FOKKER-PLANCK EQUATION FOR TRAPPED PARTICLES IN TOKAMAK WITH TOROIDAL FIELD RIPPLES

Yu.K. Moskvitina[1,2], V.O. Yavorskij[3,4], K. Schoepf[4]

[1]National Science Centre "Kharkov Physics and Technology Institute", Kharkov, Ukraine;
[2]V.N. Karazin Kharkov National University, Kharkov, Ukraine;
[3]Institute for Nuclear Research, Ukrainian Academy of Sciences, Kiev, Ukraine;
[4]Association EURATOM-OEAW, Institute for Theoretical Physics, Innsbruck, Austria
E-mail: Yu.Moskvitina@gmail.com

The kinetic description of the resonance ripple diffusion of fast ions in tokamaks with toroidal field ripples is presented for the general case of arbitrary toroidal field ripple magnitudes. The topology of banana orbits in phase space is considered. The transport coefficients of 4D Fokker-Planck equations are derived. The algorithms of averaging procedure through superbanana motion are discussed.

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1. INTRODUCTION

The analysis of high energy particle confinement in a tokamak with three dimensional (3-D) perturbations of the magnetic field is usually a difficult task because of the large number of essential phase variables. A traditional problem for tokamak reactor studies is the evaluation of fast particle losses (alpha particles in reactors). In addition to the well known first-orbit loss process, one of the possible loss mechanisms originates from the presence of ripples in the toroidal magnetic field (TF). This mechanism has been the subject of theoretical considerations since the early days of tokamak research [1-4].

The TF ripple arises from the discreteness of the toroidal coils. These ripples are generally strongest and consequential for particle transport in the low-B side of a tokamak while being of less significance at the high-B side. Field perturbations of 1% are typical in the outer edge of a tokamak plasma and appear a few orders of magnitude smaller at the magnetic axis.

Ripple losses of fusion α-particles in future tokamak-reactors are of concern first of all because the associated particle and heat fluxes may damage plasma-facing component. For these reasons, ripple-induced α-particles losses from proposed burning tokamak plasmas, in particular ITER, have been modelled by many authors [4-11].

There will be two main sources for forming TF ripples in ITER. First, the toroidal magnetic field is created by the finite number (N=18) of toroidal field coils with spaces between them large enough to accommodate ports. The magnetic field ripples generated by the discrete coil assembly will have toroidal mode numbers nN with n=1,2,…, where n=1 is the dominant mode. Placing ferritic inserts underneath the toroidal field coils can mitigate the amplitude of these ripples to a reasonable magnitude of magnetic perturbations. Second, when the test blanket modules (TBM) will be installed, their ferromagnetic materials are expected to significantly perturb the local toroidal field [12, 13].

Early experimental studies on TF ripple have been focused on energetic ion losses, as the theory [3, 4] indicated that TF ripple enhances the transport of trapped energetic ions only and that TF ripple diffusion of the trapped ions is much faster than axisymmetric neoclassical diffusion.

In fact, experiments have shown that 100 keV [14] and MeV passing ions [15] diffused radially as slowly as predicted by neoclassical diffusion [16-18]. The radial transport of MeV trapped ions was supposed to be determined predominantly by the collisionless stochastic ripple diffusion [19]. Fortunately, both analytical analysis and simulation have predicted that the ripple losses of charged fusion products are not crucial in the power balance of burning plasma whose TF ripple amplitude is as low as - 1% [20]. In spite of that good outlook, TF ripple transport of energetic ions is still of importance and is critical to the design of a fusion reactor. This is mainly because the ripple induced excursion of energetic ions can result in serious localized heat deposition on the first wall [21] and partly because the TF ripple transport probably determines the deposition profile of alpha heating in the fusion reactor when severe instabilities are suppressed.

TF ripples are known to create secondary magnetic wells at the outer plasma edge, most dominantly in the vicinity of the mid-plane [4]. Particles trapped in these wells are subject to enhanced radial transport and hence poorly confined in the plasma. The criterion for the existence of secondary ripple wells in a circular tokamak with large aspect ratio is $\alpha \equiv \varepsilon|\sin\vartheta|/(Nq\delta) < 1$, where ε denotes the local inverse aspect ratio, ϑ the poloidal angle, N the number of toroidal field coils, q the safety factor and $\delta \equiv (B_{t\max} - B_{t\min})/(B_{t\max} + B_{t\min})$ the ripple amplitude. Note that at the plasma periphery, where δ exceeds the critical value δ_{GWB} given by the Goldstone-White-Boozer stochastisity threshold, toroidally trapped particles are nearly promptly lost from the plasma during a time small in comparison with Coulomb collision times [3,4]. Here we examine the ripple impact on toroidally trapped fast ion orbits with the banana tips in the plasma region where there are no ripple wells and the ripple magnitude is below the stochasticity threshold, i.e. where $\alpha > 1$ and $\delta < \delta_{GWB}$. The most significant effect of TF ripples occurs for toroidally trapped fast ions which are in resonance with the ripple perturbations [6], i.e. for ions satisfying the resonance condition $l\omega_b - N\omega_d = 0$, $l = 0, \pm 1, \pm 2, \ldots$, where ω_b

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and ω_d are the particle's bounce and toroidal precession frequencies. Such resonant toroidally trapped particles, so-called superbananas, undergo an increased radial diffusion and thus are responsible for a substantial contribution to the TF ripple losses of energetic ions [5-6, 8, 22-25].

The effect of the ripple collisional transport on the fast ions in tokamaks in mentioned case can be adequately described by the Fokker-Planck equation in 3D constant of motion (COM) space [22-27]. The transport coefficients of this equation in the case of weak ripples, $\delta \ll \delta_1 = \varepsilon/(Nq)^{3/2}$ where ε is the flux surface toroidicity and q the safety factor, were derived in [23].

The main purpose of this paper is to present the kinetic description of transport processes (induced by collisions and ripple orbital effects) of trapped fast ions in tokamak plasmas for the general case of arbitrary toroidal field ripple magnitudes.

In the section 2 SUPERBANANA FOKKER-PLANCK EQUATION AND COM SPACE VARIABLES the outline of the approach of the Fokker-Planck equation in 3D COM space is presented. Next section SUPERBANANA ORBIT is devoted to analysis of the single banana motion near the resonance level. The main results and conclusions are presented in the last section CONCLUSIONS.

2. SUPERBANANA FOKKER-PLANCK EQUATION AND COM SPACE VARIABLES

The approach for kinetic description of resonance ripple diffusion used here was firstly proposed in [23]. The main idea of this approach is that Fokker-Planck equation for trapped fast ions could be reduced to three-dimensional equation in constants of motion (COM) space. These COM space variables could be chosen in different way. Following the approach in [23] V, λ and $p_{\beta\max}$ are chosen, where V, λ and $p_{\beta\max}$ are particle velocity, normalized magnetic moment and maximum value of angular canonical momentum p_β. It should be noted that p_β is conjugate to variable β, defined as $\beta = \varphi - q\vartheta$, where φ and ϑ are toroidal and poloidal angles.

In variables $\mathbf{c}' = \left(V, \lambda, p_{\beta\max}, \vartheta_3\right)$, the Fokker-Planck equation can be represented as

$$\left(\partial_t + \dot{\vartheta}_3 \partial_{\vartheta_3}\right) f = Cf + S\left(\mathbf{c}'\right), \tag{1}$$

with collision term C in the form

$$C = \sqrt{g_{\mathbf{c}'}^{-1}} \left\{ \sum_{i,j\le 3} \partial_{c'^i} \left[\sqrt{g_{\mathbf{c}'}} \left(d_{\mathbf{c}'}^i - D_{\mathbf{c}'}^{ij} \partial_{c'^j} \right) \right] + \right.$$
$$\left. + \partial_{\vartheta_3} \left[\sqrt{g_{\mathbf{c}'}} \left(d_{\mathbf{c}'}^4 - D_{\mathbf{c}'}^{44} \partial_{\vartheta_3} - \sum_{i\le 3} D_{\mathbf{c}'}^{i4} \partial_{c'^i} \right) \right] \right\}, \tag{2}$$

where ϑ_3 is cyclic variable determining the particle position on the superbanana orbit [23, 26], $\mathbf{d}$ – the friction force and $\ddot{\mathbf{D}}$ – the diffusion tensor, $g_{\mathbf{c}'}$ – the Jakobian for the new set of variables $\mathbf{c}'$. The subscript '3' indicates that ϑ_3 conjugate to the third COM variable $p_{\beta\max}$. The friction force $\mathbf{d}$ and the diffusion tensor $\ddot{\mathbf{D}}$ describes correspondingly the convective and diffusive collisional transport of fast ions in a tokamak with the TF ripples in the $\mathbf{c}'$-space.

The explicit expressions for $\mathbf{d}$ and $\ddot{\mathbf{D}}$ in the $\mathbf{c}'$– space can be derived using expressions for $\mathbf{d}$ and $\ddot{\mathbf{D}}$ in another $\mathbf{c}$-space of variables:

$$d_{c'}^i = d_c^j,\; D_{c'}^{ij} = D_c^{ij},\; D_{c'}^{il} = D_c^{ik} \frac{\partial c'^l}{\partial c^k}, \tag{3}$$

if $i, j = 1,2$; $k = 1,2,3,4$; $l = 3,4$;

$$d_{c'}^i = d_c^k \frac{\partial c'^i}{\partial c^k},\; D_{c'}^{ij} = D_c^{kl} \frac{\partial c'^i}{\partial c^k} \frac{\partial c'^j}{\partial c^l}, \tag{4}$$

if $i, j = 3,4$; $k, l = 1,2,3,4$. For example, in [23] $\mathbf{c} = \left(V, \lambda, p_\beta, \psi\right)$ is used as the set of the reference coordinates. The variable ψ is angular coordinate characterizing the position of the banana in toroidal angle. It should be pointed out that transition from the set of variables $\mathbf{c} = \left(V, \lambda, p_\beta, \psi\right)$ to the set $\mathbf{c}' = \left(V, \lambda, p_{\beta\max}, \vartheta_3\right)$ gives an opportunity to separate in the Fokker-Planck equation the invariant and the oscillating parts.

Further, following the approach of [23] the Fokker-Planck equation (1) can be averaged over the cyclic variable ϑ_3. Such averaging corresponds to the time averaging over the superbanana bounce period τ_3, i.e.

$$\langle ... \rangle = \frac{1}{\tau_3} \oint_{orbit} ...dt. \tag{5}$$

After averaging Fokker-Planck equation becomes three dimensional with all variables except time being the constants of motion.

To obtain the explicit analytical expression for collision operator C in $\mathbf{c}' = \left(V, \lambda, p_{\beta\max}, \vartheta_3\right)$ one should calculate the derivatives $\frac{\partial c'^l}{\partial c^k}$. It can be done in the framework of a single banana motion analysis.

3. SUPERBANANA ORBIT

This section is devoted to the investigation of the banana averaged motion in the vicinity of the *l*-th resonance. This motion can be treated as the behavior of 1D system with canonical variables $\left(p, \psi\right)$ and Hamiltonian h in the form [23]

$$h = p^2/2 - M\cos\left(\xi + p\right)\cos\psi, \tag{6}$$

where $p = g'\left(p_\beta - p_\beta^l\right)$ is the generalized momentum representing the normalized toroidal momentum, ψ is conjugate coordinate characterizing the position in toroidal angle. p_β^l is the value of p_β which exactly satisfies resonance condition for the *l*-th resonance. It should be noted, that p_β^l is function of V and λ only, i.e. $p_\beta^l = p_\beta^l\left(V, \lambda\right)$. The quantities M, p_β^l, g' and ξ are considered as constant parameters of 1D system. The explicit expression for $p, \psi, M, p_\beta^l, g'$, ξ and their relationship with the parameters of particle and magnetic configuration are defined in [23].

To carry out the transition from the set of variables $\mathbf{c} = (V, \lambda, p_\beta, \psi)$ to the set $\mathbf{c}' = (V, \lambda, p_{\beta\max}, \vartheta_3)$ in Fokker-Planck equation and find the expressions for $\partial c'^l / \partial c^k$ the relationships between the old variables and new ones should be established.

The variable $p_{\beta\max}$ can be found from the following definition of $p_{\max}$ – maximum value of variable p along the superbanana orbit, $p_{\max} = g'(p_{\beta\max} - p_\beta^l)$. Rather complex dependence $h(p,\psi)$ does not allow to find the explicit analytical expressions for p and $p_{\max}$. However, required equation can be obtained and its solution can be found numerically.

There is some freedom in definition of ϑ_3. It is convenient for averaging to define it as $\sin(\vartheta_3/2) = (p_{\max} - p)/(p_{\max} - p_{\min})$, where $p_{\min} = g'(p_{\beta\min} - p_\beta^l)$ – minimum value of variable p on the superbanana orbit. It should be mentioned that definition of cyclic variable ϑ_3 differs from that in [23], but it gives an opportunity to simplify the averaging in the strong TF ripple limit [27]. Note also that ϑ_3 is not the canonical variable.

To obtain equation for $p_{\max}$ and $p_{\min}$ one should to calculate the derivative $\partial p/\partial \psi$ taking into account that $h - const$ at the fixed orbit. The equation $\partial p/\partial\psi = 0$ defines values of $\psi = \psi_m$ corresponding to the extreme values of p,

$$\frac{M\cos(\xi + p)\sin\psi_{\mathrm{m}}}{p + M\sin(\xi + p)\cos\psi_{\mathrm{m}}} = 0\,. \tag{7}$$

Then substituting $\psi = \psi_{\mathrm{m}}$ in Eq.6 one can find the following equations for $p_{\max}$ and $p_{\min}$:

$$h_0 = \frac{p_{\mathrm{m}}^2}{2} - M\sigma_{\mathrm{m}}\cos(\xi + p_{\mathrm{m}})\,, \tag{8}$$

where p_{m} – the extreme value of p, $\sigma_m = \cos(\psi_{\mathrm{m}})$, h_0 – constant which corresponds to value of Hamiltonian $h(p,\psi)$ at the labeled orbit. To separate pairs $(p_{\max}, \psi_{\max})$ from pairs $(p_{\min}, \psi_{\min})$ one should to consider the criterions

$$\left.\frac{\partial^2 p}{\partial\psi^2}\right|_{\psi=\psi_{\max}} < 0 \text{ and } \left.\frac{\partial^2 p}{\partial\psi^2}\right|_{\psi=\psi_{\min}} > 0\,, \tag{9}$$

where $\left.\dfrac{\partial^2 p}{\partial\psi^2}\right|_{\psi=\psi_{\mathrm{m}}} = -M\dfrac{\cos(\xi + p)\cos(\psi_{\mathrm{m}})}{p + M\sin(\xi + p)\cos(\psi_{\mathrm{m}})}$.

Using these definitions, following expressions are derived

$$\frac{\partial p_{\beta\max}}{\partial p_\beta} = \frac{\dot\psi}{\dot\psi_{\max}}\,, \tag{10}$$

$$\frac{\partial p_{\beta\max}}{\partial\psi} = -\frac{1}{g'}\frac{\dot p}{\dot\psi_{\max}}\,, \tag{11}$$

$$\frac{\partial\vartheta_3}{\partial p_\beta} = \frac{2g'\sigma}{\Delta\cos(\vartheta_3/2)}\left\{1 - \eta\frac{\dot\psi}{\dot\psi_{\max}}\right\}, \tag{12}$$

$$\frac{\partial\vartheta_3}{\partial\psi} = \frac{2g'\sigma}{\Delta\cos(\vartheta_3/2)}\eta\frac{\dot p}{\dot\psi_{\max}}\,, \tag{13}$$

$$\frac{\partial p_{\beta\max}}{\partial(\upsilon,\lambda)} = \left(1 - \frac{\partial p_{\beta\max}}{\partial p_\beta}\right)\frac{\partial p_\beta^l}{\partial(\upsilon,\lambda)}\,, \tag{14}$$

$$\frac{\partial\vartheta_3}{\partial(\upsilon,\lambda)} = -\frac{\partial\vartheta_3}{\partial p_\beta}\frac{\partial p_\beta^l}{\partial(\upsilon,\lambda)}\,, \tag{15}$$

where following designations are used

$$\eta = 1 + \frac{p - p_{\max}}{\Delta}\left(1 - \frac{\dot\psi_{\max}}{\dot\psi_{\min}}\right),$$

$$\dot p = \sigma_{\dot p}\sqrt{M^2\cos^2(\xi + p) - (h_{\max} - p^2/2)^2}\,,$$

$$\dot\psi = p + (h_{\max} - p^2/2)tg(\xi + p)\,,$$

$$\sigma_{\dot p} = \mathrm{sign}(\dot p) = -\sigma_\psi\,\mathrm{sign}(\cos(\xi + p))\,,$$

$$\sigma_\psi = \mathrm{sign}(\sin(\psi))\,,$$

$$h_{\max} = p_{\max}^2/2 - M\sigma_m\cos(\xi + p_{\max})\,,$$

$$\Delta\cos(\vartheta_3/2) = \sigma_\vartheta\sqrt{\Delta^2 - (p - p_{\max})^2}\,,$$

$$\sigma_\vartheta = \mathrm{sign}(\cos(\vartheta_3)) = -\sigma_{\dot p}\,,\ \Delta = p_{\max} - p_{\min}\,,$$

$$\dot\psi_{\max} = p_{\max} + \sigma_m\sin(\xi + p_{\max})\,,$$

$$\dot\psi_{\min} = p_{\min} + \sigma'_m\sin(\xi + p_{\min})\,,$$

$$\sigma_m = \cos(\psi_{\max}),\ \sigma'_m = \cos(\psi_{\min})\,.$$

It should be noted that, as the dependence $\psi(p)$ can be expressed analytically from Eq. (6), the p variable is chosen for integration in averaging procedure,

$$\langle f(t)\rangle = \frac{1}{\tau_3}\oint\frac{f(p)}{\dot p}dp\,. \tag{16}$$

Expressions (10)-(15) allow to construct the friction force $\mathbf{d}$ and the diffusion tensor $\overleftrightarrow{\mathbf{D}}$, describing collisional transport of fast ions in a tokamak with the TF ripples of arbitrary amplitude. The form of expressions (10)-(15) gives an opportunity to carry out the averaging of Fokker-Planck equation through banana orbit.

4. CONCLUSIONS AND DISCUSSION

The transport coefficients of Fokker-Planck equation describing collisional transport of fast ions in tokamaks with the arbitrary magnitude of TF ripple amplitude are derived.

Transition from the set of COM variables in axisymmetric case to the set of COM variables in ripple perturbed case is done for the case of the arbitrary magnitude of TF ripple amplitude. Expressions for friction force $\mathbf{d}$ and diffusion tensor $\overleftrightarrow{\mathbf{D}}$, describing correspondingly the convective and diffusive collisional transport of fast ions in a tokamak can be constructed using obtained expressions. The expressions for derivatives have form which convenient for the averaging procedure through the banana orbit.

The one way for carrying out the averaging is to use numerical methods of calculating the corresponding integrals. It should be noted that most of them have singularities at the ends of integration domain, that is

$\left(p_{\min}, p_{\max}\right)$ arising from $\dot{p}\big|_{p=p_m} = 0$. Another source of singularities in the derivatives from ϑ_3 is $\Delta\cos\left(\vartheta_3/2\right)\big|_{p=p_{\min}} = 0$. These circumstances should be taken into account when choosing the numerical method of integration. On the authors' opinion the most appreciable are Gauss-Legendre formulas.

Another approach for averaging is based on using natural averaging procedure that is integrating the Hamilton equations of motion

$$\dot{p} = -\frac{\partial h}{\partial \psi} = -M\cos(\xi + p)\sin(\psi), \tag{17}$$

$$\dot{\psi} = \frac{\partial h}{\partial p} = p + M\sin(\xi + p)\cos(\psi), \tag{18}$$

and approximation the integral by sum

$$\oint f(p(t), \psi(t))\,dt \approx \sum_{i=1}^{N_{orb}} f(p_i, \psi_i)\Delta t, \tag{19}$$

where (p_i, ψ_i) values of (p, ψ) obtained at i-th step of integration of Eqs.(17-18) and Δt – step of integration. It is obviously that this method is time expensive but very simple in realization, for example using Runge-Kutta schemes.

Second method can be recommended for validation and verification of the quadrature formulas in first approach. Besides that the quadrature formulas in first approach can be verified using exact values of some integrals, for example

$\oint \dot{\psi}\,dt = 0$ for trapped bananas and

$\oint \dot{\psi}\,dt = 2\pi$ for passing bananas,

$\oint \dot{p}\,dt = 0$ for both orbits.

Using the integral properties of odd and even functions and symmetry of the diffusion tensor $\overset{\llcorner}{\mathbf{D}}$ can considerably minimize the calculations.

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УРАВНЕНИЕ ФОККЕРА-ПЛАНКА ДЛЯ ЗАПЕРТЫХ ЧАСТИЦ В ТОКАМАКЕ С ГОФРИРОВАННЫМ ТОРОИДАЛЬНЫМ ПОЛЕМ

Ю.К. Москвитина, В.А. Яворский, К. Шопф

Представлено кинетическое описание резонансной гофрировочной диффузии быстрых ионов в токамаках с гофрировкой тороидального поля в общем случае произвольных значений величины гофрировки тороидального поля. Рассмотрена топология орбит бананов в фазовом пространстве. Получены транспортные коэффициенты четырёхмерного уравнения Фоккера-Планка. Обсуждаются алгоритмы процедуры усреднения по супербанановому движению.

РІВНЯННЯ ФОККЕРА-ПЛАНКА ДЛЯ ЗАПЕРТИХ ЧАСТИНОК В ТОКАМАЦІ З ГОФРОВАНИМ ТОРОЇДАЛЬНЫМ ПОЛЕМ

Ю.К. Москвітіна, В.О. Яворський, К. Шопф

Представлено кінетичний опис резонансної гофрувальної дифузії швидких іонів в токамаках з гофрованим тороїдальним полем в загальному випадку довільних значень величини гофрування тороїдального поля. Розглянуто топологію орбіт бананів у фазовому просторі. Здобуто транспортні коефіцієнти чотиривимірного рівняння Фоккера-Планка. Розглянуто алгоритми процедури осереднення по супербанановому руху.