DYNAMIC-STATISTICAL DESCRIPTION OF CAPTURE CROSS-SECTION - INITIAL STAGE OF THE FUSION OF HEAVY NUCLEI REACTION

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To describe the capture cross-section (σc) and the initial stage of the formation of super-heavy elements dynamic-statistical model was proposed, it considers the nuclei interaction from contact time until the formation of the dinuclear system. Were made analysis of the nucleus-nucleus potential and shown that there exists a limitation of the angular momentum contributing to the capture cross-section, which is determined by the disappearance of a potential pocket. Dynamic examination allowed to reveal the main contribution of angular momentum in the capture cross-section. Were made analysis of the capture cross sections and dynamic characteristics of the reactions $^{54}$Fe + $^{244}$Pu and $^{48}$Ni + $^{238}$U dependencies of σc from the beam kinetic energy.


1. INTRODUCTION

For the experiments on obtaining of super-heavy elements in the interaction of heavy ions, with a maximum yield of interest isotopes, main parameters are isotope selection of initial combination of the beam-target and the kinetic energy of the beam. This parameters can be changed while planning and making experiments at accelerator. To describe the mechanisms of quasi-fission and fusion of heavy nuclei numerous models were developed that can qualitatively describe these processes and can predict the most probable reaction channels. The microscopic dynamic model (MDM), based on the liquid-drop nuclear model, which describes the dynamics from the contact moment until the formation of the compound nucleus were introduced by Svyateckiy [1]. In the fluctuation-dissipation model (FDM) [2] that were introduced in the calculation of the statistical fluctuation of the trajectories in the space evolution of the nuclear system and the shell corrections in the calculation of potential energy. At the model of nucleons collectivization (MNC) [3], takes into account the nucleon structure of the colliding nuclei. Due to the overlap of the nuclear surface, the nucleons can move from one core to another, it leads to a process of nucleons collectivization which starts from the top layers and then the rest.

The most realistic description of nuclei interaction, in our opinion, proposed in the concept of the dinuclear system (CDNS) [4, 5], on which built a number of models that have been successfully used to describe quasi-fission and fusion processes. CDNS is described in [5]:

\[ \sigma_{ER}(E_{cm}) = \sigma_c(E_{cm}, L)P_{CN}(E_{cm})W_{sur}(E_{CN}^*, L), \]

1. At the capture stage, after the dissipation of kinetic energy occurs DNS formation in the minimum of potential energy.

2. The process of complete fusion is realized after the transfer of nucleons from lighter to heavier nuclei. In this case there is an inherent fusion barrier ($B^*_fus$), to overcome of which requires a minimum excitation energy ($E_{exc}^* = B^*_fus$), which is formed as a result of the kinetic energy dissipation. Consequently, the evolution of formed DNS proceeds along the coordinate R - distance between the centers of the DNS fragments and coordinates A (mass number) and Z (protons number), through the transfer of nucleons from fragment to fragment. This process can be divided into three stages, so the evaporation residues cross-section leading to the formation of super-heavy elements, is written as [6, 7]:

\[ \sigma_{ER}(E_{cm}) = \sigma_c(E_{cm}, L)P_{CN}(E_{cm})W_{sur}(E_{CN}^*, L), \]

where $\sigma_c$ - effective capture cross section; $P_{CN}$ - complete fusion probability, depends on the competition of the quasi-fission and fusion channels; $W_{sur}$ - compound nucleus survival probability.

Analysis of the capture cross-section with numerous calculations is given in [8], full capture cross section is calculated as:

\[ \sigma_c = \frac{\pi h^2}{2\mu E_{cm}} \sum_L (2L+1)P_{cap}(E_{cm}, L), \]

$P_{cap}$ - full capture probability.
To calculate the complete fusion probability, which is determined by that DNS has passed internal barrier fusion probability \( B_{fas}^f = B_{fas}^r \), and formed a compound nucleus can be use the approximate expression [5]:

\[
P_{CN} \approx \frac{1.25 \cdot \exp[-(B_{fas}^f - B_{fas}^r)/(kT_{DNS})]}{1 + 1.25 \cdot \exp[-(B_{fas}^r - B_{fas}^f)/(kT_{DNS})]},
\]

where \( T_{DNS} = (E^*/\alpha)^{1/2} \), temperature of the dinuclear system, \( E^* \)- excitation energy, \( \alpha = A/12 \text{ MeV}^{-1} \); \( B_{fas} = \min(B_{fas}^r, B_{fas}^r) \), \( B_{fas}^r, B_{fas}^r \)- determined from the potential energy calculations.

The probability of the compound nucleus survival is calculated as [9, 10]:

\[
W_{sur} = \frac{P_{CN}(E^{CN}_g) \prod_{i=0}^{\infty} \frac{\Gamma_n(E^{CN}_g)}{\Gamma_n(E^{CN}_g) + \Gamma_f(E^{CN}_g)}}{n}. (4)
\]

This paper describes dynamic description of the first stage of the reaction quasi-fission-fusion - capture (\( \sigma_c \)), which mainly determines the full formation cross-section of super-heavy elements. The calculation of capture cross-section gives the range of the kinetic energy for the experimental beam and an opportunity to select the isotope configuration of the beam and target. Shows crucial role of the change of angular momentum in the two nuclei interaction. Were made the calculations of capture cross-sections for reactions \( ^{238}\text{U} + ^{1}\text{H} \rightarrow ^{239}\text{U}^* \) for stable isotopes of light nuclei in the whole possible kinetionic energy range.

### 2. FORMALISM

To describe the capture cross-section of the interacting nuclei will be used the following expression, that is averaged over all possible angular momenta (\( L_{\text{max}} \)) involved in the reaction:

\[
\sigma_c = \frac{\hbar^2 \pi}{2 \mu E_{cm}} \cdot \frac{1}{L_{\text{max}}(E_{cm})} \sum_{L_{0i}} (2L_{0i} + 1) \cdot P_{1i} \cdot P_{2i},
\]

where \( \mu = m_1 m_2 / (m_1 + m_2) \)- reduced mass, \( E_{cm} \)- beam kinetic energy, \( L_{\text{max}} \)- maximum value of the angular momentum at a given kinetic energy \( E_{cm} \), which may be involved in the reaction quasi-fission-fusion, it is carried out by averaging the capture cross-section. The maximum value of the angular momentum are easily obtained from the energy balance at the interacting nuclei contact point: \( E_{cm} = V_{cont}^f - V_{cont}^r - V_{cont}^r = \mu \hat{R}^2 / 2 \). Angular momentum is defined[11]: \( L = \mu R R \sin \theta \) where \( R = R_1 + R_2 \), \( \hat{R} \)- velocity at the contact point, \( \theta \)- angle between the beam axis and the distances between centers of the interacting nuclei, which for the maximum value is \( \pi / 2 \), then: \( L_{\text{max}} = \sqrt{\frac{2\mu R^2 (E_{cm} - V_{cont}^f - V_{cont}^r)}{(1 - \hbar^2)} + 2\hbar^2 \cdot 0.5 b^2} \),

because \( \hbar^2 \rightarrow 0 \), then to determine the maximum angular momentum can be used expression:

\[
L_{max} = R \sqrt{2\mu (E_{cm} - V_{Coul} - V_n)}.
\]

Verification of the expression (6) was carried out on the reactions given above, and confirmed the correctness of the selected assumptions. \( L_{0i} \)- the initial value of angular momentum, which contribute to the capture cross-section. Value \( L_{0i} < L_{max} \), because it is limited by the difference of excitation energies of light and heavy nuclei.

Fig.1 schematically shows the nucleus-nucleus potential from distance between centers of the nuclei, which implies that there is an energy pocket, which determines the kinetic energy \( E_{cm}^{min} \) and \( E_{cm}^{max} \) and, naturally depends on the friction coefficients. \( E_{cm}^{min} \) determined by the energy required to overcome the Coulomb barrier, and \( E_{cm}^{max} \) - overcome the Coulomb barrier and didn’t care systems to the quasi-fission channel. Therefore \( P_{1i}(L, E_{cm}) \) determined:

\[
P_{1i}(L, E_{cm}) = \text{Heaviside}(E_i(L) - E_{i \text{ min}}^f(L)) - \text{Heaviside}(E_i(L) - E_i^f_{\text{ max}}(L)),
\]

where \( E_i(L) \)- beam kinetic energy at the contact point, \( E_i^{\text{ min}}(L) \)- minimum beam kinetic energy at the contact point necessary to overcome the Coulomb barrier (Fig. 1), \( E_i^{\text{ max}}(L) \)- maximum beam kinetic energy at the contact point at which the system doesn’t go to the quasi-fission channel(Fig. 1). As shown by calculations below for the given friction coefficients energy window is large, few hundred MeV’s and at values \( E_{cm} \geq 370 \text{ MeV} \), \( P_{2i} = 0 \). Therefore it is necessary to take into account only the first term in expression (7).

![Fig.1. Dependence of nucleus-nucleus potential from distance between the centers of the nuclei](image-url)

To determine the capture probability for fixed initial angular momentum can be used expression:

\[
P_{2i}(L_i) = 1 - \exp \left( - \frac{E_i^f(L_i) - \Delta E_i^f(L_i)}{T_i(L_i)} \right),
\]

where \( T_i(L_i) = T_1(L_i) = T_2(L_i) = \sqrt{2E_i^f(L_i)/A_0} \)- temperature of the nuclei, after the dissipation of ki-
netic energy to the excitation energy. $E^*_i(L_i) - excitation energy, A_0 - nucleons amount of the interacting nuclei. $E^*_i(L_i) - energy required to overcome the quasi-fission barrier (fig. 1), determined from the interaction dynamics. $\Delta E^*_i(L_i) - difference between of excitation energies of heavy and light nuclei.

Usually the dynamic description is provided through the classical Newton’s equation for the collective coordinate of the relative distance between the centers of nuclei mass and angular momentum, which contain the radial and tangential friction forces. At this stage of the reaction emission of nucleons is absent, so the reduced mass $\mu = Const$. For spherical nuclei without deformation, the relative motion and changes of the angular momentum of the interacting fragments can be written the following system of equations [11]:

$$
\begin{align*}
\frac{d^2 R(t)}{dt^2} + \gamma_R(R) \frac{dR(t)}{dt} &= F_{nn}(R, L) , \\
\frac{dL(t)}{dt} &= \gamma_\theta(R)L(t) = 0 ,
\end{align*}
$$

where $F_{nn}(R, L) = -\frac{\partial V_{nn}(R, L)}{\partial R}$, $\gamma_R(R) = k_R \left( \frac{\partial V_{nn}(R)}{\partial R} \right)^2$, $k_R$ and $k_\theta$ - radial and tangential friction coefficients, respectively, which were selected $[4]$ $k_R = (0.5 \ldots 5) \cdot 10^{-23}$s/MeV and $k_\theta = 0.01 \cdot 10^{-23}$s/MeV. System (9) solved numerically on range by R from the contact point of the interacting nuclei ($R_{cont}$) until the capture moment. Capture process occurs in the minimum of the potential pocket, or system goes to the quasi-fission channel.

### 3. INTERACTION POTENTIAL

Important role in describing the dynamics of interaction and calculating the capture cross-section plays a selection of nucleus-nucleus potential. The potential interaction between two nuclei can be written as [5]:

$$V_{nn} = V_C + V_N + V_{rot} ,$$

where $V_C$ - Coulomb potential, for which can be used the expression:

$$V_C = \frac{Z_1 Z_2 e^2}{2(R_1 + R_2)} \left[ 3 - \left( \frac{R}{R_1 + R_2} \right)^2 \right] ,$$

for rotational potential, which is crucial in calculating of the capture cross-section, can be used the expression:

$$V_{rot} = \frac{\hbar^2 L(L+1)}{2 \mu R^2} ,$$

where $Z_1$ and $Z_2$ - number of protons in the nuclei; $R$, $R_1$, $R_2$ - distance between the centers of the interacting nuclei and their radii, respectively. To calculate the nuclear potential is used short-range forces potential (proximity), which has practically no adjustable parameters [12]:

$$V_N = V_N^{prox} = 4\pi \gamma R \bar{b}(\xi) .$$

All parameters can be found in [12, 13].

Fig.2 shows calculations of the interaction potential for the reaction $^{56}$Fe + $^{244}$Pu for different values of angular momentum. Similar dependences were obtained for all stable isotopes in the reactions $^{A_2}$Fe + $^{244}$Pu and $^{A_2}$Ni + $^{238}$U. Table 1 shows the relevant data for listed above reactions and the maximum values of angular momentum.

The calculations shows that with increasing the rotational part of the nucleus-nucleus potential (increase of the angular momentum) the depth of the pocket becomes smaller, the quasi-fission barrier ($B_{qf}$) decreases, and with certain values $L$ barrier disappears and capture becomes impossible. Therefore just limited range of the angular momentum may contribute in quasi-fission-fusion reactions for the different isotopic configurations of the beam and target.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$B_{qf}^{max}$ when $L = 0$</th>
<th>$L$ when $B_{qf} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{64}$Fe + $^{244}$Pu</td>
<td>2.6 MeV</td>
<td>60</td>
</tr>
<tr>
<td>$^{66}$Fe + $^{244}$Pu</td>
<td>3 MeV</td>
<td>65</td>
</tr>
<tr>
<td>$^{67}$Fe + $^{244}$Pu</td>
<td>3.2 MeV</td>
<td>67</td>
</tr>
<tr>
<td>$^{68}$Fe + $^{244}$Pu</td>
<td>3.4 MeV</td>
<td>68</td>
</tr>
<tr>
<td>$^{58}$Ni + $^{238}$U</td>
<td>2.1 MeV</td>
<td>59</td>
</tr>
<tr>
<td>$^{60}$Ni + $^{238}$U</td>
<td>2.3 MeV</td>
<td>62</td>
</tr>
<tr>
<td>$^{61}$Ni + $^{238}$U</td>
<td>2.5 MeV</td>
<td>63</td>
</tr>
<tr>
<td>$^{62}$Ni + $^{238}$U</td>
<td>2.7 MeV</td>
<td>65</td>
</tr>
<tr>
<td>$^{64}$Ni + $^{238}$U</td>
<td>3 MeV</td>
<td>67</td>
</tr>
</tbody>
</table>

### 4. NUCLEI DYNAMICS

Since length of the de-Broglie wave of heavy ion is much smaller than the size of the nuclei involved in the reaction, it makes possible to consider the nucleus as classical objects. The interaction of nuclei, leading to their capture is seen as a dynamic process. Solving the equations system (9) numerically, by Gear method, in the R range, from the contact moment of the interacting nuclei until the capture or
when system goes to quasi-fission channel, were calculated dependences that characterize the capture or quasi-fission process. Fig.3 shows change of kinetic energy from the distance between the interacting nuclei centers, arrows indicate the maximum and minimum values of nucleus-nucleus potential. This process is accompanied by intensive dissipation of kinetic energy that goes into internal excitation of nuclei. It should be noted that from the calculations the best value for the radial friction coefficient is $k_R \approx (1 \ldots 2) \cdot 10^{-23} s/MeV$, because in this case capture time is $(5 \ldots 50) \cdot 10^{-22} s$, which corresponds to the data in [11], where $\tau_c \approx 2.6/T_{DNS}^2 \cdot 10^{-22} s$. At higher radial friction coefficients capture time larger by orders and a situation arises by the process of stopping on the way down of the nuclei-nuclear potential after the Coulomb barrier. The remaining calculations presented in this work when $k_R = 1 \cdot 10^{-23} s/MeV$, $k_\theta = 0.01 \cdot 10^{-23} s/MeV$ and angular momentum is measured in $\hbar$ units.

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Fig.3. Dependence of the kinetic energy from distance between the interacting nuclei centers from contact point to minimum of nucleus-nucleus potential in reaction $^{56}Fe + ^{244}Pu$

Fig.4. Dependence of rotational potential from the interaction time for reaction $^{56}Fe + ^{244}Pu$

Special attention in the nuclei capture should be given to changes in rotational potential, because it determines the dynamics of height changes of quasi-fission barrier, and, consequently, the capture probability.

Fig.4. shows dependence of the rotational potential from the interaction time, which shows that at the initial time increase due to a sharp distance reducing between the nuclei centers. In the region of minimum potential energy there is a reduction of the rotational potential due to the exponential dependence of the relative angular momentum from the time. Dependence of the relative angular momentum from the distance between the nuclei centers shown at Fig.5. Since with time is established quasi-stationary process of reducing the angular momentum with an almost complete dissipation of kinetic energy, so for all analyzed reactions can use an expression:

$$ L_{\min} = 44 \cdot \ln \left(1 + \frac{L_0}{44}\right), \quad (14) $$

where $L_0$ - initial angular momentum value, $L_{\min}$ - angular momentum value at the capture time. This process explains that in capture cross-section calculation can’t be used $L = \text{Const}$, because change of quasi-fission barrier leads to changes in energy required to overcome it, and, consequently, changes in capture probability.

Fig.5. Dependence of relative angular momentum from distance between interacting nuclei centers for reaction $^{56}Fe + ^{244}Pu$

5. CAPTURE PROBABILITY

Capture process includes both dynamic and statistical parts. From Fig.1 and definitions (7) and (8), follows that it is necessary to know the energy to overcome the Coulomb barrier $E^+$ when light fragment moving from right to left and the energy $E^-$, which is required to overcome the quasi-fission barrier, when system goes into the quasi-fission channel. These energies were determined from the solution of (9), where the kinetic energy increment was chosen 1 MeV with accuracy determination of $E^+$ and $E^-$ 0.1 MeV. Figs.6 and 7 show dependences of $E^+$ and $E^-$ from the angular momentum for reactions $^{56}Fe + ^{244}Pu$ and $^{62}Ni + ^{238}U$. These kinds of dependences, in the whole kinetic energy range were
obtained for all stable isotopes of light nuclei in analyzed reactions.

From graphs follows that energy required to overcome the Coulomb barrier increases with increasing of angular momentum. This limits participation of the initial values of angular momentum in the capture cross-section. Therefore, to increase angular momentum involved in the reaction is necessary to increase beam kinetic energy. The quasi-fission barrier situation is reverse. Because depth of the potential pocket with growth of $L_0$ decreases, and energy required to overcome quasi-fission barrier with growth of $L_0$ also decreases. Therefore, the number of nuclei involved in the capture reaction, also decreases. However, as follows from (14), angular momentum during the passage of nucleus-nucleus potential is reduced, it con-tributes to the partial inclusion to the capture reaction nuclei with higher $L_0$.

Fig. 6. Energy required to overcome the Coulomb barrier ($E^+$) and quasi-fission barrier ($E^-$) from the relative angular momentum for the reaction $^{56}\text{Fe} + ^{244}\text{Pu}$

Fig. 7. Energy required to overcome the Coulomb barrier ($E^+$) and quasi-fission barrier ($E^-$) from the relative angular momentum for the reaction $^{62}\text{Ni} + ^{238}\text{U}$

Figs. 8 and 9, show dependence of capture probability for the reactions $^{56}\text{Fe} + ^{244}\text{Pu}$ and $^{62}\text{Ni} + ^{238}\text{U}$, for the angular momentum calculated by (8) for different initial kinetic energies of beam.

From the graphs follows that with increasing of kinetic energy initially increases the contribution of angular momentum in the capture reaction. However, with further growth of $E_{cm}$, value of decreases by decreasing of the $E^-$.

The capture probability at relatively small $E_{cm}$ is close to one. With increasing $E_{cm}$, excitation energy increases and decreases $E^-$, so decreases the capture probability, this process limits the values of L in reaction. Influence of temperature on the capture probability is insignificant.

Fig. 8. Dependence of capture probability from angular momentum at various beam kinetic energy for reaction $^{56}\text{Fe} + ^{244}\text{Pu}$

Fig. 9. Dependence of capture probability from angular momentum at various beam kinetic energy for reaction $^{62}\text{Ni} + ^{238}\text{U}$

6. THE CAPTURE CROSS-SECTION

To analyze the capture cross-sections of reactions $^{56}\text{Fe} + ^{244}\text{Pu}$ and $^{62}\text{Ni} + ^{238}\text{U}$ rewrite the formulas (5, 8) as:
\[ \sigma_c = \frac{\pi \hbar^2}{2\mu E_{cm}} \cdot \frac{1}{L_{\text{max}}(E_{cm})} \sum_{L_{1i}} (2L_{0i} + 1) \cdot P_{2i}, \]

\[ P_{2i}(L_i) = 1 - \exp \left( -\frac{E^{-1}_i(L_i) - [E^{-1}_1(L_i) - E^+_2(L_i)]}{T_i(L_i)} \right). \]

Term \( P_{1i} \) from (5) can be excluded, as it determines, eventually, only a minimal input kinetic energy necessary to overcome the Coulomb barrier when \( L_0 \geq 0 \) and accuracy of its determination depends on the accuracy of the solution of the system (9). The maximum kinetic energy is limited by the term \( P_{1i} \).

In part 2, of this work is defined term \( L_{\text{max}}(E_{cm}) \) - this is maximum value of the angular momentum which may be involved in capture reaction. Table 2 shows the values of \( L_{\text{max}} \) for a beam kinetic energies range, which carried the averaging of the capture cross-sections. Dependence of the \( L_{\text{max}} \) from \( E_{cm} \) practically linear and the range of \( L_{\text{max}} \) and increases with increasing of the neutrons number in the light fragment.

Table 2. Values of the \( L_{\text{max}} \) for the maximum and minimum beam kinetic energy (\( E_{cm} \))

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( L_{\text{max}}/E_{cm} ) MeV</th>
<th>( E_{\text{min}}/E_{cm} ) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{56}\text{Fe} + ^{244}\text{Pu} )</td>
<td>144/313</td>
<td>176/339</td>
</tr>
<tr>
<td>( ^{57}\text{Fe} + ^{244}\text{Pu} )</td>
<td>142/309</td>
<td>187/347</td>
</tr>
<tr>
<td>( ^{58}\text{Fe} + ^{244}\text{Pu} )</td>
<td>142/307</td>
<td>194/351</td>
</tr>
<tr>
<td>( ^{58}\text{Ni} + ^{238}\text{U} )</td>
<td>148/308</td>
<td>203/355</td>
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<td>157/332</td>
<td>172/343</td>
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<td>192/355</td>
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<td>197/359</td>
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<td>( ^{64}\text{Ni} + ^{238}\text{U} )</td>
<td>160/323</td>
<td>213/366</td>
</tr>
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</table>

Fig.10. Dependence of the angular momentum contributing to the capture cross-section, from the beam kinetic energy for the reaction \( ^{28}\text{Ni} + ^{238}\text{U} \)

Fig.11. Dependence of the angular momentum contributing to the capture cross-section, from the beam kinetic energy for the reaction \( ^{28}\text{Ni} + ^{238}\text{U} \)

\( L_{0i} \) - is the value of the angular momentum, which contributes to the capture cross section, by it is made the summation Figs.10 and 11 show dependences of the angular momentum contributing to the capture cross-section, from the beam kinetic energy for the reactions \( ^{56}\text{Fe} + ^{244}\text{Pu} \) and \( ^{58}\text{Ni} + ^{238}\text{U} \).

From the graphs follows that with increasing of the kinetic energy is increasing \( L \) contributing to the calculation of the capture cross sections, it is due to the nuclear overcome through the Coulomb barrier with higher angular momentum. Maximum of \( L \) reached after the maximum of the capture cross-sections. After that, there is increase of excitation energy and the capture probability drops to zero when \( E_{cm} = E_{cm}^\text{max} \). The excitation energy until the formation of the dinuclear system is distributed directly proportional to the nucleon composition of interacting nuclei, so the temperature of the nuclei is the same. During increasing of the \( E_{cm} \) the system temperature increases, but substantial contribution to the capture cross-section to a sharp its reduction doesn’t make. Table 3 shows the temperature values at the maximum and minimum kinetic energy.

Table 3. Values of the \( T \) (MeV) for maximum and minimum of the kinetic energy (\( E_{cm} \)) (MeV)

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( T/E_{cm} ) MeV</th>
<th>( T/E_{cm}^\text{max} ) MeV</th>
</tr>
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<tbody>
<tr>
<td>( ^{56}\text{Fe} + ^{244}\text{Pu} )</td>
<td>1.4/313</td>
<td>1.7/339</td>
</tr>
<tr>
<td>( ^{57}\text{Fe} + ^{244}\text{Pu} )</td>
<td>1.4/309</td>
<td>1.8/347</td>
</tr>
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<td>( ^{58}\text{Fe} + ^{244}\text{Pu} )</td>
<td>1.4/307</td>
<td>1.8/351</td>
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<tr>
<td>( ^{60}\text{Ni} + ^{238}\text{U} )</td>
<td>1.5/332</td>
<td>1.6/343</td>
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<td>1.5/329</td>
<td>1.7/352</td>
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<td>1.5/323</td>
<td>1.9/366</td>
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Figs.12,13 show dependence of the capture cross-section from the beam kinetic energy for the reaction \( ^{28}\text{Fe} + ^{244}\text{Pu} \) and \( ^{28}\text{Ni} + ^{238}\text{U} \).
Analyzing the dependence, we make following conclusions: with mass asymmetry increasing energy range of the kinetic energy increases. So it is increasing the maximum value of the capture cross-section. The maxima are shifted toward lower values of kinetic energy. For reaction $^{56}$Ni + $^{238}$U need a higher value of kinetic energy than for the reaction $^{56}$Fe + $^{244}$Pu, to achieve maximum value of $\sigma_c$.

Growth of the curves (see Figs.12, 13) with increasing of kinetic energy is due to the inclusion of the higher angular momentum values, which contribute to the capture cross-section. In addition, the excitation energies difference increase and, consequently, decrease the probability of capture. Almost after the maximum capture probability decreases because the quasi-fission barrier decreases. And with increasing of the initial angular momentum decreases the values of L, which contribute to the capture cross-section, and with the disappearance of quasi-fission barrier system goes to quasi-fission channel and capture cross-section becomes zero.

CONCLUSIONS

Dynamic-statistical approach can be used to describe the capture cross-section, the initial stage of super-heavy elements formation reaction. Specified range of beam energies for reactions $^{56}$Fe + $^{244}$Pu and $^{56}$Ni + $^{238}$U, in which the capture cross section is not zero, for all stable isotopes of light nuclei. Were found the optimal values of kinetic energy, leading to maximum values of capture cross-sections and the capture cross section dependences were obtained from the beam kinetic energy.

Shown decisive influence of the angular momentum change on the capture cross section and investigate the capture probability dependence from the angular momentum with different kinetic energies. Calculate the range of angular momentum contributing to the capture cross-section with change of kinetic energy.

References

ДИНАМИКО-СТАТИСТИЧЕСКОЕ ОПИСАНИЕ СЕЧЕНИЯ ЗАХВАТА - НАЧАЛЬНОЙ СТАДИИ РЕАКЦИИ СЛИЯНИЯ ТЯЖЕЛЫХ ЯДЕР

Р.А. Анохин, К.В. Павлий

Для описания сечения захвата ($\sigma_c$) начальной стадии реакции образования сверхтяжелых элементов, предложена динамико-статистическая модель, рассматривающая взаимодействие ядер с момента касания до момента образования двойной ядерной системы. Проведен анализ ядро-ядерного потенциала и показано, что существует ограничение углового момента, вносящего вклад в сечение захвата, которое определяется исчезновением потенциального кармана. Динамическое рассмотрение позволило выявить основной вклад углового момента в сечение захвата. Проведены расчеты, анализ сечения захвата и динамических характеристик реакций $A_2Fe + 244Pu$ и $A_2Ni + 238U$. Получены зависимости $\sigma_c$ от кинетической энергии пучка.

ДИНАМИКО-СТАТИСТИЧНИЙ ОПИС ПЕРЕТИНУ ЗАХОПЛЕННЯ - ПОЧАТКОВОЇ СТАДІЇ РЕАКЦІЇ ЗЛІТТЯ ВАЖКИХ ЯДЕР

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Для опису перетину захоплення ($\sigma_c$) початкової стадії реакції утворення надважких елементів, запропонована динамико-статистична модель, яка розглядає захоплення ядер з моменту дотику до моменту утворення подвійної ядерної системи. Проведено аналіз ядро-ядерного потенціалу і показано, що існує обмеження кутового моменту, який робить внесок у перетин захоплення, який визначається зникненням потенціального кармана. Динамічний розгляд дозволив виявити основний внесок кутового моменту в перетин захоплення. Проведено розрахунки, аналіз перетину захоплення і динамічних характеристик реакцій $A_2Fe + 244Pu$ і $A_2Ni + 238U$. Отримано залежності $\sigma_c$ від кінетичної енергії пучка.