INTERACTION BETWEEN TWO SOLITONS IN TWO-DIMENSIONAL COMPLEX PLASMA WITH VARIABLE DUST CHARGE

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We have studied the propagation of the dust acoustic solitary waves in two different directions for a non-magnetized, two dimensional dusty plasma including the variation of dust charges. We extend the reductive perturbation method and obtain two KdV equations for a nonlinear wave in two different directions. We also compare our results with the case of constant dust charge.
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1. INTRODUCTION

Complex (dusty) plasma has received much interest due to its importance in the space and earth environment and in the laboratory plasma.

The dust acoustic waves (DAW) in un-magnetized dusty plasma were first predicted, theoretically, by Rao et al. Later, the existence of dust ion acoustic wave (DIAW) at higher frequency was shown by Shukla and Silin. So far, the existence of DAW and DIAW has been strongly verified in laboratory experiments on complex (dusty) plasma. Generally, the problem of nonlinear wave propagation can be described by fundamental set of equations. However, for simplicity, a number of approximate models have been introduced to describe either propagation of the wave itself or propagation of slowly varying wave envelopes. As an example for the latter case we can name KdV equation. Solitons and their interactions have been observed in most nonlinear dusty plasma waves [1,2].

Most previous works on dust acoustic waves have often assumed that the dust charge is constant. The contemporary research studies have included impacts of resonant interactions of solitons within themselves and the interactions of two KdV solitons in one dimensional systems with each other [2]. Nejoh [3] has pointed out that the dust charge variation with some parameters would affect the characteristic collective motion of the plasma. Therefore, the variation of dust charge plays an important role in investigating the behavior of the system. Although, most of these works are mainly focused on 1D systems.

Based on the work of Sh. Ch Li and W. Sh Duan et al. we have studied the propagation of the dust acoustic solitary waves in 2D un-magnetized dusty plasma. The key point is that we have considered the variation of dust charges. We have extended the reductive perturbation method and obtained two KdV equations to describe solitons. This method can be used not only for studying the interaction of two solitons with an arbitrary propagation angle, but also for general circumstances in studying soliton interaction.

The organization of this manuscript is as follows. In Sec. 2 we bring up a set of normalized equations of motion for complex (dusty) plasma and in Sec. 3 we obtain the two KdV equations. A brief discussion has been brought at the end.

2. BASIC EQUATIONS

The complex plasma studied here consists of three components, extremely massive negatively charged dust grains, electrons and ions. The charge neutrality at equilibrium gives $n_{0i} = n_{0e} + z_{0d} n_{0d}$, where n_{0i}, n_{0e} and n_{0d} are ions, electrons and dust unperturbed number of densities, respectively. $z_{0\mathrm{d}}$ is the charges residing on the dust grain measured in units of electron charge. The dimensionless basic equations for this system are as follows:

$$\begin{aligned}
&\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_{xd})}{\partial x} + \frac{\partial (n_d v_{yd})}{\partial y} = 0\\
&\frac{\partial v_{xd}}{\partial t} + v_{xd}\frac{\partial v_{xd}}{\partial x} + v_{yd}\frac{\partial v_{xd}}{\partial y} = z_d \frac{\partial \varphi_d}{\partial x}\\
&\frac{\partial v_{yd}}{\partial t} + v_{xd}\frac{\partial v_{yd}}{\partial x} + v_{yd}\frac{\partial v_{yd}}{\partial y} = z_d \frac{\partial \varphi_d}{\partial y}\\
&\frac{\partial^2 \varphi_d}{\partial \mathrm{x}^2} + \frac{\partial^2 \varphi_d}{\partial \mathrm{y}^2} = z_d n_d + n_e - n_i
\end{aligned} \qquad (1)$$

The electrons and ions are all assumed Boltzman distributed $n_e = \nu \exp(\beta \mathrm{s} \varphi_\mathrm{d}), n_i = \mu \exp(-\mathrm{s}\varphi_\mathrm{d})$, with $\nu = n_{0\mathrm{e}}$, $\mu = n_{0\mathrm{i}}$ and $\beta = \mathrm{T_i}/T_\mathrm{e}$, $\mathrm{s} = 1/(\mu + \nu\beta)$ ЧT_e and T_i denote temperatures for electrons and ions, respectively. Electron and ion densities are normalized by $z_{0d} n_{0d}$. The space coordinates, time, velocities and the electrical potential are normalized by the effective Debye length $\lambda_{Dd} = T_{eff}/(4\pi \mathbf{z}_{0\mathbf{d}} n_{0\mathbf{d}} e^2)^{1/2}$, the inverse of dust plasma frequency $\omega_{pd}{}^{-1} = m_d/(4\pi \mathrm{n}_{0\mathrm{d}}\ \mathrm{z}_{0\mathrm{d}}{}^2 \mathrm{e}^2)^{1/2}$, effective dust acoustic speed $c_d = (z_{0\mathrm{d}} T_{eff}/m_\mathrm{d})^{1/2}$ and (T_{eff}/e), respectively. We note also that $T_{eff} = (\mathrm{T_i T_e}/(\nu \mathrm{T_i} + \mu \mathrm{T_e}))$. We now use reductive perturbation technique to obtain two Korteweg de Vries (KdV) equations that govern the behavior of the small amplitude dust acoustic waves. We take the following asymptotic expansions:

$$\begin{aligned} n_d &= 1+\varepsilon^2 n_1+\varepsilon^4 n_2,\\ v_{xd} &= \varepsilon^2 v_{1x}+\varepsilon^4 v_{2x},\\ v_{yd} &= \varepsilon^2 v_{1y}+\varepsilon^4 v_{2y},\\ \phi &= \varepsilon^2\phi_1+\varepsilon^4\phi_2,\\ z_d &= 1+\varepsilon^2 z_1+\varepsilon^4 z_2, \end{aligned} \tag{2}$$

where $z_1=\gamma_1\varphi_1, z_2=\gamma_1\varphi_2+\gamma_2\varphi_1{}^2$ (see ref [4]), and ε is a small parameter characterizing the strength of nonlinearity. Now we assume that $n_d, v_{xd}, v_{yd}, \varphi_d$ are functions of multiple-scaled variables defined by:

$$\begin{aligned} \xi &= \varepsilon(k_1x+l_1y-c_\xi t)+\varepsilon^2P^{(0)}(\eta,\tau)+...,\\ \eta &= \varepsilon(k_2x+l_2y-c_\eta t)+\varepsilon^2Q^{(0)}(\eta,\tau)+...,\\ \tau &= \varepsilon^3 t, \end{aligned} \tag{3}$$

where $c_\xi=c_1, c_\eta=c_2$

3. KdV EQUATIONS

Substituting Eqs.(2) into Eqs.(1), collecting terms in different powers of ε, and by assuming:

$$\begin{aligned} n_{d1} &= n_{d\xi}(\xi,\tau)+n_{d\eta}(\eta,\tau),\\ v_{xd} &= v_{xd\xi}(\xi,\tau)+v_{xd\eta}(\eta,\tau),\\ v_{dy} &= v_{yd\xi}(\xi,\tau)+v_{yd\eta}(\eta,\tau),\\ \varphi_{d1} &= \varphi_{d\xi}(\xi,\tau)+\varphi_{d\eta}(\eta,\tau), \end{aligned} \tag{4}$$

we obtain the following relations between different physical quantities at the lowest order:

$$\begin{aligned} &\phi_{d\xi}(1+\gamma_1)=-n_{d\xi},\quad v_{xd\xi}=\frac{k_1}{c_1}n_{d\xi},\\ &\phi_{d\eta}(1+\gamma_1)=-n_{d\eta},\quad v_{xd\eta}=\frac{k_2}{c_2}n_{d\eta},\\ &c_1^2(1+\gamma_1)=k_1^2+l_1^2,\quad v_{xd\xi}=\frac{k_1}{c_1}n_{d\xi},\\ &c_2^2(1+\gamma_1)=k_2^2+l_2^2,\quad v_{yd\eta}=\frac{l_2}{c_2}n_{d\eta}. \end{aligned} \tag{5}$$

By eliminating secular terms (*e.g.* $\frac{\partial^2}{\partial\eta\partial\xi}$), we obtain:

$$\begin{aligned} &\frac{\partial\phi_{d\xi}}{\partial\tau}+a_1\phi_{d\xi}\frac{\partial\phi_{d\xi}}{\partial\xi}+b_1\frac{\partial^3\phi_{d\xi}}{\partial\xi^3}=0,\\ &\frac{\partial\phi_{d\eta}}{\partial\tau}+a_2\phi_{d\eta}\frac{\partial\phi_{d\eta}}{\partial\eta}+b_2\frac{\partial^3\phi_{d\eta}}{\partial\eta^3}=0, \end{aligned} \tag{6}$$

$$\begin{aligned} &A_1\phi_{d\eta}+B\frac{\partial P^{(0)}}{\partial\eta}=0,\\ &A_2\phi_{d\xi}+B\frac{\partial Q^{(0)}}{\partial\xi}=0, \end{aligned} \tag{7}$$

where

$$\begin{aligned} &\gamma=\frac{1}{2}s^2(\beta^2\nu\_\mu), a_1=-c_1(3+2\gamma_1), b_1=-2c_1^3,\\ &a_2=-c_2(3+2\gamma_1), b_2=-2c_2^3,\\ &A_1=2c_1^2(\gamma+\gamma_2-\gamma_1(1+\gamma_1)),\\ &A_2=2c_2^2(\gamma+\gamma_2-\gamma_1(1+\gamma_1)),\\ &B=(k_1k_2+l_1l_2). \end{aligned}$$

The stationary solution of KdV equations is obtained by transforming the independent variables ξ and η to $\zeta=\xi-u_{0\xi}\tau$ **and** $\iota=\eta-\mathbf{u}_{0\eta}\tau$ **and** $\tau=\tau$ (see Ref [5] for example), where $u_{0\xi}$ and $u_{0\eta}$ are constant speeds normalized by c_1 and c_2, respectively. Imposing the appropriate boundary condition

$\phi_{d\xi}\to 0,\ (d\phi_{d\xi}/d\xi)\to 0,\ (d^2\phi_{d\xi}/d\xi^2)\to 0$ at $\xi\to\pm\Gamma$,

and also

$\phi_{d\eta}\to \mathbf{0}, (\mathbf{d}\phi_{d\eta}/d\eta)\to\mathbf{0}, (\mathbf{d}^2\phi_{d\eta}/d\eta^2)\to\mathbf{0}$ **at** $\eta\to\pm\infty$,

stationary solutions of KdV equations are emerged as:

$$\varphi_{d\xi}=\varphi_{m\xi}\sec h^2\frac{\xi-u_{0\xi}\tau}{\Delta_\xi},\ \text{and}\ \varphi_{d\eta}=\varphi_{m\eta}\sec h^2\frac{\eta-u_{0\eta}\tau}{\Delta_\eta}, \tag{8}$$

where

$$\varphi_{m\xi}=\frac{3u_{0\xi}}{a_1},\ \varphi_{m\eta}=\frac{3u_{0\eta}}{a_2},\ \text{and}\ \Delta_\xi=2\sqrt{\frac{b_1}{u_{0\xi}}},\quad \Delta_\eta=2\sqrt{\frac{b_2}{u_{0\eta}}}, \tag{9}$$

φ_{mi} and $\Delta_i\ (i=\xi,\eta)$ are the amplitude and the width of the soliton, respectively. It is noted that both amplitude and the width are functions of arbitrary constants $u_{0\xi}$ and $u_{0\eta}$.

For the case of constant dust charge, we find that amplitude of the soliton is larger than amplitude of soliton for a variable charge.

Substituting Eqs. (6) into (7) we obtain the following functions

$$\begin{aligned} P^{(0)}(\eta,\tau)&=-\frac{2c_1(\gamma+\gamma_2-\gamma_1(1+\gamma_1))}{c_2Cos\alpha}\int\phi_{m\eta}\sec h^2\frac{\eta-u_{0\eta}\tau}{\omega_\eta}d\eta,\\ Q^{(0)}(\xi,\tau)&=-\frac{2c_2(\gamma+\gamma_2-\gamma_1(1+\gamma_1))}{c_1Cos\alpha}\int\phi_{m\xi}\sec h^2\frac{\xi-u_{0\xi}\tau}{\omega_\xi}d\xi. \end{aligned} \tag{10}$$

And after carrying out the integrations, we have:

$$\begin{aligned} P^{(0)}(\eta,\tau)&=-\frac{2c_1\phi_{m\eta}\omega_\eta}{c_2Cos\alpha}(\gamma+\gamma_2-\gamma_1(1+\gamma_1))\tanh(\frac{\mu-u_{0\eta}\tau}{\omega_\eta}),\\ Q^{(0)}(\xi,\tau)&=-\frac{2c_2\phi_{m\xi}\omega_\xi}{c_1Cos\alpha}(\gamma+\gamma_2-\gamma_1(1+\gamma_1))\tanh(\frac{\xi-u_{0\xi}\tau}{\omega_\xi}). \end{aligned} \tag{11}$$

For considering interaction between two solitons described by Eqs.(6), we put

$$\begin{aligned} P_1^{(0)}&=P^{(0)}(\eta,\tau)\ \big|_{(\eta-u_{0\eta}\tau)s\ s\to-\Gamma},\\ P_2^{(0)}&=P^{(0)}(\eta,\tau)\ \big|_{(\eta-u_{0\eta}\tau)s\ s\to+\Gamma}, \end{aligned} \tag{12}$$

in which $P_1^{(0)}$ denotes the value of the phase of soliton 1 before the interaction with soliton 2, while $P_2^{(0)}$ denotes the value after the interaction with soliton 2. So the phase shift of soliton 1 due to the interaction can be written as

$$\Delta P^{(0)}=P_2^{(0)}-P_1^{(0)}=-4\frac{\phi_{m\eta}\omega_\eta c_2(\gamma+\gamma_2-\gamma_1(1+\gamma_1))}{c_1\cos\alpha}. \tag{13}$$

Similarly, the phase shift of soliton 2 reads

$$\Delta Q^{(0)}=Q_2^{(0)}-Q_1^{(0)}=-4\frac{\varphi_{m\xi}\omega_\xi c_1(\gamma+\gamma_2-\gamma_1(1+\gamma_1))}{c_2\cos\alpha}. \tag{14}$$

We consider following cases:

1. Dust charges are constant *(i.e.* $\gamma_1 = \gamma_2 = 0$).

$$\Delta P^{(0)} = P_2^{(0)} - P_1^{(0)} = -4\frac{\varphi_{m\eta}\Delta_\eta c_2\gamma}{c_1 \cos\alpha},$$

$$\Delta Q^{(0)} = \mathbf{Q_2}^{(0)} - \mathbf{Q_1}^{(0)} = -4\frac{\phi_{m\xi}\Delta_\xi c_1\gamma}{c_2 \cos\alpha}. \qquad (15)$$

$\Delta P^{(0)}$ and $\Delta Q^{(0)}$ are functions of both amplitude and width of soliton, angle α and also the soliton velocities c_1 and c_2.

2. For $\alpha = \pi$, we have:

$\Delta P^{(0)} = 4\frac{\varphi_{m\eta}\Delta_\eta c_2\gamma}{c_1}$ and $\Delta Q^{(0)} = 4\frac{\varphi_{m\xi}\Delta_\xi c_1\gamma}{c_2}$, which is spatial case of our 2D situation.

3. When amplitudes of the two solitons are the same, $c_1 = c_2$ and $u_{0\xi} = u_{0\eta}$, we find $\Delta P^{(0)} = \Delta Q^{(0)}$.

It shows that the value of phase shift is inversely proportional with $\cos(\alpha)$, $\Delta P^{(0)} = \Delta Q^{(2)} = \frac{-4\varphi_m\Delta\gamma}{\cos\alpha}$. We have a singularity at $\alpha = \pi/2$.

4. In the case $\gamma_2 - \gamma_1(1+\gamma_1) > 0$, we see that $\Delta P^{(0)}$ and $\Delta Q^{(0)}$ are larger than the case in which the charge dust is constant, otherwise they are smaller.

4. CONCLUSIONS

We have studied the propagation of the dust acoustic solitary waves in two different directions in un-magnetized dusty plasma. We have incorporated the variation of dust charges and extended the reductive perturbation method to obtain two KdV equations for describing solitons. Our results suggest that the value of phase shift is inversely proportional with $\cos\alpha$. There is a singularity for $\alpha = \pi/2$.

The effect of variable dust charge makes the amplitude of soliton smaller compared with constant dust charge case. We find that the amplitude is a function of dust charge variations. Also both the amplitude and the width are functions of the soliton velocities, angle α and arbitrary constants $u_{0\xi}$ and $u_{0\eta}$.

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ВЗАИМОДЕЙСТВИЕ МЕЖДУ ДВУМЯ СОЛИТОНАМИ В ДВУМЕРНОЙ КОМПЛЕКСНОЙ ПЛАЗМЕ С ПЕРЕМЕННЫМ ПЫЛЕВЫМ ЗАРЯДОМ

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Изучалось распространение пылевых акустических солитонных волн в двух различных направлениях для ненамагниченной двумерной пылевой плазмы, включая изменения пылевых разрядов. Был использован редуктивный метод возмущений и получено решение двух уравнений Кортевега-де Вриза (KdV) для нелинейных волн в двух различных направлениях. Полученные результаты сравнивались с таковыми для случая постоянного пылевого разряда.

ВЗАЄМОДІЯ МІЖ ДВОМА СОЛІТОНАМИ В ДВОМІРНІЙ КОМПЛЕКСНІЙ ПЛАЗМІ ЗІ ЗМІННИМ ПИЛОВИМ ЗАРЯДОМ

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Вивчалося поширення пилових акустичних солітонних хвиль у двох різних напрямках для ненамагніченої двомірної пилової плазми, включаючи зміни пилових розрядів. Був використаний редуктивний метод збурювань і отримано рішення двох рівнянь Кортевега-де Вріза (Kd) для нелінійних хвиль у двох різних напрямках. Отримані результати порівнювалися з такими для випадку постійного пилового розряду.