OSCILLATIONS AND COHERENT RADIATION OF HARMONICS IN RADIATION SPECTRUM OF SYSTEM OF ELECTRONS MOVING IN SPIRAL IN MEDIUM

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Using the Lorentz self-interaction method completing by Dirac hypothesis it is investigated the spectral distribution of the radiation power for the system of electrons moving along a spiral in transparent isotropic medium. The overlapping between neighbour harmonics as well as oscillations in the spectral distribution of one, two, three, and four electrons radiation power are studied for the case when the transversal component of electron velocity is bigger than the light phase velocity in medium but still less than the light velocity in vacuum. The effect of coherence in the spectrum of synchrotron-Cherenkov radiation for the system of two, three and four electrons is analyzed.

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1. INTRODUCTION

The properties of synchrotron radiation of charged particles moving in a circle in vacuum in framework of classical electrodynamics were studied in papers [1 - 3]. The particularities of radiation spectrum of charged particles moving in magnetic field in vacuum were examined by Ternov in report [4] and analyzed in studies [5 - 8]. The properties of electromagnetic radiation of the system of non-interacting electrons moving in a spiral in constant magnetic field in vacuum were reported in papers [9 - 17].

The radiation spectrum of one electron moving in a medium in magnetic field was under investigation in papers [18 - 24]. The oscillations in synchrotron-Cherenkov radiation spectrum of one electron were obtained at its motion in a circle [18] and in a spiral [24]. The hopping change of the function of spectral distribution of radiation power of an electron is studied in [12].

The coherence effects in the structure of radiation spectrum of a system of non-interacting electrons moving one by one along a spiral in a transparent medium were considered in papers [25 - 31]. If the dimension of a system of electrons is smaller comparing to the radiation wavelength, both for a quantum-mechanical system [32] and for a system of electrons [33 - 36] the super-radiant regime is possible.

The aim of this paper is the investigation of the oscillations and overlapping between neighbour harmonics of the synchrotron-Cherenkov radiation spectrum of two, three, and four electrons moving in a spiral in magnetic field in transparent medium for the case when the transversal component of electrons velocity is bigger than the light phase velocity in medium but still less than the light velocity in vacuum and the parallel component of electrons velocity is much smaller than the light phase velocity in medium. The spectral distributions of electrons radiation power are calculated by the instrumentality of high accuracy numerical methods and studied within the analytical methods. The coherent radiation of harmonics in the spectrum of synchrotron-Cherenkov radiation for two, three and four electrons in the case when the distance between electrons (phase shifts between electrons) is much smaller than the radiation wavelength are studied. These studies present a great interest for the investigation of radiation spectrum structure of bunches of charged particles moving in magnetic field.

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2. TIME AVERAGED RADIATION POWER OF SYSTEM OF ELECTRONS MOVING ALONG A SPIRAL IN TRANSPARENT MEDIUM

According to [12, 29] the time averaged radiation power $\overline{P}^{\,rad}$ of charged particles moving in medium is determined by the relationship:

$$\overline{P}^{\,rad} = \lim_{T\to\infty} \frac{1}{2T} \int\limits_{-T}^{T} dt \int\limits_{\tau} d\vec{r} \times$$

$$\times \left(\vec{j}\,(\vec{r},t)\, \frac{\partial \vec{A}^{Dir}\,(\vec{r},t)}{\partial t} - \rho\,(\vec{r},t)\, \frac{\partial \Phi^{Dir}\,(\vec{r},t)}{\partial t} \right). \quad (1)$$

Here $\vec{j}\,(\vec{r},t)$ is the current density and $\rho\,(\vec{r},t)$ is the charge density. The integration is over some volume τ.

According to the hypothesis of Dirac [3, 12, 29, 37, 38], the scalar $\Phi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials.

$$\Phi^{Dir}=\frac{1}{2}\left(\Phi^{ret}-\Phi^{adv}\right),\vec{A}^{Dir}=\frac{1}{2}\left(\vec{A}^{ret}-\vec{A}^{adv}\right). \tag{2}$$

Then, the sources functions of N charged point particles are defined as [12, 29]

$$\vec{j}(\vec{r},t)=\sum_{l=1}^{N}\vec{V}_l(t)\rho_l(\vec{r},t),\ \ \rho(\vec{r},t)=\sum_{l=1}^{N}\rho_l(\vec{r},t),$$

$$\rho_l(\vec{r},t)=e\delta(\vec{r}-\vec{r}_l(t)), \tag{3}$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and velocity of the l^{th} particle, respectively.

The law of motion and the velocity of the l^{th} electron in magnetic field are given by the expressions

$$\vec{r}_l(t)=r_0\cos\{\omega_0(t+\Delta t_l)\}\vec{i}+r_0\sin\{\omega_0(t+\Delta t_l)\}\vec{j}+$$

$$+V_{\|}(t+\Delta t_l)\vec{k},\quad \vec{V}_l(t)=\frac{d\vec{r}_l(t)}{dt}. \tag{4}$$

Here $r_0=V_\perp\omega_0^{-1}$, $\omega_0=c^2eB^{ext}\tilde{E}^{-1}$, $\tilde{E}=c\sqrt{p^2+m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\parallel 0Z$, $V_\perp$ and $V_\|$ are the components of the velocity, the $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass.

In this case the time averaged radiation power of the point electrons can be obtained after substitution of (2) to (4) into (1). Then, it is found [12, 29] that

$$\overline{P}^{rad}=\int_0^\infty d\omega\ W(\omega), \tag{5}$$

$$W(\omega)=\frac{2e^2}{\pi}\int_0^\infty dx\omega\mu(\omega)\frac{\mu_0}{4\pi}\frac{\sin\left(\frac{n(\omega)}{c}\omega\eta(x)\right)}{\eta(x)}\times$$

$$\times S_N(\omega)\cos(\omega x)\left\{V_\perp^2\cos(\omega_0x)+V_\|^2-\frac{c^2}{n^2(\omega)}\right\}, \tag{6}$$

where $\eta(x)=\sqrt{V_\|^2x^2+4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$, $\mu_a(\omega)=\mu(\omega)\mu_0$ is the absolute magnetic permeability, $n(\omega)$ is the refraction index, ω is cyclic frequency, c is velocity of light in vacuum.

In the case of electrons moving one by one along a spiral the coherence factor takes the form [12, 29]:

$$S_N(\omega)=\sum_{l,j=1}^{N}\cos\{\omega(\Delta t_l-\Delta t_j)\}. \tag{7}$$

The coherence factor $S_N(\omega)$ determines a redistribution of radiation power of electrons in spectral distribution of this radiation.

3. SPECTRAL-ANGULAR AND SPECTRAL DISTRIBUTION OF THE RADIATION POWER OF THE SYSTEM OF ELECTRONS MOVING ALONG A SPIRAL IN TRANSPARENT MEDIUM

After some transformations of relationships (5) and (6) the contributions of separate harmonics to the electrons radiation power can be expressed as [29]:

$$\overline{P}^{rad}=\frac{e^2}{c}\sum_{m=-\infty}^{\infty}\int_0^\infty d\omega\mu(\omega)\frac{\mu_0}{4\pi}n(\omega)\omega^2\int_0^\pi\sin\theta d\theta\times$$

$$\times S_N(\omega)\delta\left\{\omega\left(1-\frac{n(\omega)}{c}V_\|\cos\theta\right)-m\omega_0\right\}\times$$

$$\times\left\{V_\perp^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m^{'2}(q)\right]+\right.$$

$$\left.+\left(V_\|^2-\frac{c^2}{n^2(\omega)}\right)J_m^2(q)\right\}, \tag{8}$$

where $q=V_\perp\frac{n(\omega)}{c}\frac{\omega}{\omega_0}\sin\theta$, $\omega_0=\frac{eB^{ext}}{m_0}\sqrt{1-\frac{V^2}{c^2}}$, $J_m(q)$ and $J_m^{'}(q)$ are the Bessel function with integer index and its derivative, respectively, θ is the angle formed by wave vector $\vec{k}$ and axis 0Z [29].

The relationship (8) un the case of one electron was also obtained using retarded potentials within the method of enclosing surfaces [20].

Each harmonic in relationship (8) is a set of the frequencies, which are the solutions of the equations

$$\omega\left(1-\frac{n(\omega)}{c}V_\|\cos\theta\right)-m\omega_0=0. \tag{9}$$

After integrating in (8) over θ variable we have obtained the spectral distribution of the system of electrons radiation power on harmonics [29]

$$\overline{P}^{rad}=\frac{e^2}{V_\|}\sum_{m=-\infty}^{\infty}\int_0^\infty d\omega\mu(\omega)\frac{\mu_0}{4\pi}\eta\left(u^2(m)\right)\omega\times$$

$$\times S_N(\omega)\left\{V_\perp^2\left[\frac{m^2}{q_1^2u^2(m)}J_m^2(q_1u(m))+\right.\right.$$

$$\left.\left.+J_m^{'2}(q_1u(m))\right]+\left(V_\|^2-\frac{c^2}{n^2(\omega)}\right)J_m^2(q_1u(m))\right\}, \tag{10}$$

where

$$\eta\left(u^2(m)\right)=\begin{cases}1, & u^2(m)>0\\ 0, & u^2(m)<0\end{cases},\quad q_1=V_\perp\frac{n(\omega)}{c}\frac{\omega}{\omega_0}.$$

$$u^2(m)=1-\frac{c^2(\omega-m\omega_0)^2}{n^2(\omega)V_\|^2\omega^2}. \tag{11}$$

The band boundaries in the radiation spectrum are determined by the function $\eta\left(u^2(m)\right)$.

The coherence factor $S_1(\omega)$ of a single electron is defined as

$$S_N(\omega)=S_1(\omega)=1. \tag{12}$$

In the case of two electrons the coherence factor $S_2(\omega)$ is defined as [16]

$$S_2(\omega)=2+2\cos(\omega\Delta t_{12}). \tag{13}$$

Here $\Delta t_{12} = \Delta t_2 - \Delta t_1$ is the time shift between the first and second electrons moving along a spiral.

The analogous expression for the coherence factor was obtained by Bolotovskii [39]. The coherence factor of three electrons takes the form [16]

$$S_3(\omega) = 3 + 2\cos(\omega \Delta t_{12}) + 2\cos(\omega \Delta t_{23}) + \\ +2\cos\{\omega(\Delta t_{12} + \Delta t_{23})\}. \quad (14)$$

Here Δt_{23} is the time shift between the second and third electrons. The coherence factor of four electrons is de-fined as [16]

$$S_4(\omega) = 4 + 2\cos(\omega \Delta t_{12}) + 2\cos(\omega \Delta t_{23}) + \\ +2\cos(\omega \Delta t_{34}) + 2\cos\{\omega(\Delta t_{12} + \Delta t_{23})\} + \\ +2\cos\{\omega(\Delta t_{23} + \Delta t_{34})\} + \\ +2\cos\{\omega(\Delta t_{12} + \Delta t_{23} + \Delta t_{34})\}, \quad (15)$$

where Δt_{34} is the time shift between the third and fourth electrons.

4. PECULIARITIES OF THE SPECTRAL DISTRIBUTION OF RADIATION SPECTRUM OF ONE, TWO, THREE, AND FOUR ELECTRONS MOVING ALONG A SPIRAL IN TRANSPARENT MEDIUM

Our high accuracy numerical method of calculations of the radiation spectra was carried out on the basis of relationships (5) to (7) and (12) to (15). The spectral distribution of synchrotron-Cherenkov radiation power was obtained for $B^{ext} = 10^{-4}\,T$, $\mu = 1$, $\mu_0 = 4\pi \times 10^{-7}\,H/m$, $n = 2.0$, $V_{\perp med} = 0.2 \times 10^9\,m/s$, $V_{\parallel med} = 0.15 \times 10^8\,m/s$, $\omega_{0j} = 0.1307 \times 10^8\,rad/s$, $r_{0j} = 15.3m$ (j=1,2,...,21) and $V_{\perp med} = 0.2 \times 10^9\,m/s$, $V_{\parallel med} = 0.3 \times 10^8\,m/s$, $\omega_{022} = 0.1298 \times 10^8\,rad/s$, $r_{022} = 15.4m$, $c = 0.2997925 \times 10^9\,m/s$.

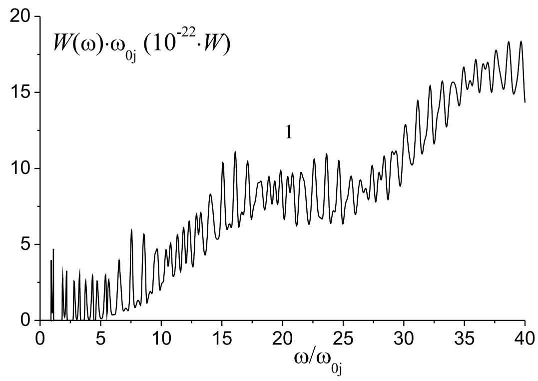

Fig.1. *Oscillations in synchrotron-Cherenkov radiation spectrum at low harmonics for* $B^{ext} = 10^{-4}\,T$, $n = 2$, $V_{\perp med} = 0.2\times10^9\,m/s$, $V_{\parallel med} = 0.15\times10^8\,m/s$. $\omega_{0j} = 0.1307\times10^8\,rad/s$, $r_{0j} = 15.3m$ (j=1,2,...,21). *Curve* 1. *One electron with radiation power* $P^{int}_{med1} = 0.315 \times 10^{-19}\,W$

The radiation power for one electron $P^{int}_{med1} = 0.315 \times 10^{-19}\,W$ in interval 0 to $40\omega_{01}$ is determined after integration of relationships (5) taking into account (6), (7) when $S_N(\omega)$ is substituted by $S_1(\omega) = 1$ (curve 1 in Figs.1 and 2).

It is interesting to compare the radiation power spectral distributions for one electron (see curve 1 in Fig.1) to that of two, three, and four electrons (curves 2 to 4 in Fig.2, respectively).

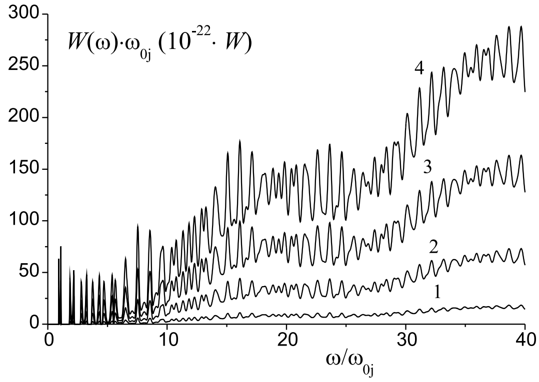

Fig.2. *Oscillations in synchrotron-Cherenkov radiation spectrum at low harmonics. Curve* 2. *Two electrons at* $\Delta t^{(2)}_{12} = 0.001\pi/\omega_{02}$ *with radiation power* $P^{int}_{med2} = 0.1257 \times 10^{-18}\,W$, $P^{int}_{med2}/P^{int}_{med1} = 3.99$. *Curve* 3. *Three electrons at* $\Delta t^{(3)}_{12} = \Delta t^{(3)}_{23} = 0.001\pi/\omega_{03}$ *with* $P^{int}_{med3} = 0.2820 \times 10^{-18}\,W$, $P^{int}_{med3}/P^{int}_{med1} = 8.95$. *Curve* 4. *Four electrons at time shifts* $\Delta t^{(4)}_{12} = \Delta t^{(4)}_{23} = \Delta t^{(4)}_{34} = 0.001\pi/\omega_{04}$ *with radiation power* $P^{int}_{med4} = 0.4999 \times 10^{-18}\,W$, $P^{int}_{med4}/P^{int}_{med1} = 15.84$

For the time shift $\Delta t^{(2)}_{12} = 0.001\pi/\omega_{02}$ (curve 2 in Figs. 2 and 3) the coherence factor $S_2(\omega) \approx 4$ at low harmonics and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four more than a single electron ($P^{int}_{med2} = 0.1257 \times 10^{-18}\,W$, $P^{int}_{med2}/P^{int}_{med1} = 3.99$).

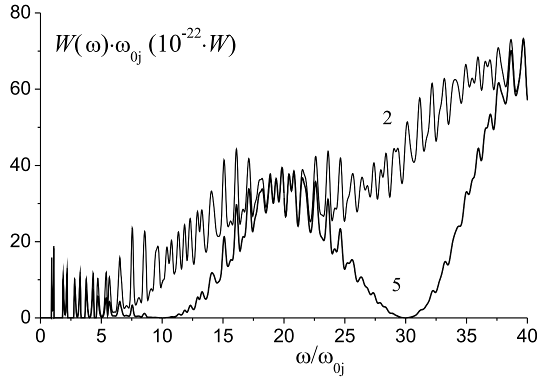

Fig.3. *Synchrotron-Cherenkov radiation spectrum at low harmonics. Curve 5. Two electrons at* $\Delta t^{(5)}_{12} = 0.1\pi/\omega_{05}$ *with radiation power* $P^{int}_{med5} = 0.6564 \times 10^{-19}$W, $P^{int}_{med5}/P^{int}_{med1} = 2.08$

For the time shifts $\Delta t^{(3)}_{12} = \Delta t^{(3)}_{23} = 0.001\pi/\omega_{03}$ the coherence factor $S_3(\omega) \approx 9$ and at low harmonics three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$(curve 3 in Figs. 2 and 4), i.e. by a factor of nine more than a single electron ($P^{int}_{med3} = 0.2820 \times 10^{-18}\,W$, $P^{int}_{med3}/P^{int}_{med1} = 8.95$).

For the time shifts $\Delta t^{(4)}_{12} = \Delta t^{(4)}_{23} = \Delta t^{(4)}_{34} = 0.001\pi/\omega_{04}$ the coherence factor $S_4(\omega) \approx 16$ and four electrons radiate as a charged particle with the charge $4e$ and the rest mass $4m_0$ (curve 4 in Figs. 2 and 5), i.e. by a factor of sixteen more than a single electron ($P^{int}_{med4} = 0.4999 \times 10^{-18}\,W$, $P^{int}_{med4}/P^{int}_{med1} = 15.84$).

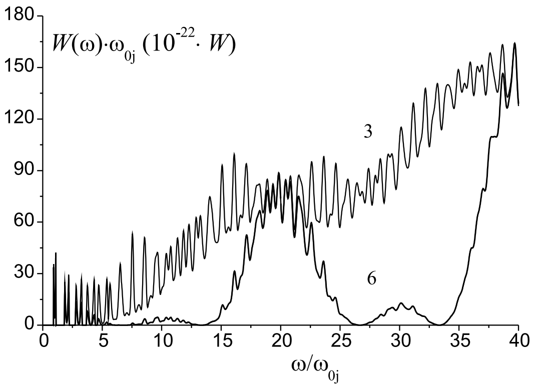

Fig.4. *Synchrotron-Cherenkov radiation spectrum at low harmonics. Curve* 6. *Three electrons at* $\Delta t^{(6)}_{12} = \Delta t^{(6)}_{23} = 0.1\pi/\omega_{06}$ *with radiation power* $P^{int}_{med6} = 0.9944 \times 10^{-19}\,W$, $P^{int}_{med6}/P^{int}_{med1} = 3.16$

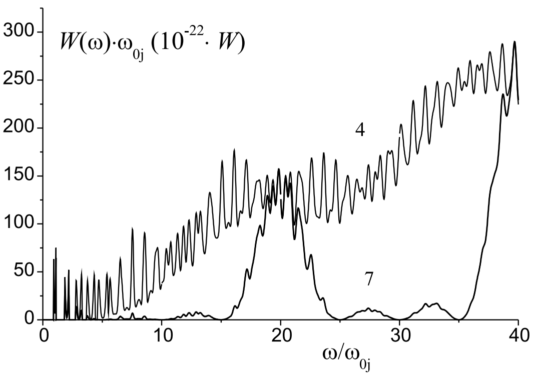

Fig.5. *Synchrotron-Cherenkov radiation spectrum at low harmonics. Curve* 7. *Four electrons at* $\Delta t^{(7)}_{12} = \Delta t^{(7)}_{23} = \Delta t^{(7)}_{34} = 0.1\pi/\omega_{07}$ *with radiation power* $P^{int}_{med7} = 0.1331 \times 10^{-18}\,W$, $P^{int}_{med7}/P^{int}_{med4} = 4.23$

In the frequency range of $0-40\omega_{0j}$ for smaller time shifts we have obtained the coherent radiation with radiation power $\overline{P}^{rad}$ proportional to N^2 (curves 2, 3, and 4 in Fig. 2) for such the electron system so far as the dimension of this system is smaller in comparison to the radiation wavelength [38], see also [32 - 36].

For the component of velocity $V_{\perp med} = 0.2 \times 10^9 m/s$, $V_{\parallel med} = 0.15 \times 10^8 m/s$ the spectral distribution of synchrotron-Cherenkov radiation power of one, two, three, and four electrons moving along spiral at the first harmonics has a form of discrete bands (see Figs.1-2).

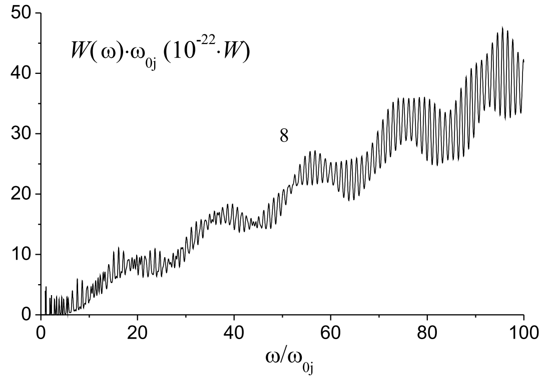

Fig.6. *Oscillations in synchrotron-Cherenkov radiation spectrum at low and middle harmonics. Curve 8. One electron with radiation power* $P^{int}_{med8} = 0.1950 \times 10^{-18}\,W$

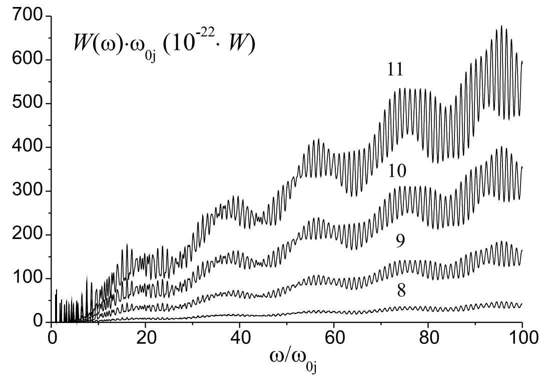

Fig.7. *Oscillations in synchrotron-Cherenkov radiation spectrum at low and middle harmonics. Curve* 9. *Two electrons at* $\Delta t^{(9)}_{12} = 0.001\pi/\omega_{09}$ *with* $P^{int}_{med9} = 0.7702 \times 10^{-18}\,W$, $P^{int}_{med9}/P^{int}_{med8} = 3.95$. *Curve* 10. *Three electrons at time shifts* $\Delta t^{(10)}_{12} = \Delta t^{(10)}_{23} = 0.001\pi/\omega_{010}$ *with radiation power* $P^{int}_{med10} = 0.1698 \times 10^{-17}\,W$, $P^{int}_{med10}/P^{int}_{med8} = 8.71$. *Curve* 11. *Four electrons at* $\Delta t^{(11)}_{12} = \Delta t^{(11)}_{23} = \Delta t^{(11)}_{34} = 0.001\pi/\omega_{011}$ *with* $P^{int}_{med11} = 0.2933 \times 10^{-17}\,W$, $P^{int}_{med11}/P^{int}_{med8} = 15.04$

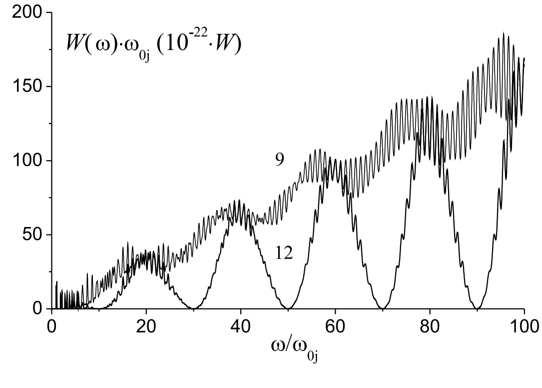

Fig.8. *Synchrotron-Cherenkov radiation spectrum at low and middle harmonics. Curve* 12. *Two electrons at* $\Delta t^{(12)}_{12} = 0.1\pi/\omega_{012}$ *with radiation power* $P^{int}_{med12} = 0.3916 \times 10^{-18}\,W$, $P^{int}_{med12}/P^{int}_{med8} = 2.02$

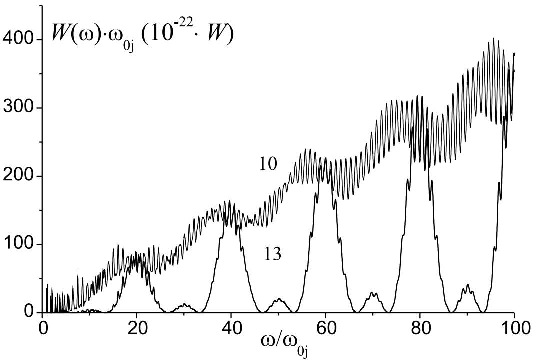

Fig.9. *Synchrotron-Cherenkov radiation spectrum at low and middle harmonics. Curve* 13. *Three electrons at* $\Delta t_{12}^{(13)} = \Delta t_{23}^{(13)} = 0.1\pi/\omega_{013}$ *with radiation power* $P_{med13}^{int} = 0.5878 \times 10^{-18}\,W$, $P_{med13}^{int}/P_{med8}^{int} = 3.01$

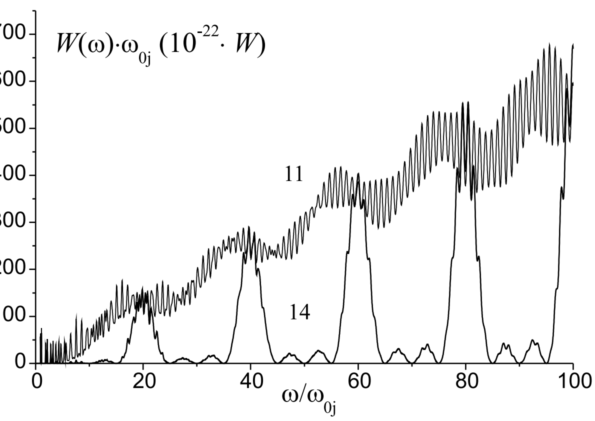

Fig.10. *Synchrotron-Cherenkov radiation spectrum at low and middle harmonics. Curve* 14. *Four electrons at* $\Delta t_{12}^{(14)} = \Delta t_{23}^{(14)} = \Delta t_{34}^{(14)} = 0.1\pi/\omega_{014}$ *with radiation power* $P_{med14}^{int} = 0.7837 \times 10^{-18}\,W$, $P_{med14}^{int}/P_{med8}^{int} = 4.02$

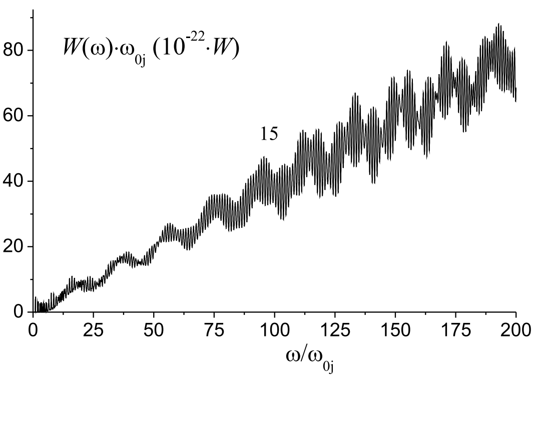

Fig.11. *Oscillations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics. Curve* 15. *One electron with radiation power* $P_{med15}^{int} = 0.7764 \times 10^{-18}\,W$

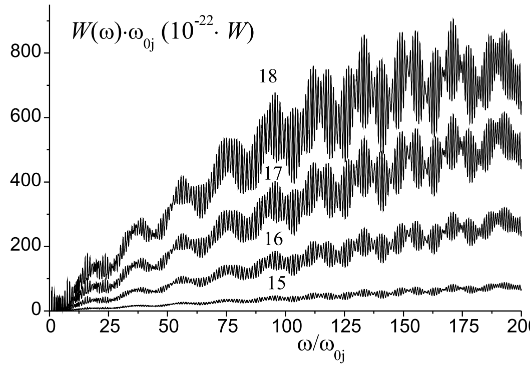

Fig.12. *Oscillations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics. Curve* 16. *Two electrons at time shift* $\Delta t_{12}^{(16)} = 0.001\pi/\omega_{016}$ *with radiation power* $P_{med16}^{int} = 0.2956 \times 10^{-17}\,W$, $P_{med16}^{int}/P_{med15}^{int} = 3.81$. *Curve* 17. *Three electrons at time shifts* $\Delta t_{12}^{(17)} = \Delta t_{23}^{(17)} = 0.001\pi/\omega_{017}$ *with radiation power* $P_{med17}^{int} = 0.6126 \times 10^{-17}\,W$, $P_{med17}^{int}/P_{med15}^{int} = 7.89$. *Curve* 18. *Four electrons at time shifts* $\Delta t_{12}^{(18)} = \Delta t_{23}^{(18)} = \Delta t_{34}^{(18)} = 0.001\pi/\omega_{018}$ *with radiation power* $P_{med18}^{int} = 0.9721 \times 10^{-17}\,W$, $P_{med18}^{int}/P_{med15}^{int} = 12.52$

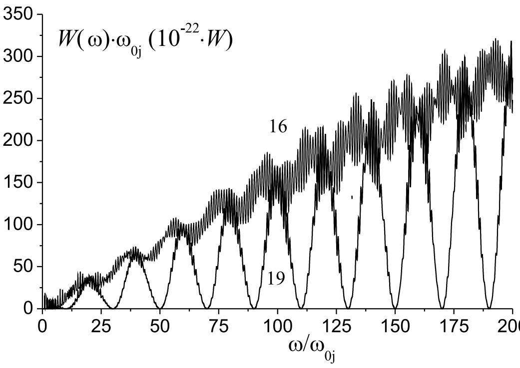

Fig.13. *Synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics. Curve* 19. *Two electrons at* $\Delta t_{12}^{(19)} = 0.1\pi/\omega_{019}$ *with radiation power* $P_{med19}^{int} = 0.1520 \times 10^{-17}\,W$, $P_{med19}^{int}/P_{med15}^{int} = 1.96$

For the velocities $c > V_{\perp med} > c/n$ ($V_{\perp med} = 0.2 \times 10^9$ m/s, $V_{\parallel med} = 0.15 \times 10^8$m/s) we have found the oscillations in the radiation spectrum of one electron (see curve 1 in Figs.1, 2, curve 8 in Figs.6, 7, and curve 15 in Figs.11, 12) as well as in that of two, three, and four electrons moving one by one along the spiral with a smaller selected time shifts $0.001\pi/\omega_{0j}$ (see curves 2 to 4 in Fig.2, curves 9 to 11 in Fig.7, and curves 16 to 18 in Fig.12).

The oscillating character of the spectral distribution of the synchrotron-Cherenkov radiation of electrons moving in magnetic field in the medium at $c > V_{\perp med} > c/n$ is defined by a properties of the Bessel functions [40] (Figs.1-16).

If $V_{\parallel med} \to 0$, then the spiral transforms into a circle and the radiation spectrum takes a discrete character. At moving in a circle at $V_{\perp med} > c/n$, the spectral distribution of the synchrotron-Cherenkov radiation of the system of electrons has also an oscillating character, too [21].

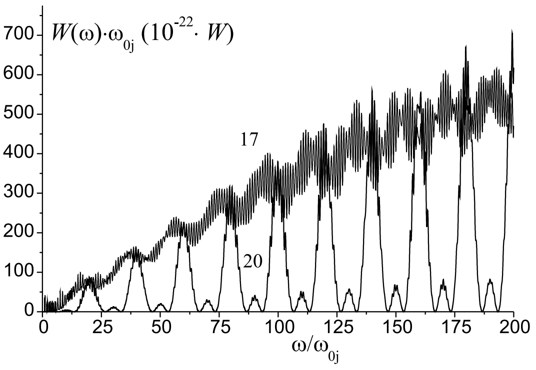

Fig.14. *Synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics. Curve* 20. *Three electrons at* $\Delta t_{12}^{(20)} = \Delta t_{23}^{(20)} = 0.1\pi/\omega_{020}$ *with radiation power* $P_{med20}^{int} = 0.2264 \times 10^{-17}\,W$, $P_{med20}^{int}/P_{med15}^{int} = 2.92$

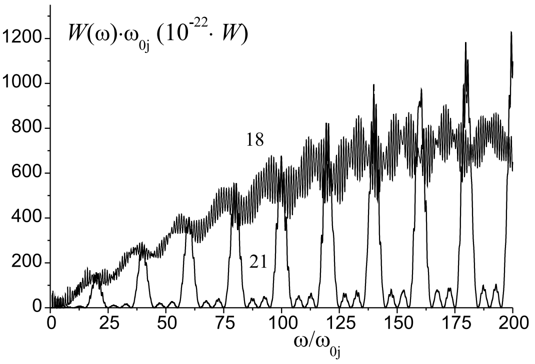

Fig.15. *Synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics. Curve* 21. *Four electrons at* $\Delta t_{12}^{(21)} = \Delta t_{23}^{(21)} = \Delta t_{34}^{(21)} = 0.1\pi/\omega_{021}$ *with radiation power* $P_{med21}^{int} = 0.3008 \times 10^{-17}\,W$, $P_{med21}^{int}/P_{med15}^{int} = 3.87$

For the time shift $\Delta t_{12}^{(19)} = 0.1\pi/\omega_{019}$ between two electrons we have found that there is no radiation at frequencies $10(2i-1)\omega_{019}$ (i=1,2,...,10) (curve 19 in Fig.13) and at the frequencies $20i\omega_{019}$ (i=1,2,...,10) the coherence factor takes the maximum value equal to four.

In the case of time shifts $\Delta t_{12}^{(20)} = \Delta t_{23}^{(20)} = 0.1\pi/\omega_{020}$ between three electrons we have found that there is no radiation at frequencies $20(3i-2)\omega_{020}/3$ and $20(3i-1)\omega_{020}/3$, (i=1,2,...,10) (curve 20 in Fig. 14) and at the frequencies $20i\omega_{020}$ (i=1,2,...,10) the coherence factor takes the maximum value equal to nine.

In the case of time shifts $\Delta t_{12}^{(21)} = \Delta t_{23}^{(21)} = \Delta t_{34}^{(21)} = 0.1\pi/\omega_{021}$ between four electrons we have found that there is no radiation at frequencies $5(4i-3)\omega_{021}$, $5(4i-2)\omega_{021}$, and $5(4i-1)\omega_{021}$, (i=1,2,...,10) (curve 21 in Fig. 15) and at the frequencies $20i\omega_{021}$ (i=1,2,...,10) the coherence factor takes the maximum value equal to sixteen.

For time shifts $0.1\pi/\omega_{0j}$ (Figs.3-5, Figs.8-10, and Figs.13-15) the coherence factor takes the maximum value $S_N(\omega) = N^2$ at frequencies $20i\omega_{0j}$ (i=1, 2,...).

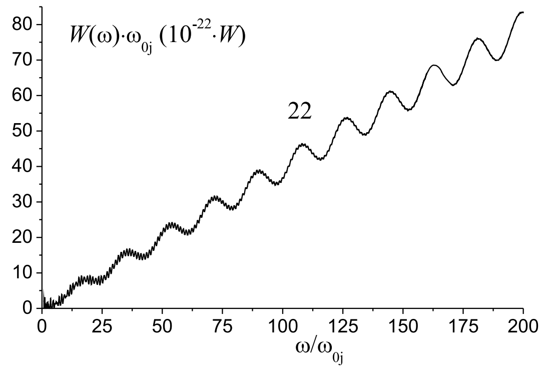

Fig.16. *Oscillations in synchrotron-Cherenkov radiation spectrum at low, middle, and high harmonics for* $B^{ext} = 10^{-4}\,T$, $n = 2$, $V_{\perp med} = 0.2 \times 10^9\,m/s$, $V_{\parallel med} = 0.3 \times 10^8\,m/s$, $\omega_{022} = 0.1298 \times 10^8\,rad/s$, $r_0 = 15.4m$. *Curve* 22. *One electron with radiation power* $P_{med22}^{int} = 0.7827 \times 10^{-18}\,W$

For high harmonics at $V_{\perp med} = 0.2 \times 10^9$m/s, $V_{\parallel med} = 0.3 \times 10^8$m/s the overlapping between neighbour harmonics does not lead, in fact, to periodical changes of the spectral distribution for synchrotron-Cherenkov radiation power. Only the oscillations of this function are observed (curve 22 in Fig.16). The obtained results are in good agreement to those obtained in [24].

5. CONCLUSIONS

The near-periodical variations of the spectral distribution function at $c > V_{\perp med} > c/n$ of synchrotron-Cherenkov radiation are preferably caused by the overlapping between the m^{th} and $(m+1)^{th}$ harmonics at some contribution of the other ones. At increasing parallel component of the velocity $V_{\parallel med}$ the near-periodical variations of the spectral distribution of the synchrotron-Cherenkov radiation power considerably decrease.

At small time shifts $0.001\pi/\omega_{0j}$ between electrons the system of two, three, and four electrons in the frequency range of $0 - 40\omega_{0j}$ there arises the coherent synchrotron-Cherenkov radiation with coherent factor $S_N(\omega) = N^2$ so far as the dimension of this system is smaller in comparison to the radiation wavelength.

For the velocities $c > V_{\perp med} > c/n$ ($V_{\perp med} = 0.2 \times 10^9$ m/s, $V_{\parallel med} = 0.15 \times 10^8$m/s) there arise the oscillations in the radiation spectrum of two, three, and four electrons moving one by one along the spiral with a smaller selected time shifts $0.001\pi/\omega_{0j}$.

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ОСЦИЛЛЯЦИИ И КОГЕРЕНТНОЕ ИЗЛУЧЕНИЕ ГАРМОНИК В СПЕКТРЕ ИЗЛУЧЕНИЯ СИСТЕМЫ ЭЛЕКТРОНОВ, ДВИЖУЩИХСЯ ВДОЛЬ ВИНТОВОЙ ЛИНИИ В СРЕДЕ

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Используя метод силы самодействия Лоренца, дополненный гипотезой Дирака, исследовано спектральное распределение мощности излучения системы электронов, движущихся вдоль винтовой линии в прозрачной изотропной среде. Перекрытие гармоник и осцилляции в спектральном распределении мощности излучения одного, двух, трех и четырех электронов исследовано для случая, когда поперечная компонента скорости электрона больше фазовой скорости света в среде, но меньше скорости света в вакууме. Исследован эффект когерентности в спектре синхротронно-черенковского излучения системы двух, трех и четырех электронов.

ОСЦИЛЯЦІЇ ТА КОГЕРЕНТНЕ ВИПРОМІНЮВАННЯ ГАРМОНІК У СПЕКТРІ ВИПРОМІНЮВАННЯ СИСТЕМИ ЕЛЕКТРОНІВ, ЩО РУХАЮТЬСЯ ВЗДОВЖ ГВИНТОВОЇ ЛІНІЇ У СЕРЕДОВИЩІ

А.В. Константинович, І.А. Константинович

Використовуючи метод сили самодії Лоренца, доповнений гіпотезою Дірака, досліджено спектральний розподіл потужності випромінювання системи електронів, що рухаются вздовж гвинтової лінії у прозорому ізотропному середовищі. Перекриття гармонік та осциляції у спектральному розподілі потужності випромінювання одного, двох, трьох і чотирьох електронів досліджено для випадку, коли поперечна компонента швидкості електрона більша від фазовоі швидкості світла у середовищі, але менша від швидкості світла у вакуумі. Досліджено ефект когерентності у спектрі синхротронно-черенковського випромінювання системи двох, трьох та чотирьох електронів.