INSTABILITY OF CYLINDRICAL RELATIVISTIC ELECTRON BEAM, PROPAGATING IN PLASMA

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The new mechanism of instability development of relativistic electron beam of finite radius in plasma has been considered. Two-dimensional linear theory of this instability has been constructed. As a result of this theory the frequencies of excited waves and growth rates of their excitation have been derived. 2.5-numerical simulation of nonlinear stage of instability development and bunching of relativistic beam has been performed. The possibility has been shown to use of this mechanism of instability for formation of train of short relativistic bunches, used in wakefield method of electron acceleration.

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1. INTRODUCTION

The possibility of modulation of relativistic electron beam in plasma due to focusing/defocusing of its electrons is shown in [1] by numerical simulation. In this paper the two-dimensional linear theory of this instability is developed and its nonlinear stage is investigated by numerical simulation.

2. THEORETICAL GROWTH RATE OF INSTABILITY DEVELOPMENT OF RELATIVISTIC BEAM IN PLASMA

We consider cylindrical REB of electron density $n_0(r)$ which is propagated in infinite homogeneous plasma. The external magnetic field is absent. The REB interaction with plasma is described by the following self-consistent equations, which include the Maxwell equations

$$
\begin{aligned}
div\vec{D} &= -4\pi en\,, \quad div\vec{H}=0\,,\\
rot\vec{E} &= -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}\,,\\
rot\vec{H} &= \frac{1}{c}\frac{\partial\vec{D}}{\partial t}-\frac{4\pi e}{c}n\vec{V}\,,
\end{aligned}
\qquad (1)
$$

and equations of REB's electron motion

$$
\begin{aligned}
&\frac{\partial\vec{p}}{\partial t}+\left(\vec{V}\vec{\nabla}\right)\vec{p}=-e\vec{E}-\frac{e}{c}\left[\vec{V}\vec{H}\right]\,,\\
&\frac{\partial n}{\partial t}+div\left(n\vec{V}\right)=0\,,
\end{aligned}
\qquad (2)
$$

where $\vec{D}$ is the electrical induction of isotropic plasma. We linearise the equations (1), (2), having presented dependence of all value perturbations on time and longitudinal coordinate as $exp(-i\omega t+ikz)$, where ω is the frequency of perturbation. As a result we obtain the following linear equations

$$
\begin{aligned}
&\frac{\partial E_z}{\partial r}=ikE_r-ik_0H_\varphi\,,\\
&kH_\varphi=k_0\epsilon E_r-\frac{4\pi ie}{c}n_0\hat{V}_r\,,\\
&\frac{1}{r}\frac{d}{dr}rH_\varphi=-ik_0\epsilon E_z-\frac{4\pi e}{c}\left(n_0\hat{V}_z+V_0\hat{n}\right)\,,\\
&\hat{V}_z=-\frac{ie}{m\gamma_0^3\Omega}E_z\,,\\
&\hat{V}_r=-\frac{ie}{m\gamma_0\Omega}E_r+\frac{ieV_0}{m\gamma_0c\Omega}H_\varphi\,,\\
&\hat{n}=n_0\frac{k\hat{V}_z}{\Omega}-\frac{i}{\Omega}\frac{1}{r}\frac{d}{dr}\left(rn_0\hat{V}_r\right)\,,
\end{aligned}
\qquad (3)
$$

where $\Omega=\omega$–kV_0, $\hat{n}$, $\hat{V_{r,z}}$ are the perturbations of density and components of electron beam velocity. The equations (3) are equivalent to the following coupled linear equations for components of an electrical field

$$
\begin{aligned}
&\frac{1}{r}\frac{d}{dr}r\epsilon_\perp E_r+ik\epsilon_zE_z=\\
&=-\frac{kV_0}{\omega}\frac{1}{r}\frac{d}{dr}\left(r\frac{\omega_b^2}{\gamma_0\Omega^2}E_r\right)-\\
&-\frac{V_0}{\omega}\frac{1}{r}\frac{d}{dr}\left(r\frac{\omega_b^2}{\gamma_0\Omega^2}\frac{dE_z}{dr}\right)\,,\\
&i\left(k-\beta_0\frac{\omega_b^2}{c\gamma_0\Omega}\right)\frac{dE_z}{dr}=\left(k_0^2\epsilon_a-k^2\right)E_r\,,
\end{aligned}
\qquad (4)
$$

where

$$\epsilon_\perp=\epsilon-\frac{\omega_b^2}{\gamma_0\Omega^2}\,,\quad \epsilon_z=\epsilon-\frac{\omega_b^2}{\gamma_0^3\Omega^2}\,,$$

$$\epsilon=1-\frac{\omega_p^2}{\omega^2}\,,\quad \epsilon_a=\epsilon-\frac{\omega_b^2}{\gamma_0\omega^2}\,,\quad k_0=\frac{\omega}{c}\,.$$

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The equations (4) describe a linear stage of instability at any dependence of REB density on radius ($n_0 \left(r \to \infty\right) \to 0$).

We consider the simple case of homogeneous REB with sharp border. Integrating the equations (4) in infinitesimal vicinity of beam border $r = a$ (a is the REB radius), we derive boundary conditions for components of an electrical field

$$\begin{aligned} &E_z^{(2)} - E_z^{(1)} = 0\,, \qquad (5)\\ &E_z^{(1)} = E_z\left(r \le a\right)\,, \quad E_z^{(2)} = E_z\left(r \ge a\right)\,,\\ &\epsilon E_r^{(2)} - \epsilon_\perp E_r^{(1)} = \frac{V_0}{\omega}\frac{\omega_b^2}{\gamma_0\Omega^2}\left(kE_r^{(1)} + i\frac{dE_z^{(1)}}{dr}\right)\,. \end{aligned}$$

Outside the beam $r \ge a$ the longitudinal component of the electrical field satisfies to the equation

$$\frac{1}{r}\frac{d}{dr}r\frac{dE_z^{(2)}}{dr} - q^2 E_z^{(2)} = 0\,, \qquad (6)$$

where $q^2 = k^2 - k_0^2\epsilon$. The solutions of the equations (4) and (6) look like

$$\begin{aligned} E_z^{(1)} &= A_1 J_0\left(\lambda r\right)\,,\\ E_r^{(1)} &= -\frac{i\lambda}{k^2}\left(k - \beta_0\frac{\omega_b^2}{c\gamma_0\Omega}\right)A_1 J_1\left(\lambda r\right)\,,\\ E_z^{(2)} &= A_2 K_0\left(qr\right), E_r^{(2)} = -\frac{ik}{q}A_2 K_1\left(qr\right)\,,\\ \lambda^2 &= \kappa^2 - \frac{\epsilon_z}{\epsilon_{\perp eff}}\,, \quad \kappa^2 = k_0^2\epsilon_a - k^2\,,\\ \epsilon_{\perp eff} &= \epsilon_\perp + \epsilon\beta_0^2\frac{\omega_b^2}{\gamma_0\Omega^2}\,. \qquad (7) \end{aligned}$$

The jump of the electrical induction on the beam border is caused by a surface charge. We use the boundary conditions (5). As a result we derive the following dispersion relation

$$\frac{\epsilon}{p}\frac{K_1\left(p\right)}{K_0\left(p\right)} + \frac{\epsilon_z}{\mu}\frac{J_1\left(\mu\right)}{J_0\left(\mu\right)} = 0\,, \qquad (8)$$

where $p = qa$, $\mu = \lambda a$.

It is necessary to note that ϵ_z takes into account longitudinal phase focusing (modulation) of the electron beam by longitudinal component of the electrical field. On the other hand $\epsilon_\perp$ takes into transversal phase focusing, caused by radial displacement of beam electrons by transversal components of the electrical E_r and magnetic H_φ fields.

We consider the Langmuir wave $\omega \simeq \omega_p$ excitation in conditions of Cerenkov resonance $\omega \simeq \omega_p = kV_0$. We look for the solution of the dispersion relation (8) in kind $\omega = kV_0 + \delta$ ($\delta \ll \omega_p$). In this approximation we have

$$\epsilon = \frac{2\delta_a}{\omega_p}\Delta\,, \quad \epsilon_z = \frac{2\delta_a}{\omega_p}\Delta\left(1 - \frac{1}{\gamma_0^2\Delta^3}\right)\,,$$

$$\Delta = \frac{\delta}{\delta_a}, \quad \delta_a = \left(\frac{\omega_b^2\omega_p}{2\gamma_0}\right)^{1/3}\,.$$

We consider that the following condition is correct

$$\Delta^3 \gg \frac{1}{\gamma_0^2}\,.$$

It means, that we neglect the longitudinal phase focusing of the beam electrons. In this case the dispersion relation (8) becomes in the kind

$$\frac{1}{\mu}\frac{J_1\left(\mu\right)}{J_0\left(\mu\right)} = -\frac{1}{p}\frac{K_1\left(p\right)}{K_0\left(p\right)}\,, \qquad (9)$$

$p = ka$.

The diagram of the function $F\left(\mu\right) = \frac{J_1(\mu)}{\mu J_0(\mu)}$ is presented in Fig.1. The horizontal straight line in figure corresponds to the right part of the equation (9), and μ_n ($n = 1, 2, 3\ldots$) are the roots of the transcendental equation (9). From this figure it is followed that the electron beam forms a discrete set of radial modes. Accordingly the dispersion relation becomes in the kind

$$1 - \Delta^3 + \beta_0^2\frac{\omega_b^2}{\gamma_0\delta_a^2}\Delta = \frac{p^2}{\mu_n^2}\Delta^3\,.$$

In the limiting case of the electron beam of small density

$$\beta_0^2\frac{\omega_b^2}{\gamma_0\delta_a^2} = \beta_0^2\left(\frac{4\omega_b^2}{\gamma_0\omega_p^2}\right)^{1/3} \ll 1$$

the third member in the left part can be neglected. As a result we obtain

$$\Delta^3 = \frac{\mu_n^2}{\mu_n^2 + p^2}\,.$$

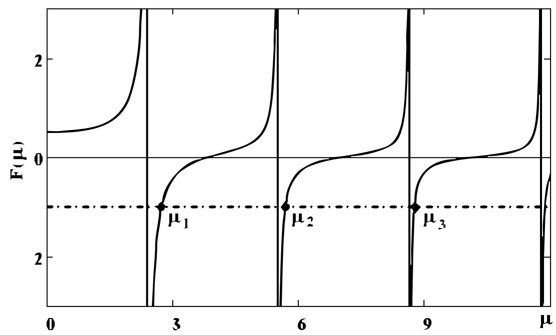

Fig.1. *The diagram of the function $F(\mu)$*

Accordingly for growth rate of beam – plasma instability we have

$$\delta_b = \frac{\sqrt{3}}{2}\left(\frac{\omega_b^2\omega_p}{2\gamma_0}\right)^{1/3}\left(\frac{\mu_n^2}{\mu_n^2 + p_e^2}\right)^{1/3}\,, \quad p_e = \frac{\omega_p a}{V_0}\,.$$

The growth rate of the instability is inversely proportional to $\gamma_0^{1/3}$. The phenomenon of the radial grouping of the beam electrons underlies instability.

We consider now the beam – plasma instability concerning the excitation of the lowest radial mode $\mu \ll 1$. For this mode the dispersion relation (8) becomes in the kind

$$\epsilon_z = -2\epsilon F\left(p\right)\,; \quad F\left(p\right) = \frac{1}{p}\frac{K_1\left(p\right)}{K_0\left(p\right)}\,.$$

In conditions of Cerenkov synchronism of the beam with the Langmuir wave $\omega \simeq \omega_p \simeq kV_0$ from the equation we derive the growth rate of the instability of the lowest radial mode

$$\delta = \frac{\sqrt{3}}{2^{4/3}\gamma_0}\left[\frac{\omega_b^2\omega_p}{2F(p_e)+1}\right]^{1/3},$$

$$p_e = \frac{\omega_p a}{V_0}.$$

It is necessary to note that the condition $\mu \ll 1$ is correct when $\frac{p_e}{\gamma_0} \ll 1$. The growth rate of the lowest mode is inversely proportional to γ_0. It is explained by that the radial electrical field E_r is small in comparison with longitudinal one E_z for this mode in the region of the beam

$$\frac{E_r}{E_z} \simeq \frac{p_e}{2\gamma_0^2}\frac{r}{a} \ll 1, \quad r \le a.$$

Therefore the radial phase grouping is not essential. The instability is caused by beam modulation by longitudinal component of the electrical field.

3. NUMERICAL SIMULATION OF INSTABILITY DEVELOPMENT OF SMALL DENSITY RELATIVISTIC BEAM IN PLASMA

We consider on time interval $t \le 975$ the interaction of continuous electron beam with current $4.47A$, radius $0.5cm$ and $\gamma_0 = 4$ with plasma of length $170\,cm$. t is normalized on ω_p^{-1}. The beam density and of plasma electron density are modulated as a result of instability development. One can see (Fig.2) that near the injection boundary, where focusing and defocusing has not sufficient time to develop, the field is not excited. On the distance $z = 90\,cm$ from the injection boundary the growth rate achieves its maximal value, equal $\gamma_s \simeq 0.096$, which is close to growth rate $\gamma_L \simeq 0.090$ of the linear theory.

The amplitude oscillations due to wave shift concerning to the beam are observed.

One can estimate that after injection into the plasma of the beam of length, approximately equal to 398 wavelengths, the observed saturation amplitude is achieved. For this estimation the ideal case is used, when defocused electrons already have left area of interaction with the excited field. Therefore in numerical simulation this number should be more. Really, in numerical simulation this number is about 500 wavelengths (see Fig.2).

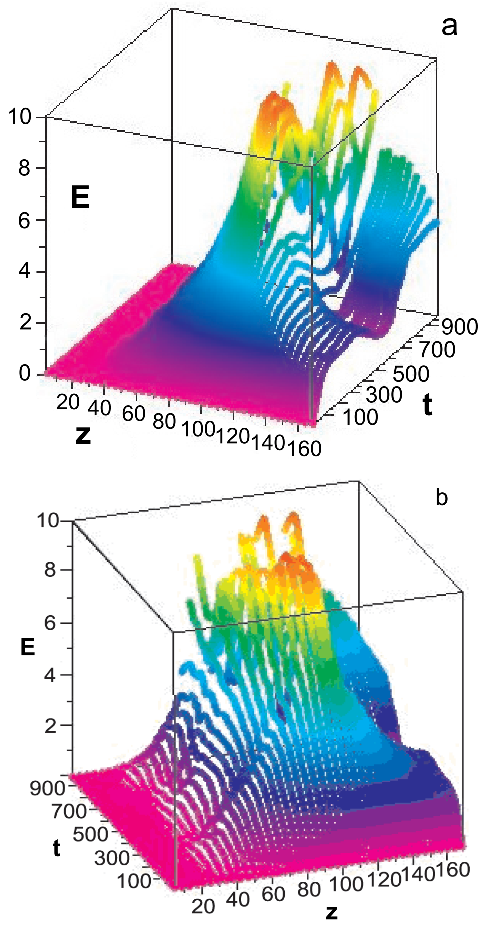

Fig.2. *The amplitude of the excited on-axis longitudinal electric field E_z as a function of the coordinate along the plasma and the time at beam – plasma instability development. E is in MV/m, z is in m, t is normalized on ω_{pe}^{-1}*

4. CONCLUSIONS

The two-dimensional linear theory of instability of the relativistic cylindrical electron beam of small density and finite radius in plasma, caused by focusing and defocusing of its electrons, is developed. The nonlinear stage of this instability is investigated by numerical simulation. Because of strong difference of longitudinal and transversal weights the instability is caused by radial dynamics of the beam electrons.

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НЕУСТОЙЧИВОСТЬ ЦИЛИНДРИЧЕСКОГО РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ПУЧКА, ДВИЖУЩЕГОСЯ В ПЛАЗМЕ

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Рассмотрен новый механизм неустойчивости радиально ограниченного релятивистского электронного пучка в плазме. Построена двумерная линейная теория этой неустойчивости, в результате которой найдены собственные частоты возбуждаемых волн и инкременты их нарастания. Проведено $2,5$-мерное численное моделирование нелинейной стадии развития неустойчивости и степени группировки релятивистского пучка. Показана возможность использования этого механизма неустойчивости для получения последовательности коротких релятивистских сгустков, используемой в схеме кильватерного метода ускорения.

НЕСТІЙКІСТЬ ЦИЛІНДРИЧНОГО РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ПУЧКА, ЯКИЙ РУХАЄТЬСЯ В ПЛАЗМІ

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Розглянутий новий механізм нестійкості радіально обмеженого релятивістського електронного пучка в плазмі. Збудовано двовимірну лінійну теорію цієї нестійкості, в результаті якої знайдені частоти збуджуваних хвиль та інкременти їх зростання. Проведено $2,5$-вимірне числове моделювання нелінійної стадії розвитку нестійкості та групування релятивістського пучка. Показано можливість використання цього механізму нестійкості для отримання послідовності коротких релятивістських згустків, яка використовується в кільватерному методі прискорення.