

# IS A FIELD-INDUCED FERROMAGNETIC PHASE TRANSITION IN THE MAGNETAR CORE ACTUALLY POSSIBLE?

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Spin polarized states in dense neutron matter with BSk20 Skyrme force are considered in magnetic fields up to  $10^{20}$  G. It is shown that the appearance of the longitudinal instability in a strong magnetic field prevents the formation of a fully spin polarized state in neutron matter, and only the states with moderate spin polarization can be developed.

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## 1. INTRODUCTION. BASIC EQUATIONS

Magnetars are strongly magnetized neutron stars with emissions powered by the dissipation of magnetic energy. The magnetic field strength at the surface of a magnetar is of about  $10^{14}$ - $10^{15}$  G. Such huge magnetic fields can be inferred from observations of magnetar periods and spin-down rates, or from hydrogen spectral lines. In the interior of a magnetar the magnetic field strength could reach values up to  $10^{20}$  G [1]. Then the issue of interest is the behavior of neutron star matter, which further will be approximated by pure neutron matter, in a strong magnetic field [2].

In particular, a scenario is possible in which a field-induced ferromagnetic phase transition occurs in the magnetar core. This idea was explored in the recent research [3], where it was shown that a fully spin polarized state in neutron matter could be formed in the magnetic field larger than  $10^{19}$  G. Note, however, that the breaking of the  $\mathcal{O}(3)$  rotational symmetry in such ultrastrong magnetic fields results in the anisotropy of the total pressure, having a smaller value along than perpendicular to the field direction [1, 4]. The possible outcome could be the gravitational collapse of a magnetar along the magnetic field, if the magnetic field strength is large enough. Thus, exploring the possibility of a field-induced ferromagnetic phase transition in neutron matter in a strong magnetic field, the effect of the pressure anisotropy has to be taken into account because this kind of instability could prevent the formation of a fully polarized state in neutron matter. In the present study, we determine thermo-

dynamic quantities of strongly magnetized neutron matter taking into account this effect.

Let us stop on the basic equations of the theory. The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons  $f_{\kappa_1\kappa_2} = \text{Tr} \varrho a_{\kappa_2}^+ a_{\kappa_1}$ , where  $\kappa \equiv (\mathbf{p}, \sigma)$ ,  $\mathbf{p}$  is momentum,  $\sigma$  is the projection of spin on the third axis, and  $\varrho$  is the density matrix of the system [5, 6]. The energy of the system is specified as a functional of the distribution function  $f$ ,  $E = E(f)$ , and determines the single particle energy [7, 8]

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}. \quad (1)$$

The self-consistent matrix equation for determining the distribution function  $f$  follows from the minimum condition of the thermodynamic potential [7] and is

$$f = \{\exp(Y_0\varepsilon + Y_i \cdot \mu_n \sigma_i + Y_4) + 1\}^{-1} \quad (2) \\ \equiv \{\exp(Y_0\xi) + 1\}^{-1}.$$

Here the quantities  $\varepsilon$ ,  $Y_i$  and  $Y_4$  are matrices in the space of  $\kappa$  variables, with  $(Y_{i,4})_{\kappa_1\kappa_2} = Y_{i,4} \delta_{\kappa_1\kappa_2}$ ,  $Y_0 = 1/T$ ,  $Y_i = -H_i/T$  and  $Y_4 = -\mu_0/T$  being the Lagrange multipliers,  $\mu_0$  being the chemical potential of neutrons, and  $T$  the temperature. In Eq. (2),  $\mu_n = -1.9130427(5)\mu_N$  is the neutron magnetic moment ( $\mu_N$  being the nuclear magneton),  $\sigma_i$  are the Pauli matrices.

Further it will be assumed that the third axis is directed along the external magnetic field  $\mathbf{H}$ . Given the possibility for alignment of neutron spins along or opposite to the magnetic field  $\mathbf{H}$ , the normal distribution function of neutrons and the matrix quantity  $\xi$

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(which we will also call a single particle energy) can be expanded in the Pauli matrices  $\sigma_i$  in spin space

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \quad (3)$$

$$\xi(\mathbf{p}) = \xi_0(\mathbf{p})\sigma_0 + \xi_3(\mathbf{p})\sigma_3. \quad (4)$$

The distribution functions  $f_0, f_3$  satisfy the normalization conditions:

$$\frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \quad (5)$$

$$\frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_{\uparrow} - \varrho_{\downarrow} \equiv \Delta\varrho. \quad (6)$$

Here  $\varrho = \varrho_{\uparrow} + \varrho_{\downarrow}$  is the total density of neutron matter,  $\varrho_{\uparrow}$  and  $\varrho_{\downarrow}$  are the neutron number densities with spin up and spin down, respectively. The quantity  $\Delta\varrho$  may be regarded as the neutron spin order parameter which determines the magnetization of the system  $M = \mu_n \Delta\varrho$ . The magnetization may contribute to the internal magnetic field  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$ . However, we will assume, analogously to the previous studies [2], that, because of the tiny value of the neutron magnetic moment, the contribution of the magnetization to the inner magnetic field  $\mathbf{B}$  remains small for all relevant densities and magnetic field strengths, and, hence,  $\mathbf{B} \approx \mathbf{H}$ . In order to get the self-consistent equations for the components of the single particle energy, one has to set the energy functional of the system. It represents the sum of the matter and field energy contributions

$$E(f, H) = E_m(f) + \frac{H^2}{8\pi}\mathcal{V}. \quad (7)$$

The matter energy is the sum of the kinetic and Fermi-liquid interaction energy terms [5, 6]

$$E_m(f) = E_0(f) + E_{int}(f), \quad (8)$$

$$E_0(f) = 2 \sum_{\mathbf{p}} \underline{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}),$$

$$E_{int}(f) = \sum_{\mathbf{p}} \{ \tilde{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}) + \tilde{\varepsilon}_3(\mathbf{p}) f_3(\mathbf{p}) \},$$

where

$$\tilde{\varepsilon}_0(\mathbf{p}) = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}), \quad \mathbf{k} = \frac{\mathbf{p} - \mathbf{q}}{2}, \quad (9)$$

$$\tilde{\varepsilon}_3(\mathbf{p}) = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_3(\mathbf{q}). \quad (10)$$

Here  $\underline{\varepsilon}_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0}$  is the free single particle spectrum,  $m_0$  is the bare mass of a neutron,  $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$  are the normal Fermi liquid (FL) amplitudes, and  $\tilde{\varepsilon}_0, \tilde{\varepsilon}_3$  are the FL corrections to the free single particle spectrum. Taking into account Eqs. (1),(2) and (8), expressions for the components of the single particle energy read

$$\xi_0(\mathbf{p}) = \underline{\varepsilon}_0(\mathbf{p}) + \tilde{\varepsilon}_0(\mathbf{p}) - \mu_0, \quad \xi_3(\mathbf{p}) = -\mu_n H + \tilde{\varepsilon}_3(\mathbf{p}). \quad (11)$$

In Eqs. (11), the quantities  $\tilde{\varepsilon}_0, \tilde{\varepsilon}_3$  are the functionals of the distribution functions  $f_0, f_3$  which, using Eqs. (2) and (3), can be expressed, in turn, through the quantities  $\xi$ :

$$f_0 = \frac{1}{2} \{ n(\omega_+) + n(\omega_-) \}, \quad (12)$$

$$f_3 = \frac{1}{2} \{ n(\omega_+) - n(\omega_-) \}, \quad (13)$$

where

$$n(\omega_{\pm}) = \{ \exp(Y_0 \omega_{\pm}) + 1 \}^{-1}, \quad \omega_{\pm} = \xi_0 \pm \xi_3.$$

Thus, Eqs. (11)–(13) form the self-consistency equations for the components of the single particle energy, which should be solved jointly with the normalization conditions (5), (6).

The pressures (longitudinal and transverse with respect to the direction of the magnetic field) in the system are related to the diagonal elements of the stress tensor whose explicit expression reads [9]

$$\sigma_{ik} = \left[ \tilde{f} - \varrho \left( \frac{\partial \tilde{f}}{\partial \varrho} \right)_{\mathbf{H}, T} \right] \delta_{ik} + \frac{H_i B_k}{4\pi}. \quad (14)$$

Here

$$\tilde{f} = f_H - \frac{H^2}{4\pi}, \quad (15)$$

$f_H = \frac{1}{\mathcal{V}}(E - TS) - \mathbf{H}\mathbf{M}$  is the Helmholtz free energy density. For the isotropic medium, the stress tensor (14) is symmetric. The transverse  $p_t$  and longitudinal  $p_l$  pressures are determined from the formulas

$$p_t = -\sigma_{11} = -\sigma_{22}, \quad p_l = -\sigma_{33}.$$

At zero temperature, using Eqs. (7), (14), one can get the approximate expressions

$$p_t = \varrho \left( \frac{\partial e_m}{\partial \varrho} \right)_H - e_m + \frac{H^2}{8\pi}, \quad (16)$$

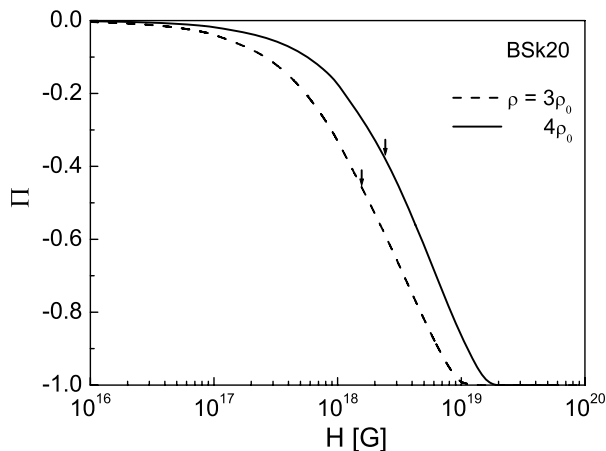
$$p_l = \varrho \left( \frac{\partial e_m}{\partial \varrho} \right)_H - e_m - \frac{H^2}{8\pi}, \quad (17)$$

where  $e_m$  is the matter energy density, and we disregarded the terms proportional to  $M$ . In ultrastrong magnetic fields, the quadratic on the magnetic field term (the Maxwell term) will be dominating, leading to increasing the transverse pressure and to decreasing the longitudinal pressure. Hence, at some critical magnetic field, the longitudinal pressure vanishes, resulting in the longitudinal instability of neutron matter. The question then is: What is the magnitude of the critical field and the corresponding maximum degree of spin polarization in neutron matter?

## 2. EOS OF DENSE NEUTRON MATTER IN A STRONG MAGNETIC FIELD

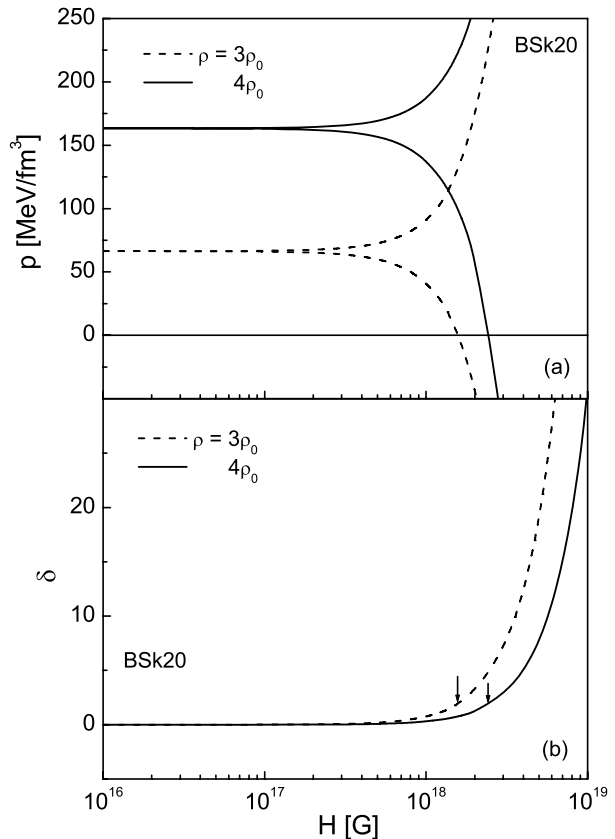
In numerical calculations, we utilize the BSk20 Skyrme force [10] constrained such as to avoid the spontaneous spin instability of neutron matter at densities beyond the nuclear saturation density and

to reproduce a microscopic EoS of nonpolarized neutron matter. Expressions for the normal FL amplitudes in Eqs. (9),(10) in terms of the parameters of the Skyrme interaction are given in Ref. [11]. Now we present the results of the numerical solution of the self-consistency equations. Fig. 1 shows the spin polarization parameter  $\Pi = \frac{\Delta \rho}{\rho}$  of neutron matter as a function of the magnetic field  $H$  at two different values of the neutron matter density,  $\rho = 3\rho_0$  and  $\rho = 4\rho_0$ , which can be relevant for the magnetar core. It is seen that the impact of the magnetic field remains small up to the field strength  $10^{17}$  G. The larger the density is, the smaller the effect produced by the magnetic field on spin polarization of neutron matter.



**Fig. 1.** Neutron spin polarization parameter as a function of the magnetic field  $H$  for the Skyrme force BSk20 at zero temperature and fixed values of the density,  $\rho = 3\rho_0$  and  $\rho = 4\rho_0$ . The vertical arrows indicate the maximum magnitude of spin polarization attainable at the given density, see further details in the text

At the magnetic field  $H = 10^{18}$  G, usually considered as the maximum magnetic field strength in the core of a magnetar (according to a scalar virial theorem, see Ref. [1] and references therein), the magnitude of the spin polarization parameter doesn't exceed 33% at  $\rho = 3\rho_0$  and 18% at  $\rho = 4\rho_0$ . However, the situation changes if the larger magnetic fields are allowable: With further increasing the magnetic field strength, the magnitude of the spin polarization parameter increases till it reaches the limiting value  $\Pi = -1$ , corresponding to a fully spin polarized state. For example, this happens at  $H \approx 1.25 \cdot 10^{19}$  G for  $\rho = 3\rho_0$  and at  $H \approx 1.98 \cdot 10^{19}$  G for  $\rho = 4\rho_0$ , i.e., certainly, for magnetic fields larger than  $10^{19}$  G. Nevertheless, we should check whether the formation of a fully spin polarized state in a strong magnetic field is actually possible by calculating the anisotropic pressure in dense neutron matter. The meaning of the vertical arrows in Fig. 1 is explained later in the text.



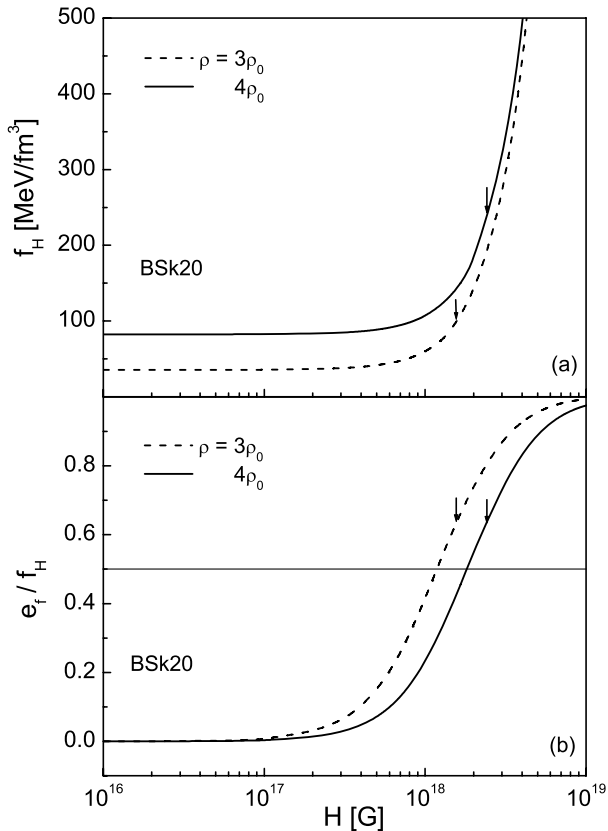
**Fig. 2.** Pressures, longitudinal (descending branches) and transverse (ascending branches), as functions of the magnetic field  $H$  for the Skyrme force BSk20 at zero temperature and fixed values of the density,  $\rho = 3\rho_0$  and  $\rho = 4\rho_0$  (a). Same as in the top panel but for the normalized difference between the transverse and longitudinal pressures (b)

Fig. 2, a shows the pressures (longitudinal and transverse) in neutron matter as functions of the magnetic field  $H$  at the same densities,  $\rho = 3\rho_0$  and  $\rho = 4\rho_0$ . First, it is clearly seen that up to some threshold magnetic field the difference between transverse and longitudinal pressures is unessential that corresponds to the isotropic regime. Beyond this threshold magnetic field strength, the anisotropic regime holds for which the transverse pressure increases with  $H$  while the longitudinal pressure decreases. The longitudinal pressure vanishes at some critical magnetic field  $H_c$  marking the onset of the longitudinal collapse of a neutron star. For example,  $H_c \approx 1.56 \cdot 10^{18}$  G at  $\rho = 3\rho_0$  and  $H_c \approx 2.42 \cdot 10^{18}$  G at  $\rho = 4\rho_0$ . In all cases under consideration, this critical value doesn't exceed  $10^{19}$  G.

The magnitude of the spin polarization parameter  $\Pi$  cannot also exceed some limiting value corresponding to the critical field  $H_c$ . These maximum values of the  $\Pi$ 's magnitude are shown in Fig. 1 by the vertical arrows. In particular,  $\Pi_c \approx -0.46$  at  $\rho = 3\rho_0$  and  $\Pi_c \approx -0.38$  at  $\rho = 4\rho_0$ . As can be inferred from these values, the appearance of the negative longitudinal pressure in an ultrastrong magnetic field prevents the formation of a fully spin polarized state in the core of a magnetar. Therefore, only the onset

of a field-induced ferromagnetic phase transition, or its near vicinity, can be caught under increasing the magnetic field strength in dense neutron matter. A complete spin polarization in the magnetar core is not allowed by the appearance of the negative pressure along the direction of the magnetic field, contrary to the conclusion of Ref. [3] where the pressure anisotropy in a strong magnetic field was disregarded.

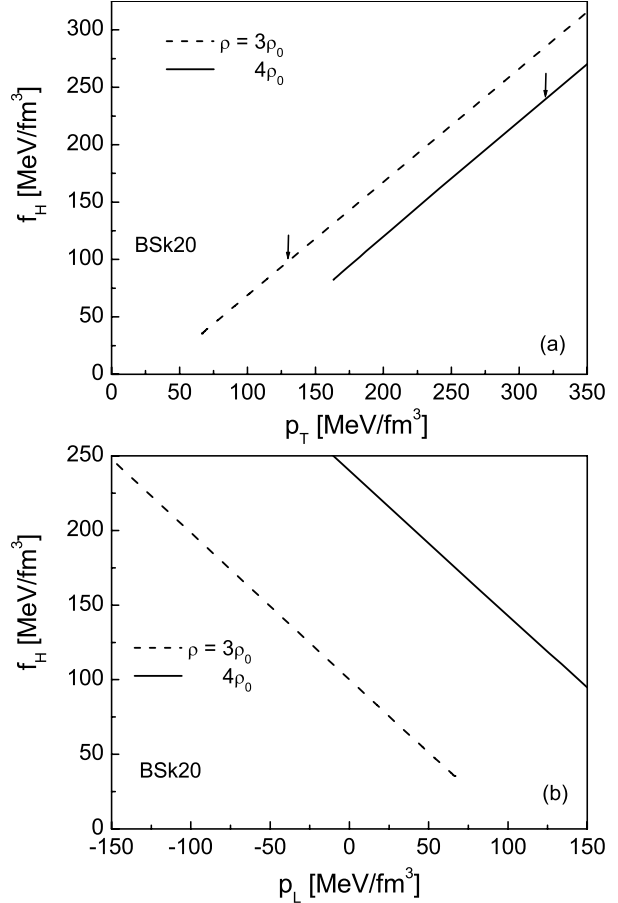
Fig. 2, b shows the difference between the transverse and longitudinal pressures normalized to the value of the pressure  $p_0$  in the isotropic regime (which corresponds to the weak field limit with  $p_l = p_t = p_0$ ) being  $\delta = \frac{p_t - p_l}{p_0}$ . Applying for the transition from the isotropic regime to the anisotropic one the criterion  $\delta \simeq 1$ , the transition occurs at the threshold field  $H_{th} \approx 1.15 \cdot 10^{18}$  G for  $\varrho = 3\varrho_0$  and  $H_{th} \approx 1.83 \cdot 10^{18}$  G for  $\varrho = 4\varrho_0$ . In all cases under consideration, the threshold field  $H_{th}$  is larger than  $10^{18}$  G, and, hence, the isotropic regime holds for the fields up to  $10^{18}$  G. The vertical arrows in Fig. 2, b indicate the points corresponding to the onset of the longitudinal instability in neutron matter. The maximum allowable normalized splitting of the pressures corresponding to the critical field  $H_c$  is  $\delta \sim 2$ .



**Fig. 3.** Same as in Fig. 2 but for: the Helmholtz free energy density of the system (a); the ratio of the magnetic field energy density to the Helmholtz free energy density of the system (b)

Fig. 3, a shows the Helmholtz free energy density of the system as a function of the magnetic field  $H$ . It is seen that the magnetic fields up to  $H \sim 10^{18}$  G have practically small effect on the Helmholtz free

energy density  $f_H$ , but beyond this field strength the contribution of the magnetic field energy to the free energy  $f_H$  rapidly increases with  $H$ . However, this increase is limited by the values of the critical magnetic field corresponding to the onset of the longitudinal instability in neutron matter. The respective points on the curves are indicated by the vertical arrows.



**Fig. 4.** The Helmholtz free energy density of the system as a function of: the transverse pressure  $p_t$  (a), the longitudinal pressure  $p_l$  for the Skyrme force BSk20 at zero temperature and fixed values of the density,  $\varrho = 3\varrho_0$  and  $\varrho = 4\varrho_0$  (b)

Fig. 3, b shows the ratio of the magnetic field energy density  $e_f = \frac{H^2}{8\pi}$  to the Helmholtz free energy density at the same assumptions as in Fig. 2. The intersection points of the respective curves in this panel with the line  $e_f/f_H = 0.5$  correspond to the magnetic fields at which the matter and field contributions to the Helmholtz free energy density are equal. This happens at  $H \approx 1.18 \cdot 10^{18}$  G for  $\varrho = 3\varrho_0$ , and at  $H \approx 1.81 \cdot 10^{18}$  G for  $\varrho = 4\varrho_0$ . These values are quite close to the respective values of the threshold field  $H_{th}$ , and, hence, the transition to the anisotropic regime occurs at the magnetic field strength at which the field and matter contributions to the Helmholtz free energy density become equally important. It is also seen from Fig. 3, b that in all cases when the longitudinal instability occurs in the magnetic field  $H_c$  the contribution of the magnetic field energy to the Helmholtz free energy density of the system dominates over the matter contribution.

Because of the pressure anisotropy, the EoS of neutron matter in a strong magnetic field is also anisotropic. Fig. 4 shows the dependence of the Helmholtz free energy density  $f_H$  of the system on the transverse pressure (top panel) and on the longitudinal pressure (bottom panel) at the same densities considered above. Since in an ultrastrong magnetic field the dominant Maxwell term enters the pressure  $p_t$  and free energy density  $f_H$  with positive sign and the pressure  $p_l$  with negative sign, the free energy density  $f_H$  is the increasing function of  $p_t$  and decreasing function of  $p_l$ . In the bottom panel, the physical region corresponds to the positive values of the longitudinal pressure.

The obtained results can be of importance in the structure studies of magnetars. It would be also of interest to extend this research to finite temperatures relevant for proto-neutron stars which can lead to a number of interesting effects, such as, e.g., an unusual behavior of the entropy of a spin polarized state [12, 13].

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## References

1. E.J. Ferrer, V. de la Incera, J.P. Keith, I. Portillo, and P.L. Springsteen. Equation of state of a dense and magnetized fermion system // *Phys. Rev. C*. 2010, v. 82, 065802, 15 p.
2. A.A. Isayev and J. Yang. Spin-polarized states in neutron matter in a strong magnetic field // *Phys. Rev. C*. 2009, v. 80, 065801, 7 p.
3. G.H. Bordbar, Z. Rezaei, and A. Montakhab. Investigation of the field-induced ferromagnetic phase transition in spin-polarized neutron matter: A lowest order constrained variational approach // *Phys. Rev. C*. 2011, v. 83, 044310, 7 p.
4. V.R. Khalilov. Macroscopic effects in cold magnetized nucleons and electrons with anomalous magnetic moments // *Phys. Rev. D*. 2002, v. 65, 056001, 6 p.
5. A.A. Isayev and J. Yang. Spin polarized states in strongly asymmetric nuclear matter // *Phys. Rev. C*. 2004, v. 69, 025801, 8 p.
6. A.A. Isayev. Spin ordered phase transitions in isospin asymmetric nuclear matter // *Phys. Rev. C*. 2006, v. 74, 057301, 4 p.
7. A.I. Akhiezer, A.A. Isayev, S.V. Peletminsky, A.P. Rekalov, and A.A. Yatsenko. On a theory of superfluidity of nuclear matter based on the Fermi-liquid approach // *JETP*. 1997, v. 85, p. 1-12.
8. A.I. Akhiezer, A.A. Isayev, S.V. Peletminsky, and A.A. Yatsenko. Multi-gap superfluidity in nuclear matter // *Phys. Lett. B*. 1999, v. 451, p. 430-436.
9. L.D. Landau, E.M. Lifshitz, and L.P. Pitaevskii. *Electrodynamics of Continuous Media*. New York: Pergamon, 1984, 2nd ed.
10. S. Goriely, N. Chamel, and J.M. Pearson. Further explorations of Skyrme-Hartree-Fock-Bogoliubov mass formulas. XII. Stiffness and stability of neutron-star matter // *Phys. Rev. C*. 2010, v. 82, 035804, 18 p.
11. A.A. Isayev and J. Yang. Phase transition to the state with nonzero average helicity in dense neutron matter // *JETP Lett.* 2010, v. 92, p. 783-787.
12. A.A. Isayev. Finite temperature effects in antiferromagnetism of nuclear matter // *Phys. Rev. C*. 2005, v. 72, 014313, 4 p.
13. A.A. Isayev. Unusual temperature behavior of the entropy of the antiferromagnetic spin state in nuclear matter with an effective finite range interaction // *Phys. Rev. C*. 2007, v. 76, 047305, 4 p.

## ДЕЙСТВИТЕЛЬНО ЛИ ВОЗМОЖЕН ИНДУЦИРОВАННЫЙ ПОЛЕМ ФЕРРОМАГНИТНЫЙ ФАЗОВЫЙ ПЕРЕХОД В СЕРДЦЕВИНЕ МАГНЕТАРА?

*А.А. Исеев, Дж. Янг*

Рассматриваются спиновопорядоченные состояния в плотной нейтронной материи с силой Скирма BSk20 в магнитных полях вплоть до  $10^{20}$  Гс. Показано, что появление продольной неустойчивости в сильном магнитном поле препятствует формированию полностью поляризованного состояния и возможны только состояния с умеренной поляризацией.

## ДІЙСНО ЛЬ МОЖЛИВИЙ ІНДУКОВАНИЙ ПОЛЕМ ФЕРРОМАГНІТНИЙ ФАЗОВИЙ ПЕРЕХІД У СЕРЦЕВИНІ МАГНЕТАРА?

*О.О. Исеев, Дж. Янг*

Розглядаються спіновпорядковані стани в густій нейтронній матерії з силою Скирма BSk20 в магнітних полях до  $10^{20}$  Гс. Показане, що поява поздовжньої нестійкості в сильному магнітному полі перешкоджає формуванню повністю поляризованого стану і можливі тільки стани з помірною поляризацією.