RADIATION LOSSES OF ELECTRON ENERGY IN MULTILAYER BIMETALLIC MEDIA

B.V. Borts, I.V. Tkachenko, V.I. Tkachenko

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

E-mail: tkachenko@kipt.kharkov.ua

Evaluation of radiation losses of the electron in inhomogeneous media is presented. Such media may appear as a composition of material layers with different dielectric constants or it may be modeled with the materials which dielectric permeability varies in the space. It is shown that in inhomogeneous media with dielectric permeability variable by harmonic law in space, the radiation losses of electron are proportional to square of parameter of inhomogeneity, that is are low. It is shown that in the inhomogeneous media with dielectric permeability varying in the space under the harmonic law, the radiation losses of the electron are proportional to square of the parameter of inhomogeneity i.e. are low. In the case when the conditions of parametric relation between eigen waves of the medium are satisfied radiation losses of electron are proportional to the parameter of inhomogeneity and are comparable to the losses during the acts of scattering.

PACS: 41.60.-m

INTRODUCTION

It is known that accelerated electron during its movement in medium losses energy for ionization and radiation deceleration [1]. For electron energy exceeding critical one, its total losses are determined by the certain radiation mechanisms. Therefore further we will consider the energy of electron to be near-critical or supercritical and we will investigate losses of its energy on passing through multilayer bimetallic medium.

Radiation of charged fast particles in periodic media attracts the researchers attention for a long time [2,3]. Along with the investigation of hard short-wave length radiation when the wave length λ is considerably less than the medium period *d: λ << d/2γ*, where γ – relativistic factor, the investigation of radiation in long-wave part of spectrum, the case when *λ >> d* is also interesting. Such coherent radiation of fast charged particles caused by the crystal spatial periodicity is commonly named the parametric Cherenkov radiation (PCR). PCR likewise the Cherenkov radiation appears as the effect of medium polarization caused by the electric field of moving particle. But unlike Cherenkov radiation which condition is the excess of particle velocity over the light velocity in the medium, PCR doesn't need to meet this condition. In the last case the characteristic angle of radiation θ and the frequency of coherently released photon ω in periodic medium are related by the equation.

At the same time with investigation of hard short-wave length radiation when the wave length λ is considerably less than the medium period *d: λ << d/2γ*, where γ – relative factor, the investigation of radiation in long-wave part of spectrum, when *λ >> d* also is interesting. Such coherent radiation of fast charged particles caused by the spacing of crystal is usually named Cerenkov radiation. Also as Cerenkov radiation parametric Cherenkov radiation (PCR) is the consequence of medium polarization caused by the electric field of moving particle. But unlike Cherenkov radiation the condition for which is the excess of particle velocity over the velocity of light in medium, for PCR the satisfaction of this inequality is not necessary. In the last case the characteristic angle of radiation θ and frequency of coherently released photon ω in periodic medium are related by relationship:

$$\omega \cdot \frac{d}{c_p} \cdot \left(\cos\theta - \frac{c_p}{\mathrm{v}} \right) = 2 \cdot \pi \cdot n , \qquad (1)$$

where: d – constant of lattice; c_p – velocity of light in medium; v – velocity of charged particle; n – integral number.

The left part of equation (1) is the difference of phases of photons released by two successive lattice sites on a distance d one from another. Relation (1) for $n = 0$ corresponds to the zero difference of phases of interfering waves and is the condition of Cherenkov radiation initiation. The case of $n \neq 0$ corresponds to PCR which is possible only in a medium with a spatial periodicity. Results of experiments carried out until recently confirm the main properties of parametric radiation of Cherenkov radiation (see, for example, [4]).

Evaluation of power losses for radiation is of special interest during investigation of electron passing through the periodic media. So, on passing of the low-energy electron beam through multilayer nanostructures composed by two alternating materials with close dielectric permeability ε_1 and ε_2 the narrow band X-ray transient irradiation is generated, the power of which is proportional to the square of a small parameter $q=(\varepsilon_1 - \varepsilon_2)$ [5]. But under certain conditions the power of losses for radiation in such periodic media may be considerably higher, because in [5] the possibility of coupling of eigen electromagnetic waves of medium excited by electron was not taken into account.

Paper [6] is devoted to the investigation of this effect influence on the value of power losses. In this paper it is shown that the power of energy loss of electron for radiation is proportional to the small parameter q, that is, increases considerably. Those investigations were carried out considering medium inhomogeneity of the type $\varepsilon(\vec{r}) = \varepsilon_0 + q\cos(\vec{\kappa} \cdot \vec{r})$, where : $\vec{\kappa}$ – vector of inverse lattice of medium; $\vec{r}$ – space coordinate; $q \ll \varepsilon_0$.

In the presented paper the radiation losses of the relativistic electron energy in the multilayer bimetallic medium are investigated on the base of the proposed theory of the eigen waves' parametric correlation; characteristic angles of relativistic electron radiation are also determined.

RADIATION LOSSES OF ELECTRON ENERGY IN MULTILAYER PERIODIC MEDIA

Let us consider the radiation of non-relativistic electron moving in infinite periodic medium one period of which consists of two layers of different metals. We will describe the electromagnetic properties of the medium by introduction of high frequency dielectric permeability of metals composing the medium.

$$\varepsilon_{1,2}(\omega) = 1 - \frac{\omega_{p1,p2}^2}{\omega\left(\omega + i\nu_{eff}^{1,2}\right)}, \qquad (2)$$

where: $\omega_{pi} = \sqrt{\frac{4\pi e^2 Z_i n_i}{m_i}}$ – plasma frequency of electron gas of i – metal; e – electron charge; m – its mass; n_i – concentration of atoms of i – metal, Z_i – number of free electron per one atom of i – metal; $\nu_{eff}^{1,2} = \tau_{1,2}^{-1}$ – effective dissipation of i – metal determined through the time of relaxation $\tau_{1,2}$ [7].

For such metals as aluminum and tungsten the relaxation times at room temperatures appear to be $\tau \approx 10^{-14}\ldots10^{-15}$ s [7]. Therefore in the region of high frequencies ($\omega^2 \gg \omega_{p1,p2}^2$) the relation $\omega \gg \nu_{eff}^{1,2}$ is fulfilled.

Now we will consider the radiation of charged particle moving with a constant velocity $\vec{v}_0$ in infinite medium formed by alternating alloys of different thickness, for instance, of tungsten l_1 and aluminum l_2 (see Fig.1). Electromagnetic properties of these metals may be represented as (2).

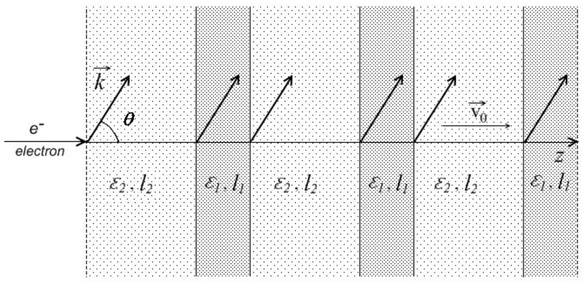

Fig.1. The configuration of resonant interaction of electron with a multilayer bimetallic medium

RADIATION LOSSES OF ELECTRON ENERGY IN HARMONIC PERIODIC MEDIA

Let us evaluate the energy losses of the electron in such periodic media on the base of approximation of the laminar bimetallic medium by its harmonic analogue.

We regard the metal boundaries to be not sharp and we will represent the effective high frequency dielectric permeability of such medium in the following model form:

$$\varepsilon(\vec{r}) = \varepsilon_0 + q\cos(\vec{\kappa}\cdot\vec{r}), \qquad (3)$$

where:

$$\varepsilon_0 = 1 - \frac{4\pi e^2}{m_e\omega^2}\frac{Z_1 n_1 l_1 + Z_2 n_2 l_2}{l_1 + l_2} = 1 - \frac{4\pi e^2}{m_e\omega^2}N_0; \qquad (4)$$

$$q = \frac{4\pi e^2\left|Z_1 n_1 - Z_2 n_2\right|}{m\omega^2}\frac{2}{\pi}sin\left(\frac{\pi l_1}{l_1 + l_2}\right) = \frac{4\pi e^2}{m\omega^2}\Delta N; \qquad (5)$$

$Z_\alpha n_\alpha$ – mean density of electron conduction in a metal of kind α [7]; $\vec{\kappa} = \frac{2\pi}{l}\vec{e}_z$ – vector of inverse lattice of periodically inhomogeneous medium; $l = l_1 + l_2$ – lattice period, $\vec{e}_z$ – unit vector directed along the axe *oz*; $|q| << |\varepsilon_0| < 1$ – index of space inhomogeneity of medium, $\vec{r}$ – space coordinate. The expression (3) represents first two terms of expansion into a Fourier series of mean electron density of alternating layers of aluminum and tungsten. For ratio of layers thickness in the range $0,5 < l_2/l_1 < 2$ the other terms of expansion in Fourier series may be neglected.

Representation of the medium as alternating layers of form (3) may be useful for qualitative determination of the radiation power of electromagnetic waves including the soft X-ray range.

Let us firstly determine the radiation power of the relativistic electron in the harmonic inhomogeneous medium of form (3) basing on noted known results [8]. In this case the transient radiation's power of relativistic electron in the medium with dielectric permeability: $\varepsilon = \varepsilon_0 + \Delta\varepsilon\cdot\sin\left(\vec{\kappa}\cdot\vec{r}\right)$ is determined by the expression

$$\frac{dW}{dt} = \frac{\left(e\Delta N\omega_p^2\right)^2}{32c\left(\kappa c N_0\right)^2}\int_0^\pi \theta^2 d\theta^2 \int_0^\infty \delta\left(\frac{\omega}{2}\left(\gamma^{-2} + \theta^2 + \frac{\omega_p^2}{\omega^2}\right) - \kappa v\right)d\omega =$$
$$= \frac{\left(e\Delta N\omega_p\right)^2}{16c\left(\eta N_0\right)^2}\int_{\omega_{\min}}^{\omega_{\max}}\left(2\eta\frac{\omega_p}{\omega} - \gamma^{-2} - \frac{\omega_p^2}{\omega^2}\right)\frac{d\omega}{\omega}, \qquad (6)$$

where: $\Delta\varepsilon = -\frac{4\pi e^2}{m\omega^2}\Delta N$; $\varepsilon_0 = 1 - \frac{4\pi e^2}{m\omega^2}N_0$;

$N = N_0 + \Delta N\cdot\sin\left(\vec{\kappa}\cdot\vec{r}\right)$; $k \approx \frac{\omega}{c} \approx 2\kappa\gamma^2 \gg \kappa$;

$\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}$ – relativistic factor; $\beta = \frac{v_0}{c}$.

The angle between the wave vector of the emitted wave $\vec{k}$ and the vector of the reverse lattice $\vec{\kappa} \parallel \vec{v}_0$ is small, which means $\theta^2 \ll 1$;

$$\omega_{\max,\min}\cdot\left(\eta \mp \sqrt{\eta^2 - \gamma^{-2}}\right) = \omega_p; \quad \eta = \frac{\kappa c}{\omega_p}.$$

It follows from (6) that the radiation power is low (6) because it is proportional to the square of the parameter of the medium inhomogeneity q^2 <<1. The effective length of losses for radiation in the band of frequencies $\omega_{\min} \le \omega \le \omega_{\max}$ for γ = 6 is determined by expression $l_{eff}^{-1} \equiv \frac{1}{\tilde{n}W\kappa}\frac{dW}{dt} \approx \frac{0,5\cdot10^{-2}}{\kappa}\left(\frac{\Delta N}{N_0}\right)^2 \ll 1$ and exceeds considerably the mean path of the electron in the periodic tungsten-aluminum medium of $\bar{R} \approx 0,315$ cm [9,10].

So, such process can`t be considered as concurrent for energy loss' evaluation of the charged particle in the harmonic inhomogeneous medium.

Analyzing the results obtained in [6] shows that energy losses of relativistic electron for radiation are of the same order:

$$\frac{dW_w}{dt} = \frac{\pi}{2}\frac{q^2 e^2}{\varepsilon_0^{3,5}}\kappa^2 \tilde{n}\cdot B\left(\beta_{\text{ô}}\right)\cdot M\,, \tag{7}$$

where: $B\left(\beta_{\text{ô}}\right) = \left(1-\beta_{\text{ô}}^2\right)^2\beta_{\text{ô}}^4$, $\beta_{\text{ô}} = \beta\sqrt{\varepsilon_0}$ – ratio of particle's velocity to the phase velocity of wave's propagation in medium; M-number of medium layers, intersected by oscillator ($M \gg 1$).

In the investigated case the radiation angles are determined by the expression: $\cos\theta_w = \pm\frac{m}{\sqrt{m^2+\beta_{\text{ô}}^2}}$. Because in a solid state $\left|1-\varepsilon_0\right| \ll 1$ the expression for non-relativistic electron $\beta_{\text{ô}}$ <<1 is valid. It follows from this that all emitted waves like in the expression (6) are concentrated in narrow cones with divergence angles: $\theta_w \approx \pm\beta_{\text{ô}}$.

From above evaluations it follows that in harmonic inhomogeneous laminar metals the power of electron losses for radiation is low, because it is proportional to small value: q^2.

RADIATION LOSSES OF ELECTRON IN THE LAMINAR PERIODIC MEDIUM

As it follows from the results obtained in paper [2] the power of electron losses of energy for radiation in the laminar medium may be represented as:

$$\frac{dW}{dt} \approx \frac{\left|\varepsilon_2-\varepsilon_1\right|}{\bar{\varepsilon}_0^2}\cdot\frac{e^2\kappa^2 \text{v}_0}{2\cdot\pi^2}(-1)^{n+p}\frac{1+2p}{1+2n}\int_0^\infty \frac{x^3\left(2\bar{\varepsilon}_0-\beta^{-2}-x^2\right)^2 dx}{\left(\bar{\varepsilon}_0-\beta^{-2}-x^2\right)^4}\,. \tag{8}$$

This expression is maximal from possible radiation losses of electron energy because is proportional to the parameter of inhomogeneity in first degree:

$$\frac{\left|\varepsilon_2-\varepsilon_1\right|}{\bar{\varepsilon}_0} = \frac{\pi}{2}q\,.$$

Let us indicate the main assumptions which allowed to obtain the expression for power of electron losses (8).

1. Expression (8) is valid at equal thickness of layers $l_1 \equiv \grave{a} = l_2 \equiv b$ (a and b note the papers [2] and under condition of parametric relation between eigen waves of medium (see Fig.2)

$$\vec{k}_m - \vec{k}_{m-1} = \vec{\kappa}\,;\ \left|\vec{k}_m\right| = \left|\vec{k}_{m-1}\right|, \tag{9}$$

where: $\vec{k}_m \equiv \vec{k} + m\vec{\kappa} = \left\{\frac{\omega}{c}\sqrt{\bar{\varepsilon}_0}\cos\left(\theta\right) + m\cdot\kappa, k_\perp\right\}$,

$\vec{k}_{m-1} = \left\{\frac{\omega}{c}\sqrt{\bar{\varepsilon}_0}\cos\left(\theta\right) + \left(m-1\right)\cdot\kappa, k_\perp\right\}$ wave vectors of periodical medium's eigen waves; θ – angle between vector of reverse lattice of medium and wave $\vec{k}_m$ or $\vec{k}_{m-1}$; $k_\perp = \frac{\omega}{c}\sin\left(\theta\right)$ – horizontal wave number; $\varepsilon_1 \approx \varepsilon_1 = \bar{\varepsilon}_0$.

From the first expression (9) we can obtain that the condition of radiation is similar to (1):

$$\sqrt{\bar{\varepsilon}_0}\cdot\frac{\omega}{c}\cdot\cos\theta - \frac{\kappa}{2} = \kappa\cdot n\,. \tag{10}$$

On formulation of (9) resonance relation $\omega = \frac{\kappa \text{v}_0}{2}$, was used which corresponds to the condition of parametric relation of eigen waves frequencies and frequency of periodic lattice (second expression (9)).

2. With the use of (10) it is easy to find the solution of dispersion equation of paper [2] (in original symbols)

$$\cos\left(\frac{2\cdot\omega\cdot a}{\text{v}_0}\right) = \cos(p_1 a)\cdot\cos(p_2 a) -$$
$$-\frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1} + \frac{p_2\varepsilon_1}{p_1\varepsilon_2}\right)\sin(p_1 a)\cdot\sin(p_2 a)\,, \tag{11}$$

in [11] this solution is considered as solution that can`t be solved in obvious form.

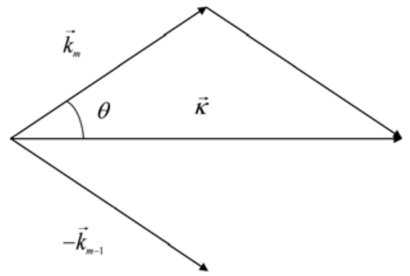

Fig.2. The scheme of interaction between the wave vectors of the self-waves of periodical medium $\vec{k}_m, \vec{k}_{m-1}$ and vector of inverse lattice of periodically inhomogeneous medium $\vec{\kappa}$

In considered case the solution of equation (11) allowing for (10), may be obtained in following way.

Assume in (11) that $\omega = \omega_0 + \Delta\Omega$; $\left|\Delta\Omega\right| \ll \omega_0$; $p_{10}a = \frac{\pi}{2}\left(2n+1\right)$ and $p_{20}b = \frac{\pi}{2}\left(2l+1\right)$, where n, l are integer numbers (from condition $a=b$ it follows $l = n$). Then it is ease to obtain the desired solution of equation (11):

$$\omega_0 = \frac{\text{v}_0}{2a}\pi\left(2p+1\right);\ \Delta\Omega = \mp i\frac{\text{v}_0}{2a}\frac{\varepsilon_2-\varepsilon_1}{\sqrt{\varepsilon_1\varepsilon_2}}\,,$$

where p – integer numbers. Relations $p_{10} = \sqrt{\varepsilon_1\frac{\omega_0^2}{c^2} - k_\perp^2}$, $p_{20} = \sqrt{\varepsilon_2\frac{\omega_0^2}{c^2} - k_\perp^2}$ and the condition that for high values of n in (11) the terms of first and second infinitesimal order of parameter $\omega_o^{-1}\Delta\Omega$ in expansions of p_1 and p_2 may be neglected were used while finding these solutions.

Under the condition of satisfaction of equations (9)-(11) integral losses of electron for radiation may be represented as:

$$\left|\frac{dW}{dt}\right| \approx \frac{\left|\varepsilon_2-\varepsilon_1\right|}{\varepsilon_0^2}\cdot\frac{e^2\kappa^2 \text{v}_0}{4\cdot\pi}\int_0^{\lambda_{\max}} \frac{x^3\left(2\varepsilon_0-\beta^{-2}-x^2\right)^2 dx}{\left(\varepsilon_0-\beta^{-2}-x^2\right)^4}\,, \tag{12}$$

where: $\lambda_{\max} = \frac{k_\perp c}{\omega_0} \approx \frac{L}{d_{\grave{a}}}$; $d_{\grave{a}}$ – mean distance between atoms of medium; $\beta = \frac{\text{v}_0}{c} \ll 1$.

Characteristic angles of radiation θ are in the range $[0,\pi]$ and may be determined from relation:

$$\cos\left(\theta\right) = \frac{2\left(1+2n\right)}{\beta\left(1+2p\right)}\,. \tag{13}$$

It must be noted that radiation is possible under the condition $2\left(1+2n\right) \le \beta\left(1+2p\right)$ that is true for comparatively high-frequency waves.

Evaluation of effective length of losses for electron's radiation with energy 2,0 (6,0) MeV in composition of tungsten-aluminum for value of period $L \sim 0{,}3\cdot10^{-6}$ cm from relation (12) gives the value $l_{eff} \approx 0{,}17(0{,}5)$ cm that is comparable with mean path of electron in the medium, calculated by traditional methods [1]: $\bar{R} \approx 0{,}1(0{,}32)$ cm.

CONCLUSIONS

In this paper evaluation of electron energy's radiation loss in inhomogeneous media is presented; investigated media may be formed by layers of materials with different dielectric constants or may be modeled introducing dielectric permeability varying in space by harmonic law.

It is shown that in inhomogeneous media with varying dielectric permeability the radiation losses of electron are proportional to the square of parameter of inhomogeneity i.e. are low.

In the case when the conditions of parametric relation of medium' eigen waves are satisfied the radiation losses of electron are proportional to the parameter of inhomogeneity in first degree and are comparable with losses which are caused by the elementary events of scattering.

Effective length of losses for radiation of electron with energy 2,0 (6,0) MeV in multi layer bimetallic tungsten-aluminum medium with value of period $L \sim 0{,}3\cdot10^{-6}$ cm is comparable with mean path of electron in such medium.

Characteristic angles of radiation have discrete character and are directed from 0 to 180^{o}. With increase of angle of radiation the losses increase but only up to certain determined value, because with the approach to 180° the theory becomes inapplicable and must be revised.

REFERENCES

1. A.P. Chernyaev. *Interaction of ionizing radiation with matter.* M.: "Physmatlit", 2004, p.152.
2. J.B. Fayinberg, N.A. Khijnyak. Energy losses by particle on passing through laminar dielectric// *JETPh*. 1957, v.32, Issue 4, p.883-895.
3. M.L. Ter-Mikaelyan. *Influence of medium on electromagnetic processes under high energies*. Erevan: "AS of Arm. SSR", 1969, p.459.
4. Y.N. Adishev, V.N. Zabaev, Y.L. Pivovarov, et al. // *International Worshop "Many-particle Effects in Radiation Physics*". September 7-10, 2004, Belgorod, Russia, p.40.
5. A.E. Kaplan, C.T. Law, P.L. Shkolnikov. X-ray narrow-line transition radiation source based on low-energy electron beam traversing a multilayer nanostructure // *Phys. Rev. E.* 1995, v.52, №6, p.6795-6808.
6. V.I. Tkachenko, I.V. Tkachenko. Radiation of oscillating charge moving with non-relativistic rate in periodically inhomogeneous medium // *Problems of Atomic Science and Technology. Series "Plasma electronics and new methods of acceleration".* 2008. №4(6), p.242-244.
7. N. Ashkroft, N. Mermin. *Physics of solid state*. M.: "Mir". 1975, v.1, p.399.
8. V.L. Ginzburg, V.N. Tsitovich. Some questions of the theory of transient radiation and ransient scattering // *UPhN*. 1978, v.126, issue 12, p.553-608.
9. *Tables of physical values. Reference book* / Edited by academician I.K. Kikoin, M.:-Atomizdat, 1976, p.1008.
10. Nuclear physics in Internet with support of SRIPh MSU: http://nuclphys.sinp.msu.ru/
11. G.M. Garibyan, Yan Shi. X-ray transient radiation / Executive editor Y.G. Shakhnazaryan. Erevan: Ed. AS Arm. SSR, 1983, p.320.

Статья поступила в редакцию 02.11.2009 г.

РАДИАЦИОННЫЕ ПОТЕРИ ЭНЕРГИИ ЭЛЕКТРОНА В МНОГОСЛОЙНЫХ БИМЕТАЛЛИЧЕСКИХ СРЕДАХ

Б.В. Борц, И.В. Ткаченко, В.И. Ткаченко

Проведена оценка радиационных потерь энергии электрона в неоднородных средах, которые могут быть сформированы либо слоями материалов с различными диэлектрическими постоянными, либо смоделированы изменяющейся по гармоническому закону в пространстве диэлектрической проницаемостью. Показано, что в неоднородных средах с изменяющейся по гармоническому закону в пространстве диэлектрической проницаемостью радиационные потери электрона пропорциональны квадрату параметра неоднородности, т.е. малы. В случае, когда в среде выполняются условия параметрической связи собственных волн среды, которые излучаются электроном, радиационные потери электрона пропорциональны параметру неоднородности в первой степени и сравнимы с потерями, которые обусловлены элементарными актами рассеяния.

РАДІАЦІЙНІ ВТРАТИ ЕНЕРГІЇ ЕЛЕКТРОНА В БАГАТОШАРОВИХ БІМЕТАЛІЧНИХ СЕРЕДОВИЩАХ

Б.В. Борц, І.В. Ткаченко, В.І. Ткаченко

Проведена оцінка радіаційних втрат енергії електрона в неоднорідних середовищах, які можуть бути сформовані або шарами матеріалів з різними діелектричними постійними, або змодельовані діелектричною проникністю, що змінюються за гармонійним законом у просторі. Показано, що в неоднорідних середовищах з діелектричною проникністю, що змінюється за гармонійним законом у просторі, радіаційні втрати електрона пропорційні квадрату параметра неоднорідності, тобто малі. У випадку, коли в середовищі виконуються умови параметричного зв'язку власних хвиль середовища, які випромінюються електроном, радіаційні втрати електрона пропорційні параметру неоднорідності в першому ступені і порівнянні із втратами, які обумовлені елементарними актами розсіювання.