THE PREWAVE ZONE EFFECT IN TRANSITION
RADIATION AND BREMSSTRAHLUNG BY RELATIVISTIC ELECTRON

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The problem of measurement of bremsstrahlung characteristics in the process of sharp scattering of relativistic electron in the case when transverse distances responsible for radiation process have macroscopic size is considered. It is shown that in this case the results of measurements substantially depend on the size of detector and its position relatively to the scattering point. The problem of transition radiation by relativistic electron with nonequilibrium own field which was formed in the result of sharp scattering is considered. It is shown that the state of electron with nonequilibrium field manifests itself by suppression of transition radiation and by oscillation type of its characteristics dependence on the distance between the plate on which the radiation occurs, and the scattering point.

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1. INTRODUCTION

During the process of electron’s scattering the reconstruction of the field around it occurs, which leads to radiation of electromagnetic waves. For ultrarelativistic particles the radiation process forms on distances along the particle’s initial velocity which considerably exceed radiated wavelength. Such distances are called coherence lengths of radiation process (or longitudinal radiation formation lengths)[1-3].

The same distances are responsible for rebuilding of the initial field of the scattered particle while it moves along the direction of scattering. Within these lengths the field around the electron considerably differs from the coulomb one. The radiation process is also described by characteristic transversal distances which are responsible for radiation formation. Both longitudinal and transversal radiation formation lengths can have macroscopic size not only in the case of ultra high particle energies but for electron energies of several tens Mev in the millimeter wavelength region as well. In the present paper we show that under such conditions in the considered case bremsstrahlung characteristics can substantially depend on both the size of the used detector and its position relatively to the scattering point.

The effects in bremsstrahlung that occur in this case are similar to the analogous effects in transition radiation in prewave zone [4-7]. We also show that the state of electron with nonequilibrium field substantially manifests itself in the process of further transition radiation by such electron. In this case the effect of transition radiation suppression as well as the oscillation type of radiation characteristics dependence on distance between the plate (on which the transition radiation occurs) and the electron’s scattering point takes place. The causes of such effects are discussed.

2. BREMSSTRAHLUNG

Let us consider the process of instant scattering of relativistic electron to a large angle at which the particle’s velocity changes from the initial value v to the final v at the moment of time t = 0. As it was shown in [3] the retarded solution for the total field scalar potential in the space after the scattering moment can be presented in the following form (here and further the speed of light c is considered to equal unit):

\[ \varphi(r, t) = \theta(r - t)\varphi_v(r, t) + \theta(t - r)\varphi_{v'}(r, t), \]

where \( \varphi_v \) and \( \varphi_{v'} \) are the coulomb potentials of the electrons which move uniformly straightforward along the axes z and z’ (see Figure). The Fourier-expansion of (1) over the plane waves with wave vectors \( \mathbf{k} \) is:

\[ \varphi(r, t) = \frac{e}{2\pi^2} Re \int \frac{d^3k}{k} e^{i\mathbf{k}\cdot\mathbf{r}} \times \left[ e^{-i\mathbf{k}v't} \left( 1 - e^{i\mathbf{k}(\mathbf{v'} - \mathbf{v})t} \right) - e^{-ikt} \right], \]

where \( k^2 = q^2 + k_z^2 \), \( k_z \) and q are respectively the components of wave vector \( \mathbf{k} \) along the z-axis and orthogonal to it (z-axis is chosen to be the direction of the initial electron’s velocity \( \mathbf{v} \)).
The first item in square brackets in (2) corresponds to the nonequilibrium field which the scattered electron has already managed to rebuild around itself by the moment of time \( t \). This field vanishes in the region which the signal about the scattering has not yet reached, which is outside the sphere of radius \( R = ct \) with the center in the scattering point (\( \theta \)-sphere). Inside the sphere it coincides with equilibrium coulomb field of the electron. As we can see from (2), the main contribution to the integral of the first item is made by \( k \) with the directions nearly parallel to \( v' \). Thus by the moment of time \( t \sim 1/k_0(1 - v') \) (which in ultra relativistic case is \( t \sim 2\gamma^2/k_0 \), \( \gamma \) is the particle’s lorentz-factor) after the scattering the Fourier-harmonics of this part of total field which have absolute values of wave vector \( k < k_0 \) are still highly suppressed. The electron with such field is known as the ‘half-bare’ electron [8].

The momentary scattering of relativistic electron to a large angle in the point \( z = z' = 0 \)

The second item in square brackets in (2) describes the field which as though ‘tears away’ from the electron at the scattering moment. It is a packet of free electromagnetic waves which moves in the direction of the initial electron’s velocity \( v \) and gradually transforms into bremsstrahlung. This field is different from zero outside the \( \theta \)-sphere and vanishes inside it. Now we will consider more closely this part of the total field (2) in order to derive the observable spectral-angular characteristics of bremsstrahlung, which takes place in the considered process of electron scattering.

At first let us consider the total energy of radiation of certain frequency \( \omega \) which traverses small element of surface which is situated in the point with radius-vector \( r = (\rho, z) \), and seen at the solid angle \( d\Omega \) from the scattering point (here \( \rho \) is the polar coordinate in the plane orthogonal to the \( z \)-axis).

Such consideration corresponds to the measurement by point detector situated in the point \( r = (\rho, z) \).

Making the variable substitution \( k_z \rightarrow k \) by \( k_z = \sqrt{k^2 - q^2} \) and denoting \( k = \omega \), it is possible to present the Fourier-expansion of the ‘torn-away’ field scalar potential in the following form:

\[
\varphi(r, t) = \frac{e}{\pi v^2} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_{0}^{\infty} dq \frac{q J_0(q\rho)}{q^2 + \frac{q^2}{\gamma^2}} A(z),
\]

where

\[
A(z) = \frac{\omega \cos(\sqrt{\omega^2 - q^2}z)}{\sqrt{\omega^2 - q^2}} + i e^{i\omega\pi/2} \sin(\sqrt{\omega^2 - q^2}z)
\]

and \( \rho = |\rho| \).

In (3) and (4) the square root \( \sqrt{\omega^2 - q^2} \) is considered to be a single-valued branch of the analytical function which is equal to \( |\sqrt{\omega^2 - q^2}| \) for \( \omega > q \) and \( -|\sqrt{\omega^2 - q^2}| \) for \( \omega < -q \).

In ultra relativistic case (\( \gamma \gg 1, v \rightarrow c \)) the range of \( q \), which makes the main contribution to the integral (3) is \( q \leq \omega/\gamma < \omega \) and it is possible to expand the square roots \( \sqrt{\omega^2 - q^2} \) in (4) by the small factor \( q/\omega \). Let us leave the items proportional to the second power of \( q/\omega \) in the arguments of sinus and cosinus while in the other parts of the expression (4) neglect them. Moreover the integration over \( q \) can be extended to the region \( 0 < q < \infty \). This leads to the following expression for the ‘torn-away’ field potential in ultra relativistic case:

\[
\varphi(r, t) = \frac{e}{\pi} \int_{-\infty}^{\infty} d\omega \int_{0}^{\infty} dq \frac{q J_0(q\rho)}{q^2 + \frac{q^2}{\gamma^2}} e^{i\omega(z-t)} e^{-\frac{q^2}{\gamma^2}}.
\]

In this case the ‘torn-away’ electric field can be considered as transverse having only component \( E_\perp \) orthogonal to \( z \)-axis. Moreover, for \( \gamma \gg 1 \) the radiation is mainly concentrated in the directions in the vicinity of the initial electron’s velocity \( v \). For such small angles between the radiation direction and the \( z \)-axis we can present the expression for bremsstrahlung spectral-angular density in the following form:

\[
\frac{d\varepsilon}{d\omega d\Omega} = \frac{v^2}{4\pi^2} |E_\perp(r, \omega)|^2,
\]

which is valid for arbitrary distances \( z \) from the scattering point.

Using (5) we can derive the Fourier-component of electric field orthogonal to the \( z \)-axis. Substituting it into (6) for spectral-angular distribution of bremsstrahlung we achieve:

\[
\frac{d\varepsilon}{d\omega d\Omega} = \left(\frac{e\gamma v}{\pi}\right)^2 \int_{0}^{\infty} dq \frac{q^2 J_1(q\rho)}{q^2 + \frac{q^2}{\gamma^2}} e^{-\frac{q^2}{\gamma^2}}.
\]

For large distances from the scattering point, namely in the wave zone of the radiation process (\( z \gg 2\gamma^2/\omega \)) the integral in (7) can be calculated with the use of stationary phase method [9]. It leads
to the well known expression for radiation distribution from the Bremsstrahlung theory [10]:

$$\frac{d\xi}{d\omega d\sigma} = \frac{e^2}{\pi^2} \frac{\theta^2}{(\theta^2 + \gamma^{-2})^2},$$  \hspace{1cm} (8)$$

where $\theta = \rho/z$ is the angle between the direction of radiation and the z-axis. As we can see from (8) in the wave zone the radiation is mainly concentrated within the characteristic angles $\theta \sim 1/\gamma$.

In the prewave zone ($z \ll 2\gamma^2/\omega$) of the radiation process it is not possible to use the stationary phase method for the analysis of radiation characteristics. Here, making the substitutions $q = \omega x/\gamma$ and $\rho = z\theta$, we can present the integral (7) in the following form:

$$\frac{d\xi}{d\omega d\sigma} = \left(\frac{e\omega z}{\pi \gamma}\right)^2 |I_1 - I_2|^2,$$  \hspace{1cm} (9)$$

where

$$I_1 = \int_0^\infty dx J_1(\omega z \gamma^{-1} x\theta) \exp\left(-i \frac{\omega z}{2\gamma^2} x^2\right),$$

$$I_2 = \int_0^\infty dx J_1(\omega z \gamma^{-1} x\theta) \exp\left(i \frac{\omega z}{2\gamma^2} x^2\right).$$

In the case $\gamma \gg 1$ the absolute value of the integral $I_2$ is negligibly small comparing to the corresponding value of $I_1$ and for spectral-angular density of Bremsstrahlung in the prewave zone we obtain:

$$\frac{d\xi}{d\omega d\sigma} = \left(\frac{e\omega z}{\pi}\right)^2 |I_1|^2 = \frac{4e^2}{\pi^2 \gamma^2} \frac{1}{\theta^2} \sin^2\left(\frac{\omega z \theta^2}{4}\right).$$  \hspace{1cm} (10)$$

From (10) we can conclude that in the prewave zone the radiation is mainly concentrated within the angles $\theta \sim 1/\sqrt{\omega \gamma}$, which exceed the characteristic angles $\theta \sim 1/\gamma$ of the wave zone. Therefore in the prewave zone ($z \ll 2\gamma^2/\omega$) the point detector gives broader angular distribution of radiation than in the wave zone ($z \gg 2\gamma^2/\omega$). Moreover this distribution depends on the frequency $\omega$ of the radiated waves.

By the point detector we mean here the detector of the smaller size $\delta \rho$ than the transversal radiation length of the process $l_T \sim \gamma/\omega$ which is the characteristic transversal distance on which at the moment of time $t = 0$ the Fourier-harmonics of frequency $\omega$ are concentrated in the wave packet (5). Such detector registers the radiation of frequency $\omega$, which falls on the small domain of space, where the detector is situated.

The measurements however can be made by the extended detector of the larger size than the characteristic transversal length of the radiation process, so that $\delta \rho \gg l_T$. Such detector registers not only the waves of frequency $\omega$ which fall on the small element of surface with coordinates $\rho$ and $z$, as the point detector does, but all the electromagnetic waves of frequency $\omega$ which propagate in the direction of wave vector $k$ ($|k| = \omega$). In order to calculate the bremsstrahlung spectral-angular distribution, which is registered by extensive detector, which is a plate of large size, we need to integrate the expression (7) over the entire considered plate and express the obtained result in the form of an integral over the directions of wave vectors of radiated waves. The integrand at that will be nothing else than the required distribution. In our case after performing the procedures described above we can present the expression (7) in the following form:

$$\frac{d\xi}{d\omega} = \left(\frac{e}{\pi}\right)^2 \int \frac{d^2\theta_\gamma \theta^2_\gamma}{(\theta^2_\gamma + \gamma^{-2})},$$  \hspace{1cm} (11)$$

where $\theta_\gamma = q/\omega$ is the angle between the direction of the wave vector $k$ and the z-axis. Hence the Bremsstrahlung spectral-angular distribution obtained by the extended detector coincides with the one (8) obtained by the point detector in the wave zone. But unlike the case with point detector this distribution does not depend on the distance from the scattering point and is the same both in the wave and the prewave zones.

The analogous effects concerning large transversal radiation lengths take place as well for backward transition radiation during the electron’s traverse of thin metallic plate [4-7]. It is explained by the fact that the structure of the fields which arise in this case is analogous to the one which takes place at the momentary scattering of the electron to a large angle. Indeed, solving the boundary problem for the total electric field on the surface of the plate we can finally derive the explicit expression for the scalar potential of the field reflected from the plate, which gradually transforms into backward transition radiation [11]:

$$\varphi(r, t) = [\varphi_\nu(r, t) - \varphi_\nu(r, t)] \theta(r - t),$$  \hspace{1cm} (12)$$

where $\varphi_\nu$ and $\varphi_\nu$ are the coulomb potentials of the electron and its image inside the plate. Comparing (12) with (1) we can see that in ultra relativistic case after the electron’s traverse of the plate the structure of the field on the left of the plate (we assume that electron traverses the plate from left to right) is analogous, but not identical, to the structure of the field which is ‘torn away’ from the electron at its scattering. Such structural similarity of the fields explains the existence of analogous effects in bremsstrahlung and transition radiation in the considered cases.

3. TRANSITION RADIATION

Let us consider the backward Transition radiation which occurs when the scattered electron normally traverses thin ideally conducting plate situated in the direction of scattering in the plane $z' = z'_0$ (see Figure). The Fourier-expansion of the field around the scattered electron

$$\varphi(r, t) = \frac{e}{2\pi^2} Re \int \frac{d^3k}{k} \left[ \frac{e^{ik(r-v't)}}{k - kv'} - \frac{e^{ikr-ikt}}{k - kv'} \right],$$  \hspace{1cm} (13)$$

consists of two parts, the first of which describes the equilibrium coulomb field of the electron which moves
with the velocity \( \mathbf{v}' \) along the direction of scattering, while the second part is the nonequilibrium field, which is structurally equal to the ‘torn away’ field (it is equal to equilibrium coulomb field outside the \( \theta \)-sphere and vanishes inside it). Hence, the second part of the field (13) can be presented in the analogous form by making the substitution \( \mathbf{k}_2 \rightarrow \mathbf{k} \) from \( k = \sqrt{k_1^2 + q^2} \) and denoting \( k_2v' = \omega \). From the expression for scalar potential obtained by the considered transformations we can derive the expression for the Fourier-component of the electric field perpendicular to \( z' \)-axis, which in ultra relativistic case is:

\[
E_\perp (r, \omega) = 2e \int_{-\infty}^{\infty} d\omega e^{-i\omega/\omega_0} \int_{0}^{\infty} dq \frac{q^2 J_1(q\rho)}{q^2 + \frac{\omega^2}{v^2}} \times \left[ 1 - e^{-i\omega'2/(\gamma^2 + z'/z^2)} \right]. \tag{14}
\]

From (14) it follows that the rebuilding of the field around the electron occurs in such way that each Fourier harmonic of frequency \( \omega_0 \) totally reconstructs and becomes the harmonic of equilibrium coulomb field on the distance from the scattering point which coincides with radiation formation length for this \( \omega_0 \) \( (|z'| \sim 2\gamma^2/\omega_0) \). It is possible to place the plate quite close to the scattering point so that at the moment of electron’s traverse of the plane the Fourier-harmonics of certain frequencies \( \omega < \omega_0 \) will have not yet reconstructed. In other words it is possible to place the plate in the prevave zone for these frequencies. In this case the incident electron will be ‘half-bare’ and its transition radiation should differ from such radiation by electron with equilibrium field.

The total field of the electron-plate system consists of the field of ‘half-bare’ electron \( E_\perp \) and the field \( E^{(f)}_\perp \) of currents induced on the surface of the plate. Applying the boundary condition for electric field on the surface of the plate \( E_\perp (z' = 0) + E^{(f)}_\perp (z' = 0) = 0 \), we can find the expression for the Fourier-harmonic of the field of induced surface currents:

\[
E^{(f)}_\perp (r, \omega) = 2e \frac{e^{i\omega R}}{R} \frac{\vartheta}{\vartheta^2 + \gamma^2} [F(r, \omega) - 1], \tag{15}
\]

where

\[
F(r, \omega) = \frac{1}{v^2} + \frac{\gamma^2(z' - 2z_0)^2}{v^2 + \gamma^2(z' - z_0)^2} \times \exp \left\{ \frac{i\omega z'_0}{2} \left( \frac{1}{v^2 - \gamma^2} - \frac{\rho^2}{(z' - z_0)^2} \right) \right\},
\]

\( R \) is the distance between the point of the electron’s traverse of the plate and the point where the field is considered, \( R \approx z'_0 - z' + \rho^2 / [2(z'_0 - z')] \) and \( \vartheta \) is counted from the direction of \(-\mathbf{v}'\). This field gradually transforms into backward transition radiation.

The expression (15) can be simplified for \( -z' \gg 2\gamma^2/\omega \). In this case:

\[
E^{(f)}_\perp (r, \omega) = 2e \frac{e^{i\omega R}}{R} \frac{\vartheta}{\vartheta^2 + \gamma^2} \left\{ 1 - e^{-\frac{i\omega R}{\omega}(1 + \gamma^2 \vartheta^2)} \right\} - 1 \tag{16}
\]

and using (6) for spectral-angular density of transition radiation by ‘half-bare’ electron we obtain:

\[
\frac{d\xi}{d\omega d\vartheta^2} = \frac{e^2}{\pi^2 (\vartheta^2 + \gamma^2)^2} \left\{ 1 - \cos \left( \frac{\omega z'_0}{2} (\gamma^2 + \vartheta^2) \right) \right\}. \tag{17}
\]

The expression (17) differs from the corresponding expression for transition radiation by electron with equilibrium field by the interference factor inside the braces and the coefficient two in front of them. As we can see from (17), when the distance \( z'_0 \) between the scattering point and the plate is much less than the radiation formation length \( (l_C \sim 2\gamma^2/\omega) \) the radiation is highly suppressed. For larger values of \( z'_0 \) the dependence of the radiation intensity on \( z'_0 \) has the oscillation type with the period of the order of the formation length:

\[
\Lambda = 4\pi/\omega(\vartheta^2 + \gamma^2). \tag{18}
\]

Due to the nonzero frequency resolution \( \Delta \omega \) of the detector it is possible to observe such oscillations only in the area limited by the condition

\[
\frac{\omega z'_0}{2} < \pi/\gamma \Delta \omega (\vartheta^2 + \gamma^2). \tag{19}
\]

Also due to the nonzero size and, therefore, angular resolution of the detector the oscillations can be observed only inside the region

\[
\frac{\omega z'_0}{2} < \pi/\omega \vartheta \Delta \vartheta. \tag{20}
\]

For large distances \( z'_0 \gg \Lambda \) the considered oscillations disappear and the detector registers an incoherent sum of contributions to transition radiation by electron’s own field reflected from the plate and by the field of bremsstrahlung in this direction.

4. CONCLUSIONS

In the present paper the process of momentary scattering of relativistic electron to a large angle is considered. It is shown that the observable spectral-angular characteristics of bremsstrahlung which takes place in this process substantially depend on the position of the used detector relatively to the scattering point if the size of the detector is smaller than the characteristic transversal distance of the radiation process \( l_T \sim \gamma/\omega \). Namely in the prevave zone \( z \ll l_C \) such detector gives broader angular distribution than in the wave zone \( z \gg l_C \) and such distribution depends on the frequency of the registered waves in contrast to the case of measurement in the wave zone. If the size of the used detector exceeds \( l_T \) the results obtained by such detector do not depend on its position and coincide with the results obtained by the point detector in the wave zone. In the present paper it is also shown
that the state of electron with nonequilibrium field can be manifested by its transition radiation if the plate on which the radiation occurs, is situated (relatively to the scattering point) in the prewave zone of the frequencies which are detected. Namely the dependence of transition radiation intensity on the distance between the scattering point and the plate in this case will be of oscillation type with the period of the order of the formation length. For even closer position of the plate to the scattering point the radiation will be highly suppressed.

References


Эффект предволновой зоны в переходном и тормозном излучении релятивистского электрона

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Рассмотрена проблема измерения характеристик тормозного излучения при мгновенном рассеянии релятивистского электрона на большой угол в условиях, когда поперечные рассеяния, отнесенные за процесс излучения, имеют макроскопические размеры. Показано, что в этом случае результаты измерений существенно зависят от размеров детектора и его положения относительно точки рассеяния. Рассмотрена задача о переходном излучении электрона с неравновесным собственным полем, который образовался в результате его резкого рассеяния. Показано, что состояние электрона с неравновесным полем приводится в подавлении переходного излучения и оцилляторном характере зависимости его характеристик от рассеяния между пластинкой, на которой происходит излучение, и точкой рассеяния.

Ефект предхвильовой зоны у переходному та гальмівному випромінюванню релятивістського електрона

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Розглянуто проблему вимірювання характеристик гальмівного випромінювання при миттєвому розсіянні релятивістського електрона на великий кут в умовах, коли поперечні відстані, що відповідають за процес випромінювання, мають макроскопічні розміри. Показано, що в цьому випадку результати вимірювань залежать від розміру детектора та його положення відносно точки розсіяння. Розглянута задача про переходне випромінювання електрона з нерівноважним квазин станий полем, що утворився в результаті його різкого розсіяння. Показано, що стан електрона з нерівноважним полем виражається в приглушеній переходному випромінюванні за оцилляторному характері залежності його характеристик від відстані між пластинкою, на якій відбувається випромінювання, і точкової розсіяння.