QUANTUM-CONSISTENT SUPERSTRING MODELS IN FOUR DIMENSIONAL SPACE-TIME

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Extra space-time dimensions are predicted by String theory. However, up to date there are not any experimental signals in favor of their existence. It forces to search for consistent string theory formulations in four space-time dimensions. The task can be completed with extending the standard vector-type coordinates of four-dimensional space-time with additional tensorial-type bosonic coordinates. The reason of introducing the new set of coordinates is discussed, and calculations of the critical dimension in the Neveu-Schwarz-Ramond tensorial superstring formulation are performed. It is also discussed the role of the new coordinates in the construction of the consistent five-dimensional superstring formulation solely in terms of the tensorial-type coordinates and their world-sheet superpartners. Properties of massless modes casting an open and a closed five-dimensional superstrings spectra are considered in brief.

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1. INTRODUCTION

One of the most fascinating predictions of String theory is the existence of extra dimensions required for quantum consistency of strings. Extra dimensions, the presence of which was very sceptically conceived at early days of String theory, have become the standard *de facto* in the construction of various Unification models of fields and have received a lot of attention on the particle physics community side.

Searching for signals from extra dimensions on modern experimental factories is a good way to put String theory on the test. Unfortunately, up to date there are not any experimental observations in favor of extra dimensions. Therefore, a possibility of living in the World without extra dimensions has not a priori to be ruled out. This fact stimulates searching for String based Unification models being already consistent in the observed number of space-time dimensions.

In this notes I discuss models of supersymmetric strings which fall into such a criterion. To make the notes self-contained I begin with a brief discussion of extra dimensions in frameworks of String theory. Then, I discuss benefits and drawbacks of two most popular scenarios of Unification models with extra dimensions, the Kaluza-Klein scenario and the Brane World model. Next, I consider a formulation of String theory which is consistent, i.e. anomaly free, in $D = 4$, and outline some of features of the construction. My conclusions with summary of the results are collected in the end of the paper.

2. EXTRA DIMENSIONS FROM STRINGS

As I have noted in the above, the quantum consistency of String theory implies living in extra-dimensional world. This conclusion comes from an infinite-dimensional algebra of quantum operators [1]

$$[\hat{L}_m, \hat{L}_n] = (m-n)\hat{L}_{m+n} + A(m)\delta_{m+n}, \qquad (1)$$

$$A(m) = \frac{1}{12}D(m^3 - m),$$

which corresponds to a classical infinite-dimensional algebra

$$\{L_m, L_n\} = i(m-n)L_{m+n}, \qquad (2)$$

generating conformal transformations on a two-dimensional strings' world-sheet.

The difference between quantum and classical Virasoro algebras (1), (2) consists in the central element of the algebra $A(m)$. This term is absent in (2), and appears after the normal ordering of quantum oscillators entering the Virasoro generators $\hat{L}_m$. The central extension of (1) encodes the conformal anomaly in the quantized theory, and it manifestly depends on the space-time dimension. In fact, $A(m)$ is only a part of the total anomaly coefficient, since the classical Virasoro operators generate the residual world-sheet symmetry after the conformal gauge fixing. Following the Faddeev-Popov recipe ghosts have to be introduced, and their contribution into the anomaly coefficient is [1]

$$A^{b,c}(m) = \frac{1}{6}(m - 13m^3).$$

The total anomaly coefficient

$$A^{total}(m) = A(m) + A^{b,c}(m) + 2a_{open}m. \qquad (3)$$

also contains the contribution from the open string intercept a_{open}.

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The anomaly absence, i.e. $A^{total}(m) = 0$, implies the space-time dimension D=26 and the string intercept $a_{open} = 1$. The same result holds for a closed bosonic string [1].

In the case of Neveu-Schwarz-Ramond (open) superstring (NSR superstring [2]) the Virasoro algebra is modified with two additional graded commutators, that leads to the set of two total anomaly coefficients. Depending on boundary conditions on world-sheet fermions two sectors of quantum oscillators appears: the Ramond (R) sector with

$$\begin{cases} A^{total}(m) = \frac{D}{8}m^3 + \frac{1}{6}(m - 13m^3) + \\ \quad + \frac{1}{12}(11m^3 - 2m) + 2a^R_{open}m, \\ \\ B^{total}(m) = \frac{D}{2}m^2 - 5m^2 + 2a^R_{open}\,, \end{cases} \tag{4}$$

and the Neveu-Schwarz (NS) sector, where

$$\begin{cases} A^{total}(m) = \frac{D}{8}(m^3 - m) + \frac{1}{6}(m - 13m^3) + \\ \quad + \frac{1}{12}(11m^3 + m) + 2a^{NS}_{open}m, \\ \\ B^{total}(m) = \frac{D}{2}(m^2 - \frac{1}{4}) + (\frac{1}{4} - 5m^2) \\ \quad + 2a^{NS}_{open}\,. \end{cases} \tag{5}$$

From $A(m) = 0$, $B(m) = 0$ it follows the critical dimension D=10 and the string intercepts $a^R_{open} = 0$, $a^{NS}_{open} = 1/2$.

Therefore, the presence of extra dimensions is an intrinsic property of the consistently quantized String theory. We get 22 space-like extra dimensions in bosonic string theory and 6 extra spatial dimensions for superstrings. The superstring case is more preferable since we have not the tachyonic vacuum state here: $a^{NS}_{open} = 1/2$ corresponds to the tachyonic vacuum, while $a^R_{open} = 0$ leads to a well-defined zero-energy vacuum state. Though we have $a^{NS}_{open} = 1/2$ in the Neveu-Schwarz sector of superstring, we get rid off the tachyon state taking the Gliozzi-Scherk-Olive (GSO) projection [3]. Once the GSO projection is applied the spectrum of states in NS+R sectors possesses D=10 space-time supersymmetry. In what follows I will mainly focus on the superstring case, where the number of extra dimensions is 6.

3. LIVING WITH EXTRA DIMENSIONS

From the String theory point of view, our World looks as follows (see Fig.1):

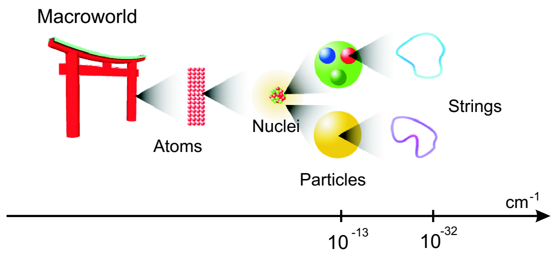

Fig.1. *The Stringy World*

To make a contact of String theory living in ten-dimensional world to observable physics in D=4 a special procedure of compactifying the extra dimensions has to be realized. There are many ways to this end, but the right way, which would reproduce main properties of the Standard Model (or its minimal supersymmetric extension), is up to date missed.

Dealing with extra dimensions one may wonder what is their size and what is the nature of extra dimensions? In the Kaluza-Klein picture [4] the extra dimensions have a small size that leads to appearing very massive particles after the compactification, with masses $M \sim 1/l_{comp.}$ ($l_{comp.}$ is a characteristic length of a compactified dimension).

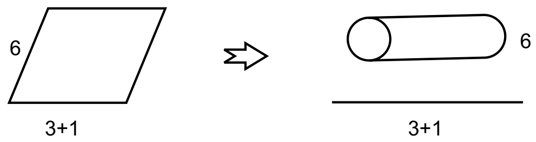

Fig.2. *The Kaluza-Klein picture*

Massive modes coming from the Kaluza-Klein compactification are too massive to be ever experimentally observed, so the best one can do is to consider massless modes, corresponding in part to the gauge bosons of the SM symmetry group $SU(3) \times SU(2) \times U(1)$, or an extended symmetry group including the SM group as a subgroup (see Fig.2).

Common drawbacks of the Kaluza-Klein scheme consist in:

- *Unsatisfactory spectrum of particles appearing upon the reduction which does not fit well the spectrum of the SM fields.* The desired spectrum of the Kaluza-Klein massless modes has to be realistic. A part of this spectrum should correspond to the gauge bosons of the Standard Model that puts restrictions on the type of the internal six-dimensional manifolds. However, the way of getting masses for the rest of the modes and establishing their correspondence to other Standard Model fields is an open task [5].

- *Gauge hierarchy problem still takes place.* The Kaluza-Klein scenario does not resolve the hierarchy problem, the gravity scale still remains near the Plank scale.

- *Typically four rather than three generations of quarks and leptons.* The exact number of generations coming after the dimensional reduction is strongly depended on geometrical and topological characteristics of internal six-dimensional manifolds. Roughly speaking, the number of fermion generations is twice less than the main topological number of the internal manifold (the Betti number). It turns out that the minimal Betti number for phenomenologically relevant internal manifolds is equal to 8 (Calabi-Yau manifolds), hence the number of generations is 4. A way to resolve this problem is to consider special manifolds of a Calabi-Yau type with the relevant Betti number [6], or to reduce on orbifolds [7] which are not manifolds in a common sense. Another perspective direction is to consider branes intersections [8] within the Brane World scenario (see below).

- *Masses and chiralities of fermions.* After the reduction fermions received masses of the compactification scale order, i.e. huge masses. Massless fermions comes from the zero-eigenvalue states of the Dirac operator on a compact internal manifold. In most phenomenologically interesting cases such zero-eigenvalue states do not exist. Another problem is to recover chiral fermions after the reduction. It may not be correctly resolved within the standard Kaluza-Klein scheme (see e.g. [9] and Refs. therein).

- *Large cosmological constant.* This point becomes important in context of String theory application to Cosmology and astrophysics, since we have definitely known that the right cosmological constant is small.

Problems with Kaluza-Klein motivated searching for other scenarios. One of them became popular last decade is the Brane World scenario [10].

Within the Brane World (BW) scenario it is supposed that fields of the SM do confine on a 4-dimensional brane (3-brane). A 3-brane is embedded in a higher-dimensional World. Gravity takes a special place in the BW picture since gravity does not confine on a 3-brane and gets trapped in high dimensions.

From the String theory point of view the BW picture looks like (see Fig.3):

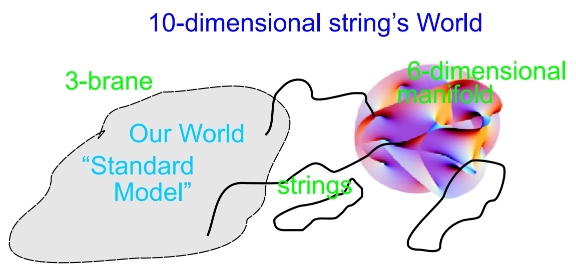

Fig.3. *The Brane-World picture*

A 3-brane is embedded into ten-dimensional space-time, a connection between 3-brane and extra dimensions is realized through strings. What is important in such a scheme is that extra dimensions are large. It leads to essential decreasing of the effective Plank scale on a Brane, that resolves the hierarchy problem.

Substantial progress in the Stringy BW has been achieved, nevertheless several important problems still remain open:

- *How to break Supersymmetry in a correct way?* Indeed, once we are talking about a 3-brane, it naturally appears in type IIB supersymmetric String theory. One may wonder why it is so necessary to deal with Superstring theory? The answer is we would like to have a joint coupling constant in high energies that provides by supersymmetry, and we would like to have a unified theory of gravity and the SM fields that is realized in String theory. However, the SM is not a supersymmetric theory, hence the way of supersymmetry breaking has to be found.

- *How to set up the right cosmological constant in the end?* I recall that *Anti-de-Sitter* space is actively exploited within the BW. Hence, all the machinery of the AdS/CFT correspondence is applied here. But we have to recover the right, *de-Sitter* space, cosmological constant in the end, which is the experimentally verified cosmological constant driving the late-time acceleration of Universe.

- *The predicted gravity scale is over TeV, but should we believe in that?* The BW scenario is a proposal for the resolving the hierarchy problem. However, we have not any signals on TeV quantum gravity (as well as on extra dimensions) up to date that makes the point questionable.

4. LIVING WITHOUT EXTRA DIMENSIONS

Living in extra dimensional World makes possible to resolve some of the fundamental problems of the Standard Model. At the same time the major worry on extra-dimensions is the absence of any experimental signals in favor of their existence. Once living in extra-dimensional World will be experimentally verified, it will get rid of any doubts on them, and on String theory, which predicts extra dimensions, as well. Currently, all possible ways of constructing Unification models, with or without extra dimensions, are needed to be taken into account on equal footing.

4.1. SUSY algebra and supersymmetric strings in extended superspaces

I have noted String theory is good enough to unify gravity with other interactions. But could we find a comprehensive String theory with realistic critical dimensions?

To get an answer let me begin with reviewing an irrelevant at first sight subject. In 1988 Curtright [11] made an analysis of the maximally extended SUSY algebra in D=11 (M-theory algebra [12], [13]). The algebra in particular includes

$$\{Q, Q\} = \gamma^a P_a + \gamma^{ab}\mathfrak{P}_{ab} + \gamma^{abcde}\mathfrak{P}_{abcde}. \quad (6)$$

The right hand side of (6) contains different types of "momenta". Dynamical charge P_a corresponds to the standard momenta, other 'momenta' are topological charges corresponding to 'electrically' charged membrane and 'magnetically' charged 5-brane. Clearly,

$$\mathfrak{P}_{ab} = -\mathfrak{P}_{ba}, \qquad \mathfrak{P}_{abcde} = \mathfrak{P}_{[abcde]}.$$

Membranes and five-branes appear in eleven-dimensional M-theory, however there is not a room for strings there.

What happens if charges on the r.h.s. of (6) will be treated in more democratic way? They are different, of course, the dynamical momenta have the conjugated coordinates, whilst topological charges have not. To reach charges democracy Curtright proposed, instead of the standard D=11 superspace (X^a, θ^α), an 'extended' D=11 superspace $(X^a, Z^{ab}, Z^{abcde}, \theta^\alpha)$ [11], where Z^{ab}, Z^{abcde} are "coordinates" conjugated to topological charges $\mathfrak{P}_{ab}$, $\mathfrak{P}_{abcde}$. Curiously enough, there exists a room for superstrings in such an extended superspace.

A general form of the Curtright's superstring action with unit tension looks as follows [11]

$$S = \int d^2\xi \sqrt{-\det(\omega^a_\mu\omega_{\nu a} + \alpha\omega^{ab}_\mu\omega_{\nu ab} + \beta\omega^{abcde}_\mu\omega_{\nu abcde})} \quad (7)$$
$$+S_{WZ}.$$

The building blocks of the action consist of the pull-back of D=11 Volkov-Akulov superform $\omega^a_\mu = \partial_\mu X^a + i\bar\theta\gamma^a\partial_\mu\theta$, its extensions to tensorial-type coordinates $\omega^{ab}_\mu = \partial_\mu Z^{ab} + i\bar\theta\gamma^{ab}\partial_\mu\theta$ and $\omega^{abcde}_\mu = \partial_\mu Z^{abcde} + i\bar\theta\gamma^{abcde}\partial_\mu\theta$. Two parameters α, β are constants fixed by supersymmetry in the end, and the last terms of the action is the Wess-Zumino term. The term by Wess and Zumino was introduced in (7) to reach the invariance of the action under a local fermionic symmetry, the so-called kappa-symmetry, taking an important place in theory of supersymmetric extended objects. Nevertheless, in the original Curtright's paper the kappa-invariance of the action was rather claimed than exactly proved.

Now what about D=4? A similar extension of D=4 superspace was considered by Amorim and Barcelos-Neto [14], and a line of they reasoning was almost the same.

The maximally extended N=1 D=4 superalgebra in particular contains [12]

$$\{Q, Q\} = \gamma^a P_a + \gamma^{ab}\mathfrak{P}_{ab}. \quad (8)$$

Adding new tensor-type coordinates $Z^{ab} = -Z^{ba}$, which are conjugated to "momenta" $\mathfrak{P}_{ab}$, we get an extended superspace $(X^a, Z^{ab}, \theta^\alpha)$. $\mathfrak{P}_{ab}$ is commonly treated as a topological charge (due to a D=4 membrane), but treating it dynamically it's possible to construct a Green-Schwarz-type superstring in the extended superspace

$$S = \int d^2\xi \sqrt{-\det(\omega^a_\mu\omega_{\nu a} + \alpha\omega^{ab}_\mu\omega_{\nu ab})} + S_{WZ}. \quad (9)$$

The notation in (9) is that of (7). I postpone the discussion of (9) to the end of the paper, currently focussing on the superconformal algebra in the extended superspace and on the superstring critical dimension.

4.2. Superconformal algebra in tensorial superspace and superstring's critical dimension

To calculate the critical dimension of tensorial superstring let us turn back to the bosonic string case. As it has been noted in the above the total conformal anomaly coefficient (eq. (3))

$$A^{total}(m) = \frac{1}{12}D(m^3 - m) + \frac{1}{6}(m - 13m^3) + 2a_{open}m$$

contains contributions from bosonic fields X^a, conformal (anti)ghosts and the string intercept. One could notice that

- The critical dimension is calculated from setting the terms proportional to m^3 to zero.
- D bosonic coordinates X^a contribute the relative coefficient D.
- The conformal (anti)ghosts contribute the relative coefficient "-26" independently on the number of space-time dimensions.

Hence, we need 26 bosonic coordinates X^a to compensate the ghosts contribution, $+26 - 26 = 0$.

In the NSR superstring case one of the total superconformal anomaly coefficients has the following form

$$A^{total}(m) = \left(\frac{D}{12}\cdot 1 + \frac{D}{12}\cdot\frac{1}{2}\right)m^3 + \frac{1}{6}(m - 13m^3) +$$
$$+\frac{1}{12}(11m^3 - 2m) + 2a^R_{open}m\,. \quad (10)$$

It's easy to recognize the contributions of bosonic X^a, fermionic conformal (anti)ghosts, bosonic superconformal (anti)ghosts and the contribution of the string intercept. But what about the second term of (10)?

This term contains the contribution of the world-sheet fermionic superpartners ψ^a of the bosonic coordinates X^a. Clearly, the fermionic superpartners contribute only 1/2 of the corresponding bosonic coefficient.

Hence, to calculate the critical dimension the following mnemonic rule may be used [1]:

- D bosons get the coefficient D.
- The input of D fermions (boson's superpartners) is $D/2$.
- The (super)conformal ghosts give '-26' for conformal (fermionic) ghosts, and superconformal (bosonic-type) ghosts contribute "+11".

The difference '$-26+11=-15$' has to be compensated with contributions of additional bosonic and/or fermionic fields, one of the realizations of which are bosonic coordinates X^a and their world-sheet superpartners ψ^a.

Let us fix the space-time dimension $D=4$. Four bosonic coordinates X^m and their four world-sheet superpartners ψ^m contribute the coefficient $4+4/2=+6$. On account of ghosts contribution it is necessary to compensate the coefficient equal to $-15+6=-9$. There are different routes to this end. Say, if one were to use 6 'internal' coordinates y^i and their superpartners ψ^i $(i=1,\ldots,6)$, this choice would be transformed into the standard $4+6=10$ set of coordinates of the NSR superstring in the end.

Another productive choice suggested by $D=4$ $N=1$ superalgebra structure is to consider 6 additional tensorial-type coordinates $Z^{mn}=-Z^{nm}$ together with their world-sheet superpartners $\Psi^{mn}=-\Psi^{nm}$ [15]. This set of coordinates contribute the required coefficient $+9$, hence we arrive at the consistent quantum formulation of superstring in the *observable number of space-time dimensions.*

5. NEW SET OF COORDINATES AND NEW FIELDS

We have established the existence of a NSR-type superstring formulation with realistic critical dimension. The price we paid to this end is the extension of the conventional space-time with additional tensorial-type bosonic coordinates. Let me take an extensive treatment of new coordinates in the so extended space and give more strong evidence for the quantum consistency of the superstring.

Note to this end the bosonic subset (X^m, Z^{mn}) could be embedded into the unique set of tensorial coordinates Z^{MN}, but in $D=5$,

$$Z^{\tilde m 5} \rightsquigarrow X^m, \qquad Z^{\tilde m \tilde n} \rightsquigarrow Z^{mn}, \quad \tilde m = 0,\ldots,3\,. \tag{11}$$

Such a coordinates embedding of (X^m, Z^{mn}) can be done in any space-time dimension $(D-1)$ thus leading to D-dimensional space parameterized by Z^{MN}.

After that, instead of the standard Nambu-Goto string action functional in the conventional space-time with coordinates $X^m(\xi)$

$$S_{NG} = \frac{T}{2}\int d^2\xi\, \sqrt{-\det \partial_\mu X^m \partial_\nu X_m}\,,$$

we have the action in terms of solely tensorial coordinates $Z^{MN}(\xi)$

$$S = \frac{T}{2}\int d^2\xi\, \sqrt{-\det \partial_\mu Z^{MN} \partial_\nu Z_{MN}}\,. \tag{12}$$

Here, as well as in the action before, T is the string tension.

The action (12) possesses the same world-sheet symmetries as that of the Nambu-Goto action, hence after the appropriate gauge fixing the equation of motion of $Z^{MN}(\xi)$ is reduced to

$$\partial^\mu \partial_\mu Z^{MN} = 0. \tag{13}$$

In what follows I will consider the closed tensorial string, then the solution to eq. (13) satisfying the closed string boundary conditions is

$$\begin{aligned} Z^{MN}_R &= \frac12 z^{MN} + \frac12 l^2 p^{MN}(\tau-\sigma) + \\ &+ \frac{l}{2}\sum_{n\neq 0}\frac1n \alpha^{MN}_n e^{-2in(\tau-\sigma)}\,, \\ Z^{MN}_L &= \frac12 z^{MN} + \frac12 l^2 p^{MN}(\tau+\sigma) + \\ &+ \frac{l}{2}\sum_{n\neq 0}\frac1n \tilde\alpha^{MN}_n e^{-2in(\tau+\sigma)}\,. \end{aligned} \tag{14}$$

Here, as usual, I set $l=\sqrt{2\alpha'}=1/\sqrt{\pi T}$, where T is the string tension. $Z^{MN}_{R,L}$ are supposed to be real that leads to $\alpha^{MN}_{-n}=(\alpha^{MN}_n)^\dagger$, $\tilde\alpha^{MN}_{-n}=(\tilde\alpha^{MN}_n)^\dagger$.

Now we have to define the Poisson brackets between canonical variables. They are

$$\begin{aligned} \{\dot Z^{MN}(\sigma), Z^{PQ}(\sigma')\}_{PB} &= \frac12 T^{-1}\times \\ \times \left(\eta^{MP}\eta^{NQ} - \eta^{MQ}\eta^{NP}\right)&\delta(\sigma-\sigma')\,, \end{aligned} \tag{15}$$

and the overall factor in the r.h.s. of the Poisson brackets has chosen to be one half to get the right canonical Poisson brackets between $D-1$ vector coordinates and their momenta

$$\{\dot X^m(\sigma), X^n(\sigma')\}_{PB} = T^{-1}\eta^{mn}\delta(\sigma-\sigma') \tag{16}$$

after identifying $\sqrt2 Z^{m(D)} = X^m$.

Substituting (14) into (15) we derive, by use of

$$\sum_{n=-\infty}^{\infty} e^{in(\sigma-\sigma')} = 2\pi\delta(\sigma-\sigma')\,,$$

the following non-trivial Poisson brackets

$$\{\alpha^{MN}_n, \alpha^{KL}_m\}_{PB} = \frac{i}{2} n\delta_{n+m}\left(\eta^{MK}\eta^{NL} - \eta^{ML}\eta^{NK}\right)\,, \tag{17}$$

$$\{\tilde\alpha^{MN}_n, \tilde\alpha^{KL}_m\}_{PB} = \frac{i}{2} n\delta_{n+m}\left(\eta^{MK}\eta^{NL} - \eta^{ML}\eta^{NK}\right)\,. \tag{18}$$

Next, we construct the Virasoro generators

$$L_m = \frac{T}{2}\int_0^\pi e^{-2im\sigma}\dot Z^2_R\, d\sigma|_{\tau=0} = \frac12\sum_{-\infty}^{\infty}\alpha^{MN}_{m-k}\alpha^{MN}_k\,. \tag{19}$$

Here we have substituted

$$\dot Z^{MN}_R = l\sum_{-\infty}^{\infty}\alpha^{MN}_m e^{-2im(\tau-\sigma)}\,,$$

and have used

$$\int_0^\pi e^{2im\phi}e^{-2in\phi}d\phi = \pi\delta_{mn}\,.$$

Clearly, $\alpha^{MN}_0 = \frac12 l p^{MN}$. The left moving Virasoro generators $\tilde L_m$ are identical to (19) with relpacing $\alpha^{MN}_m \to \tilde\alpha^{MN}_m$, and $\alpha_0 = \tilde\alpha_0$.

Taking into account the Poisson brackets (17), (18) one may calculate the Poisson brackets between the Virasoro generators

$$\{L_m, L_n\}_{PB} = i(m-n)L_{m+n},$$
$$\{\tilde{L}_m, \tilde{L}_n\}_{PB} = i(m-n)\tilde{L}_{m+n}. \qquad (20)$$

There is no difference between the Poisson brackets in (20) and those of the standard bosonic string theory. The same happens for the quantum Virasoro algebra, which is

$$[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n},$$
$$[\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n} + A(m)\delta_{m+n}, \qquad (21)$$

and $A(m) = c_3 m^3 + c_1 m$. Two unfixed coefficients are calculated in the standard way [1] that results in

$$A(m) = \frac{1}{24}D(D-1)(m^3 - m). \qquad (22)$$

The conformal anomaly coefficient possesses the same dependence on m as its standard counterpart entering eq. (1). This is not a surprise since the result is based on properties of a two-dimensional world-sheet of any string. The difference comes from the contribution of the world-sheet coordinates: it is equal to D in the conventional bosonic string theory, while for the tensorial string case it becomes $D(D-1)/2$ (as it should follow from the mnemonic rule considered in the previous section).

The $b-c$ ghosts system contributes

$$A^{b,c}(m) = \frac{1}{3}(m - 13m^3), \qquad (23)$$

where we have summed over the left and the right modes contributions. Summing up, the total anomaly, which has to be canceled, is

$$A^{total}(m) = \left[\frac{1}{2}D(D-1)\right] \times \frac{1}{6}(m^3 - m) +$$
$$+\frac{1}{3}(m - 13m^3) + 2a_{cl.}m. \qquad (24)$$

The expression we got is very similar to that of the total anomaly of a bosonic string with vector-type coordinates (cf. eq. (1)). The difference is just in replacing D in the standard bosonic string case with $\frac{1}{2}D(D-1)$. From (24) one may notice that there is not a (integer) critical dimension where anomaly is canceled, and the ordering constant is the same as for a closed bosonic string, $a_{cl.} = 2$.

Let me turn to the NSR-type tensorial superstring. The world-sheet superpartners of X^m and Z^{mn} can also be recast into the single world-sheet fermion Ψ^{MN}

$$\Psi^{\tilde{m}5} \rightsquigarrow \psi^m, \qquad \Psi^{\tilde{m}\tilde{n}} \rightsquigarrow \psi^{mn}, \quad \tilde{m} = 0, \ldots, 3\,.$$

The gauged fixed action of the tensorial superstring (compare to [16], [17]) is in the case

$$S = \frac{T}{2}\int d^2\xi \,\left(\partial_\mu Z^{MN}\partial^\mu Z_{MN} - i\bar{\Psi}^{MN}\rho^\mu\partial_\mu\Psi_{MN}\right). \qquad (25)$$

In the NSR-type formulation we are dealing with tensorial-type coordinates Z^{MN} which are scalars w.r.t. the world-sheet diffeomorphisms, and with their superpartners Ψ^{MN} which are world-sheet spinors. If we calculate the Virasoro-like superalgebra of the NSR-type string following standard methods of [1], the difference in the total anomaly coefficients, in compare to the standard NSR superstring case, consists just in replacing D in (4), (5) with $\frac{1}{2}D(D-1)$ [15]. This result is very expected from the previous calculations of the bosonic tensorial string anomaly coefficient. We have

$$\begin{cases} A^{\text{total}}(m) = \frac{1}{12}(\frac{D(D-1)}{2} + \frac{D(D-1)}{4})m^3 + \\ +\frac{1}{6}(m - 13m^3) + \frac{1}{12}(11m^3 - 2m) + 2a^R m, \\ \\ B^{\text{total}}(m) = \frac{D(D-1)}{4}m^2 - 5m^2 + 2a^R\,, \end{cases} \qquad (26)$$

in the Ramond sector and

$$\begin{cases} A^{total}(m) = \frac{1}{12}(\frac{D(D-1)}{2} + \frac{D(D-1)}{4})(m^3 - m) + \\ +\frac{1}{6}(m - 13m^3) + \frac{1}{12}(11m^3 + m) + 2a^{NS}m, \\ \\ B^{total}(m) = \frac{D(D-1)}{4}(m^2 - \frac{1}{4}) + (\frac{1}{4} - 5m^2) + \\ \qquad +2a^{NS} \end{cases} \qquad (27)$$

in the Neveu-Schwarz sector.

The critical dimension is $D = 5$ in the case, and we arrive at new formulation of superstring theory living in five-dimensional space-time endowed with coordinates Z^{MN}. This parametrization includes, as a four-dimensional part, the standard set of vector-type coordinates X^m, hence coordinates Z^{MN} seem to be more fundamental than X^m, and the tensorial string theory in $D = 5$ becomes more fundamental than its four-dimensional analog. All of that reminds of the M-theory–String theory relation, when the more fundamental theory is formulated in a space-time of one spatial dimension higher.

Another consequence of introducing the new space-time parametrization may be viewed from the following observation. Recall that calculating the Virasoro-like superalgebra of the NSR-type $D = 5$ tensorial string theory we have used the canonical relation between Z^{MN} and their conjugate generalized momenta $\mathfrak{P}_{MN}$

$$\{\mathfrak{P}^{MN}(\sigma), Z^{KL}(\sigma')\}_{PB} = \frac{1}{2}T^{-1}\times$$
$$\times\left(\eta^{MK}\eta^{NL} - \eta^{ML}\eta^{NK}\right)\delta(\sigma - \sigma')\,.$$

When we pass to operators, the generalized momenta become $\hat{\mathfrak{P}}_{MN} = -i\partial_{MN}$, $\partial_{MN}Z^{MN} = \frac{1}{2}D(D-1)$. Hence, in tensorial space there is an exotic *one-form* of the Yang-Mills-type $\mathcal{A} = \mathcal{A}_{MN}dZ^{MN}$. Its strength tensor is

$$\mathcal{F}_{MN,KL} = \partial_{MN}\mathcal{A}_{KL} - \partial_{KL}\mathcal{A}_{MN} + i[\mathcal{A}_{MN}, \mathcal{A}_{KL}] \qquad (28)$$

and the action functional for such a field looks like

$$S = \frac{1}{4}\mathrm{Tr}\int d\Omega_D\, \mathcal{F}_{MN,KL}\mathcal{F}^{MN,KL},$$

where $d\Omega_D$ is the invariant volume form in D-dimensional tensorial space.[1] The one-form field $\mathcal{A}$ is an analog of a non-abelian Yang-Mills gauge field in the conventional space-time. It corresponds to the massless mode in the spectrum of open tensorial string. Indeed, let me define the vacuum state $|0\rangle$ as $\hat\alpha_n^{MN}|0\rangle = 0$ for $n > 0$, where $\hat\alpha_n^{MN}$ are the annihilation operators. They are quantum analog of classical oscillators α_n^{MN} entering (14). The first exited level in the momentum representation is described by

$$\mathcal{A}_{MN}(k)\hat\alpha_{-1}^{MN}|0\rangle, \qquad k^{MN}\mathcal{A}_{MN}(k) = 0,$$

which is nothing but the above mentioned one-form gauge field.

In the spectrum of the closed tensorial string there are other exotic massless modes $G_{MN|PQ}$ and $B_{MN|PQ}$ corresponding to graviton and the Kalb-Ramond antisymmetric tensor fields of the standard superstring spectrum:

$$G_{MN|PQ}\,\hat{\hat{\alpha}}_{-1}^{\{MN}\,\hat\alpha_{-1}^{PQ\}}|0\rangle, \qquad B_{MN|PQ}\,\hat{\hat{\alpha}}_{-1}^{[MN}\,\hat\alpha_{-1}^{PQ]}|0\rangle.$$

The 'graviton' mode $G_{MN|PQ}$ possesses the following properties

$$G_{MN|PQ} = -G_{NM|PQ} = -G_{MN|QP} = G_{PQ|MN}$$

which are formally the same as that of the curvature tensor in the conventional space. For a "Kalb-Ramond" field we have

$$B_{MN|PQ} = -B_{NM|PQ} = -B_{MN|QP} = -B_{PQ|MN}.$$

The remaining massless mode in the spectrum is a 'dilaton' Φ

$$\Phi\,\hat{\hat{\alpha}}_{-1}^{MN}\hat\alpha_{MN\ -1}|0\rangle.$$

Having such exotic modes it is important, from the point of view of various applications, to understand the dynamics of these fields. The action functional for the 'dilaton' and the 'Kalb-Ramond' fields is more or less predictable. It is likely

$$S = \int d\Omega_5\,(\frac{1}{2}\,\partial_{MN}\Phi\partial^{MN}\Phi +$$
$$+\frac{1}{12}\,H_{MN,\ KL|PQ}H^{MN,\ KL|PQ}),$$

where the "Kalb-Ramond" field strength is defined by

$$H_{MN,\ KL|PQ} = \partial_{MN}B_{KL|PQ} +$$
$$+\partial_{PQ}B_{MN|KL} + \partial_{KL}B_{PQ|MN}.$$

As for the effective action of 'graviton' $G_{MN|PQ}$, its structure is unclear. It could be recovered from calculations of the 3-point tree amplitude of interacting strings in the low-energy approximation (as, for instance, in [19]) and I postpone this task for further studies.

6. SUMMARY AND CONCLUSIONS

We have discussed a reformulation of superstring theory, the critical dimension of which coincides with the observable space-time dimension.

To recover the critical dimension $D = 4$ an extension of the standard space-time is required. New elements which have to be taken into account are tensorial-type bosonic coordinates. From the point of view of the string world-sheet theory, it does not matter what kind of bosonic coordinates need to be added to compensate the superconformal anomaly. They could be scalars, vectors or tensors under the space-time Poincare. The main point is that they are scalars with respect to the world-sheet diffeomorphisms.

One may wonder, that is a rule for selecting new coordinates then? What kind of the coordinates have to be selected to parameterize the target space? It turns out that the choice of the string's coordinates describing an immersion of the string world-sheet into a target superspace is governed by the structure of a target space superalgebra.

Let me discuss the target-space – world-sheet correspondence in more detail. There are two independent formulations of superstrings:

1. Neveu-Schwarz-Ramond with the world-sheet supersymmetry;

2. Green-Schwarz with the manifest target-space supersymmetry.

As I have noted these formulations are equivalent in $D = 10$, since their quantum spectra coincide (after truncation of the NSR spectrum with the GSO projection (see Fig.4)) and their critical dimensions are the same.

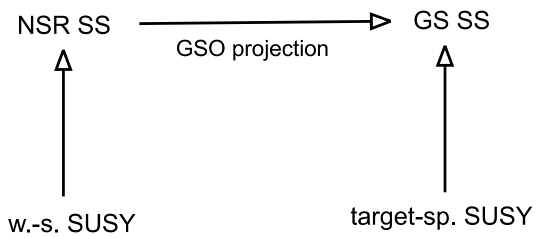

Fig.4. *NSR-GS connection*

The world-sheet SUSY in the NSR formulation just says that there are world-sheet scalars and their superpartners under the world-sheet supersymmetry. However, it doesn't say anything on properties of these variables under the target-space Poincare transformations.

In its turn, properties of the string coordinates in the Green-Schwarz formulation are fixed. Indeed, a part of the space-time SUSY algebra is

$$\{Q, Q\} = \gamma^a P_a + \ldots$$

[1] I should recall that the number of coordinates of D-dimensional tensorial space does not coincide with D.

and string coordinates are defined as ones conjugated to P_a. They are vector-type coordinates X^a with respect to the target-space Poincare. Precisely this type of the coordinates enter the standard NSR string action. Extending the space-time to superspace recovers the rest of the coordinates entering the Green-Schwarz superstring action, the space-time fermions θ^α. They are the target-space superpartners of X^a.

Hence, the relation between NSR and GS superstrings observes an independent interpretation, in which properties of the space-time SUSY, manifest in the GS formulation, govern the choice of the space-time coordinates to describe the NSR string.

Let me now turn to the Green-Schwarz-type action (9). This action is based on the target space supersymmetry algebra involving the supercharges anticommutator (8). If we give a credit to having a correspondence between NSR and GS formulations in the extended superspace $(X^m, Z^{mn}, \theta^\alpha)$, the NSR-type tensorial superstring variables are (X^m, Z^{mn}) together with their world-sheet superpartners (ψ^m, Ψ^{mn}). Indeed, these variables casting Z^{MN} and Ψ^{MN} enter the gauge fixed action (25).

As for the NSR-type tensorial superstring the bosonic subset (X^m, Z^{mn}) of the Green-Schwarz-type tensorial superstring coordinates could be embedded into the unique set of tensorial coordinates Z^{MN}, but in $D = 5$.

$$Z^{\tilde{m}5} \rightsquigarrow X^m, \quad Z^{\tilde{m}\tilde{n}} \rightsquigarrow Z^{mn}, \quad \tilde{m} = 0, \ldots, 3\,. \quad (29)$$

The # of fermionic target-space superpartners θ^α is the same in $D = 5$ and $D = 4$. Therefore, it is possible to reformulate the superstring model solely in terms of (Z^{MN}, θ^α) coordinates that essentially simplifies the Green-Schwarz-like tensorial superstring action [18] and proving its kappa-invariance. Moreover, the consistency of the Green-Schwarz-type superstring model in $D = 5$ tensorial superspace (Z^{MN}, θ^α) (kappa-invariance of the action) also requires [18]

$$G_{M[N|PQ]} = 0\,.$$

This condition just says that the field $G_{MN|PQ}$ is in the [2,2] irreducible rep. over the Lorentz in $D = 5$ tangent space.

At the same time I should note that the Green-Schwarz-type formulation of tensorial superstring in $D = 4$ (or equivalently in $D = 5$) extended superspace faces with several questionable points. First of all one may encounter an apparent mismatch between bosonic and fermionic degrees of freedom in the case. Hence, it is necessary to understand the root of the problem. A helpful way to this end is to recover the spectrum of open/closed tensorial strings in different formulations and to figure out an analog of the GSO projection to relate spectra of tensorial superstrings. Perhaps, applying the machinery of the twistor-like superembedding approach [20], [21] (and Refs. therein), which 'closes' the diagram on Fig.4, may be useful to this end. Another intriguing problem is to construct the effective action of massless modes to check a correspondence of the approach to that of [22] where a new concept of the area metric was introduced.

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О КВАНТОВО-СОГЛАСОВАННЫХ МОДЕЛЯХ СУПЕРСТРУН В ЧЕТЫРЕХМЕРНОМ ПРОСТРАНСТВЕ-ВРЕМЕНИ

А.Ю. Нурмагамбетов

Наличие дополнительных измерений пространства-времени предсказывается теорией струн. Однако, на сегодняшний день не существует каких-либо экспериментальных подтверждений в пользу их существования. Данное обстоятельство дает толчок к поиску последовательных формулировок теории суперструн в четырехмерном пространстве-времени. Одним из решений проблемы является расширение стандартного набора четырехмерных пространственно-временных координат векторного типа дополнительными бозонными координатами тензорного типа. В работе обсуждается причина введения именно такого набора дополнительных координат, а также приводятся вычисления критической размерности для формулировки тензорной суперструны типа Невье-Рамона-Шварца. Также обсуждается роль новых координат в построении последовательной формулировки суперструны в пространстве-времени размерности пять исключительно в терминах координат тензорного типа и их суперпартнеров относительно преобразований ‘суперсимметрии’ на мировом листе струны. Кратко рассмотрены свойства безмассовых мод в спектрах открытой и замкнутой пятимерных суперструн.

ПРО КВАНТОВО-УЗГОДЖЕНІ МОДЕЛІ СУПЕРСТРУН У ЧОТИРИВИМІРНОМУ ПРОСТОРІ-ЧАСІ

О.Ю. Нурмагамбетов

Наявність додаткових просторо-часових вимірів передбачено теорією струн. Проте, на сьогоднішній день не існує яких-небудь експериментальних підтверджень на користь їх існування. Ця обставина дає поштовх до пошуку послідовних формулювань теорії суперструн у чотиривимірному просторі-часі. Одним з вирішень проблеми є розширення стандартного набору чотиривимірних просторово-часових координат векторного типу додатковими бозонними координатами тензорного типу. У роботі обговорюється причина введення саме такого набору додаткових координат, а також приводяться обчислення критичної вимірності для формулювання тензорної суперструни типу Невье-Рамона-Шварца. Також обговорюється роль нових координат в побудові послідовного формулювання суперструни у просторі-часі вимірності п’ять виключно в термінах координат тензорного типу і їх суперпартнерів щодо перетворень ‘суперсиметрії’ на світовому листі струни. Стисло розглянуті властивості безмасових мод в спектрах відкритої і замкнутої п’ятивимірних суперструн.