ROLE OF EXCHANGE INTERACTION OF IDENTICAL PARTICLES OF THE DISCRETE SPECTRUM AT SIMULTANEOUS THEIR TUNNELING THROUGH THE POTENTIA BARRIER DIVIDING THESE PARTICLES

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Work is devoted to theoretical research of influence of exchange interaction of identical particles on processes of simultaneous their tunneling through a rectangular potential barrier. Expression for exchange integral has been received and the analysis of dependence on parameters of identical particles and characteristics of a quantum barrier is lead. It is shown, that as including of exchange energy changes phase characteristics of the wave functions describing process of tunneling, exchange processes influence on the time characteristics of tunneling.

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1. INTRODUCTION

As is known [1, 2], processes of interaction between quantum objects in some cases can be approximated precisely enough, having presented potential of interaction as a rectangular quantum barrier that allows to simplify mathematical calculations considerably.

Such approximation has allowed solving a number of the important problems of quantum mechanics, and also welling to describe behavior of some quantum objects [2]. Therefore attractive is the use of it and for the decision of other tasks which yet it is not enough investigated.

To the number questions which to the present tense in general was not explored, a question on influence of exchange interaction on the processes of particles tunneling in the field of rectangular potential barrier belongs to theoretical development of which the present work is devoted.

2. PASSAGE OF PARTICLES THROUGH A RECTANGULAR QUANTUM BARRIER: THE GENERAL CONSIDERATION

In the beginning, let us consider a variant when on a rectangular potential barrier in width $2a$ and height U particles with a pulse p from left to right fall. Equation of Schrödinger for particles looks like:

$$H\Psi(x,t) = -\frac{p^2}{2m}\Delta\Psi + U\Psi = E\Psi\,,$$

or

$$\Delta\Psi + \frac{2m}{\hbar^2}\left[E-U\right] = 0.$$

Time dependence of own functions is described by expression:

$$\Delta\Psi + \frac{2m}{\hbar^2}\left[E-U\right] = 0.$$

Therefore for various areas decisions can be functions:

$$\Psi_T = A_T(x)\exp ip_1 r/\hbar \tag{1}$$

at $r > a$,

$$\Psi = B\,\exp p_2 r/\hbar + C\,\exp -p_2 r/\hbar \tag{2}$$

at $r \in [-a, a]$,

$$\Psi = D\exp ip_1 r/\hbar + E\exp -ip_1 r/\hbar \tag{3}$$

at $r < -a$, where

$$p_1 = \sqrt{2mE}, \quad p_2 = \sqrt{2m\,(U-E)}. \tag{4}$$

Into a barrier to one particle correlations take place [2]:

$$B = \frac{A}{2}\left\{1+\frac{ip_1}{p_2}\right\}\exp\Big[(ip_1-p_2)\frac{2a}{\hbar}\Big], \tag{5}$$

$$C = \frac{A}{2}\left\{1-\frac{ip_1}{p_2}\right\}\exp\Big[(ip_1+p_2)\,\frac{2a}{\hbar}\Big]. \tag{6}$$

Provided that these functions and their first derivatives are continuous on border of a barrier, it is possible to get the exact decision of a task which is resulted in [1, 2] and other the sources.

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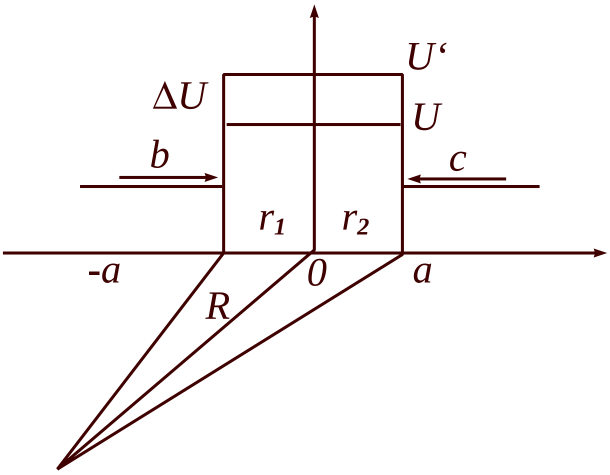
3. EXCHANGE INTERACTION OF IDENTICAL PARTICLES IN A FIELD OF A RECTANGULAR QUANTUM BARRIER

Processes of collision are considered in system of the center of weights, for which speed of two particles before collision and after are opposite. Interaction of particles takes place in the field of a rectangular potential barrier (see figure).

Let us consider two cases. The 1-st case: let us assume that the barrier is opaque to tunneling. In this case A_t in a formula (1) equals a zero. We will assume that a barrier becomes opaque to tunneling at some value of potential corresponding U'. We have: $\Psi_{b,c} = \Psi_b(r_1)\Psi_c(r_2)$, where $\Psi_b(r_1)$ is a solution of equation of: $H\Psi_b(r_1) = E_b\Psi_b(r_1)$, answering a value E_b, and, similarly, for $\Psi_c(r_2)$, corresponding to a value E_c. Energy of the system: $E_0 = E_c + E_b$. Because at a change placed of particles nothing changes, $\Psi_{c,b} = \Psi_b(r_2)\Psi_c(r_1)$ will correspond to the same energy E_0, i.e. we have double degeneration of power level of the system.



The circuit of interaction of two identical particles in a field of rectangular potential barrier

The 2-nd case: a barrier is transparent for tunneling. This case is realized, when the additional constant field is enclosed to area of a barrier in opposite to first direction, creating additional potential $|\Delta U|$. Then the size of potential of barrier will fall down on a size $|\Delta U|$, and have:

$$U = U' - |\Delta U|.$$

Let us consider additional potential $|\Delta U|$ as small indignation in system of two particles, originally between itself not interactive because of opacity of a barrier (a case 1).

The additional probability of particles to pass through a barrier leads to removal of degeneration.

At simultaneous approaching of particles b and c to the borders of a barrier there can be the effects connected to exchange interaction of particles. Such effects will take place only for identical particles and their size is determined by "overlapping" of wave functions, i.e. they are realized when wave functions of particles will be distinct from zero in the same area of space.

Taking to account that at interaction of identical particles symmetric or antisymmetric states are realized only [1, 2], we shall have for symmetric or antisymmetric wave function of expression:

$$\begin{aligned}\Psi_{1,2} = \Psi_\pm = \tfrac{1}{\sqrt{2}}(\Psi_1 \pm \Psi_2) = \\ = \tfrac{1}{\sqrt{2}}\left[\Psi_b(r_1)\Psi_c(r_2) \pm \Psi_c(r_1)\Psi_b(r_2)\right],\end{aligned} \tag{7}$$

Size $|\Psi_\pm|^2$ - expresses probability of finding of the system in the symmetric or antisymmetric state depending on what particles are bosons or fermions.

If particles were not identical, the system would be described a non symmetric function and we would have:

$$|\Psi_{1,2}|^2 = |\Psi_b(r_1)\Psi_c(r_2)|^2.$$

For identical particles there is an additional element: $|\Psi_b(r_1)\Psi_c(r_2)|^*\Psi_c(r_1)\Psi_b(r_2)$, and, consequently, additional energy, proper to energy of interaction of identical particles, appears. This energy, taking to account that a potential barrier has a form, indicated on figure, it is possible to calculate, in accordance with [1] on a formula:

$$V = |\Delta U| \int_r \Psi_b^*(r_1)\Psi_c^*(r_2)\Psi_b(r_2)\Psi_c(r_1). \tag{8}$$

From equalization (2) evidently, that a size V is determined by "overlapping" the wave functions Ψ_b and Ψ_c.

For values $r < -a$ and $r > a$ the value of integral is identically equally to zero, because for these areas $U = 0$, therefore will conduct integration for $-a \le r \le a$.

Into a barrier for one particle the ratio (2), (5) and (6) are fair.

At $p_2a/\hbar \gg 1$ (i.e. provided that parameters an exhibitor strongly change from one border of a barrier to another) $C \gg B$ [2].

Then it is possible to do such approaching:

$$\Psi_b^*(r_1) = C^* \exp(\frac{p_2r_1}{\hbar}),$$

$$\Psi_c^*(r_2) = C^* \exp(-\frac{p_2r_2}{\hbar}),$$

$$\Psi_b(r_2) = B\exp(\frac{p_2r_2}{\hbar}) + C\exp(-\frac{p_2r_2}{\hbar}),$$

$$\Psi_c(r_1) = B\exp(-\frac{p_2r_1}{\hbar}) + C\exp(\frac{p_2r_1}{\hbar}).$$

In accordance with (8) have:

$$
\begin{aligned}
V \;&=\; |\Delta U| \int\!\!\int C^* \exp\frac{p_2 r_1}{\hbar}\cdot C^* \exp-\frac{p_2 r_1}{\hbar}\cdot\Big\{B^* \exp\frac{p_2 r_1}{\hbar} + C^* \exp-\frac{p_2 r_1}{\hbar}\Big\}\times \\
&\times\;\; \Big\{B \exp-\frac{p_2 r_1}{\hbar} + C\,\exp\frac{p_2 r_1}{\hbar}\Big\}\, dr_1\, dr_2 = |\Delta U|\cdot\Big\{\big[aBC^*\big]^2 - \Big[CC^*\,\frac{\hbar}{2\,p_2}\,\exp-\frac{2p_2 a}{\hbar}\Big]^2\Big\}.
\end{aligned} \tag{9}
$$

From (9) follows, that $V = 0$ at

$$|B| = \Big|\frac{C\hbar}{2p_2 a}\,\exp-\frac{2p_2 a}{\hbar}\Big|,$$

i.e. exchange interaction at such parameters of tunneling is absent.

More exact values it is possible to get for the proper parameters of tunneling, if to present the wave function of a moving particle as a wave package [3].

During [3] time of tunneling it is determined as:

$$\tau_T^{Ph} = \hbar\frac{\partial \arg A_T}{\partial E}. \tag{10}$$

As to exchange energy V there corresponds a certain value ω' ($V = \hbar\omega'$) and, accordingly, Ψ_T passes in $\Psi'_T = A'_T \exp(\frac{i}{\hbar}\sqrt{2m(E \pm V)}$ that, hence, is received also change of value of time of tunneling.

4. CONCLUSION

Within the framework of approximation of potential of interaction of particles as a rectangular potential barrier expression for exchange integral of identical particles, simultaneously tunneling through a barrier towards each other is found.

It is shown, that the account of exchange interaction changes time of tunneling. Such analysis of processes of simultaneous tunneling is conducted first.

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РОЛЬ ОБМЕННОГО ВЗАИМОДЕЙСТВИЯ ТОЖДЕСТВЕННЫХ ЧАСТИЦ ДИСКРЕТНОГО СПЕКТРА ПРИ ОДНОВРЕМЕННОМ ТУННЕЛИРОВАНИИ ИХ ЧЕРЕЗ ПОТЕНЦИАЛЬНЫЙ БАРЬЕР, РАЗДЕЛЯЮЩИЙ ЭТИ ЧАСТИЦЫ

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Работа посвящена теоретическому исследованию влияния обменного взаимодействия тождественных частиц на процессы одновременного туннелирования их через прямоугольный потенциальный барьер. Было получено выражение для обменного интеграла V и проведен анализ зависимости V от параметров тождественных частиц и характеристик квантового барьера. Показано, что, поскольку включение обменной энергии изменяет фазовые характеристики волновых функций, описывающих процесс туннелирования, то обменные процессы влияют на временные характеристики туннелирования.

РОЛЬ ОБМІННОЇ ВЗАЄМОДІЇ ТОТОЖНИХ ЧАСТОК ДИСКРЕТНОГО СПЕКТРА ПРИ ОДНОЧАСНОМУ ТУННЕЛЮВАННІ ЇХ ЧЕРЕЗ ПОТЕНЦІЙНИЙ БАР'ЄР, ЩО РОЗДІЛЯЄ ЩІ ЧАСТИНКИ

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Робота присвячена теоретичному дослідженню впливу обмінної взаємодії тотожних часток на процеси одночасного туннелювання їх через прямокутний потенційний бар'єр. Був отриманий вираз для обмінного інтеграла V й проведений аналіз залежності V від параметрів тотожних часток і характеристик квантового бар'єра. Показано, що, оскільки включення обмінної енергії змінює фазові характеристики хвильових функцій, що описують процес туннелювання, то обмінні процеси впливають на часові характеристики туннелювання.