducing the actions. We have demonstrated this procedure for maximal higher-dimensional supergravities, though the prescription works for other supergravities as well.

Surely, there are some open questions. For instance, it is unclear, how the prescription does work in the case of non-toroidal reduction, and it is in general correct to talk about hidden symmetries of supergravities compactified on Calabi-Yau spaces in situation when the consistency of the compactification fails. Another interesting question concerns the role of twisted Kac-Moody algebras within M-theory. Recall that Kac-Moody-type algebras we considered here were of the untwisted type.

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REFERENCES

1. M.B. Green, J.H. Schwarz, E. Witten. *Superstring theory.* Cambridge: "CUP", 1987, v. 1, 469 p.
2. J. Polchinski. *String theory.* Cambridge: "CUP", 1999, 933 p.
3. J. Fuchs, C. Schweigert. *Symmetries, Lie algebras and representations.* Cambridge: "CUP", 1997, 438 p.
4. A.J. Nurmagambetov. Duality-symmetric approach to general relativity and supergravity //*SIGMA.* 2006, v. 2, 020, 34 p.
5. S. de Buyl. *Kac-Moody algebras in M-theory.* hep-th/0608161, 2006, 227 p.
6. D.R. Morrison. *TASI lectures on compactification and duality.* hep-th/0411120, 2004, 66 p.
7. E. Cremmer, B. Julia, H. Lu, C.N. Pope. Dualisation of dualities I //*Nucl. Phys. B*. 1998, v. 523, p. 73-144.
8. E. Cremmer, B. Julia, H. Lu, C.N. Pope. Dualisation of dualities II //*Nucl. Phys. B*. 1998, v. 535, p. 242-292.
9. N.D. Lambert, P.C. West. Coset symmetries in dimensionally reduced bosonic string theory //*Nucl. Phys. B*. 2001, v. 615, p. 117-132.
10. A.J. Nurmagambetov. Diagrammar and metamorpho-sis of coset symmetries in dimensionally reduced type IIB supergravity //*Pis'ma v ZhETF*. 2004, v. 79, iss. 7, p. 436-440.

О E(11) В М-ТЕОРИИ: 1. СКРЫТЫЕ СИММЕТРИИ МАКСИМАЛЬНЫХ СУПЕРГРАВИТАЦИЙ И ЛЕГО ДИАГРАММ ДЫНКИНА

А.Ю. Нурмагамбетов

Представлен обзор графического метода классификации алгебр и групп скрытых симметрий D=11, 10 максимальных супергравитаций в терминах диаграмм Дынкина, форма которых определяется составом бозонных полей супергравитационного супермультиплета. Предлагаемый подход тесно связан с гипотезой Веста о скрытой симметрии М-теории, и обсуждаются преимущества данного подхода по сравнению с другими способами нахождения скрытых симметрий Теории Струн.

ОБ E(11) В М-ТЕОРІЇ: 1. ПРИХОВАНІ СИМЕТРІЇ МАКСИМАЛЬНИХ СУПЕРГРАВІТАЦІЙ ТА ЛЄГО ДІАГРАМ ДИНКІНА

О.Ю. Нурмагамбетов

Подано огляд графічного методу класифікації алгебр і груп прихованих симетрій D=11, 10 максимальних супергравітацій в термінах діаграм Динкіна, форма яких визначається складом бозоних полів супергравітаційного супермультиплету. Запропонований підхід тісно пов'язаний з гіпотезою Веста про приховану симетрію М-теорії, і обговорюються переваги даного підходу в порівнянні з іншими способами знаходження прихованих симетрій Теорії Струн.