LOW BOUND ON A MAGNETIC FIELD STRENGTH IN THE HOT UNIVERSE

E. Elizalde [1] ***and V. Skalozub*** [2*]

[1] *Institute for Space Science, ICE-CSIC and IEEC, Campus UAB, 08193, Bellaterra, Barcelona, Spain*

[2] *Dnipropetrovsk National University, 49010, Dnipropetrovsk, Ukraine*

(Received October 24, 2011)

It is assumed that the intergalactic magnetic fields were spontaneously generated in the early Universe due to vacuum polarization of non-Abelian gauge fields at high temperature T . Here, a procedure to estimate the field strengths $B(T)$ at different T is developed and the value of $B(T_{ew}) \sim 10^{14}$ G, at the electroweak phase transition temperature, is derived by taking into consideration the present intergalactic magnetic field strength $B_0 \sim 10^{-15}$ G.

PACS: 98.62.En, 12.38.-t, 11.15.Ex

1. INTRODUCTION

Recent experimental discovery of intergalactic magnetic fields having the field strength of the order $B \sim 10^{-15}$ G [1, 2] is one of the most interesting events of modern cosmology. In Ref. [3] a model-independent 95 per sent CL interval 1×10^{-17} G $\leq B \leq 3 \times 10^{-14}$ G is determined. This discovery, in particular, restricts the possible processes resulting in the generation of fields in the hot Universe [2, 4], and stimulates further investigations.

In the present report we discuss a mechanism based on non-Abelian magnetic fields. As it was shown recently, a spontaneous magnetization happens in non-Abelian gauge theories at high temperature T. This phenomenon is the extension of the Savvidy [9] vacuum $B(T) = \text{const} \neq 0$ to the finite temperature case. In contrast to the zero temperature, the state $B(T)$ appears to be stable due to a magnetic mass of the color charged gluon and a A_0-condensate. Its energy is below the perturbative vacuum one, and the minimum of the effective potential is reached for a field of order $gB \sim g^4T^2$. Although this phenomenon was discovered in $SU(2)$ gluodynamics, it is common for other $SU(N)$ gauge fields.

An important property of such magnetic fields is the vanishing of their magnetic mass, $m_{magn.} = 0$ [8, 11]. The mass parameter describes the inverse spatial scales of the transverse field components.The absence of the screening mass means that the spontaneously generated Abelian chromomagnetic fields are long range at high temperature. Hence, it is reasonable to believe that, in the hot Universe, at each stage of its evolution spontaneously created, strong, long-range magnetic fields of different types have been present. Since they are constant ones, their scale is coinciding with the horizon scale at a particular temperature.

In what follows, in the frameworks of the standard model (SM), we estimate the strength of the magnetic field at the temperature T_c^{ew} of the electroweak phase transition (EWPT), assuming the mechanism as described above. We carry out an actual calculation in the frame of a consistent effective potential (EP) accounting for the one-loop, $V^{(1)}$, and the daisy (or ring), V^{ring}, diagram contributions. In Sec. 2 the EP of an Abelian constant electromagnetic B field at finite temperature is obtained. It is used, in Sec. 3, to estimate the magnetic field strength at the (EWPT). In Sec. 4 the discussion of the results is given.

2. EFFECTIVE POTENTIAL AT HIGH TEMPERATURE

The complete EP for the standard model is given in the review [10]. In the present investigation we are interested in two limits:

1. Weak magnetic field and large scalar field condensate, $h = eB/M_w^2 < \phi^2$, $\phi = \phi_c/\phi_0$, $\beta = 1/T$;
2. Case of the restored symmetry, $\phi = 0$, $gB \neq 0$, $T \neq 0$.

For the former case we show the absence of spontaneous vacuum magnetization at finite temperature. For the latter one we estimate the field strength at high temperature. Here M_w is the W-boson mass at zero temperature, ϕ_c is a scalar field condensate, and ϕ_0 its value at zero temperature.

To demonstrate the first property we consider the one-loop contribution of W-bosons:

*Corresponding author E-mail address: skalozubv@daad-alumni.de

$$V_w^{(1)}(T,h,\phi)=\frac{h}{\pi^2\beta^2}\sum_{n=1}^{\infty}\Big[\frac{(\phi^2-h)^{1/2}\beta}{n}K_1(n\beta(\phi^2-h)^{1/2})-\frac{(\phi^2+h)^{1/2}\beta}{n}K_1(n\beta(\phi^2+h)^{1/2})\Big], \quad (1)$$

where n labels discrete energy values and $K_1(z)$ is the MacDonald function.

The main goal of our investigation is the restored phase of the SM. So, we adduce the high temperature contribution of the complete effective potential relevant for this case using the results in Ref. [10]. First we write down the one-loop W-boson contribution as the sum of the pure Yang-Mills weak-isospin part ($\tilde{B}\equiv B^{(3)}$),

$$V_w^{(1)}(\tilde{B},T)=\frac{\tilde{B}^2}{2}+\frac{11}{48}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2}-\frac{1}{3}\frac{(g\tilde{B})^{3/2}T}{\pi} - i\frac{(g\tilde{B})^{3/2}T}{2\pi}+O(g^2\tilde{B}^2), \quad (2)$$

where τ is a temperature normalization point, and the charged scalars,

$$V_{sc}^{(1)}(\tilde{B},T)=-\frac{1}{96}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2} + \frac{1}{12}\frac{(g\tilde{B})^{3/2}T}{\pi}+O(g^2\tilde{B}^2), \quad (3)$$

describing the contribution of longitudinal vector components. This representation is convenient for the case of extended models including other gauge and scalar fields. In the SM, the contribution of Eq. (3) has to be taken with a factor 2, in the case of the Two-Higgs-Doublet SM, this factor must be 4, etc. The imaginary part is canceled by the term appearing in the contribution of the daisy diagrams for the unstable mode [6],

$$V_{unstable}=\frac{g\tilde{B}T}{2\pi}[\Pi(\tilde{B},T)-g\tilde{B}]^{1/2}+i\frac{(g\tilde{B})^{3/2}T}{2\pi}. \quad (4)$$

Here $\Pi(\tilde{B},T)$ is the mean value for the charged gluon polarization tensor taken in the ground state of the spectrum. If this value is sufficiently large, spectrum stabilization due to radiation correction takes place. This possibility formally follows from the temperature and field dependence of the polarization tensor in the high temperature limit $T\to\infty$ [12]: $\Pi(\tilde{B},T)=c\ g^2T\sqrt{g\tilde{B}}$, where $c>0$ is a constant which must be calculated explicitly. At high temperature the first term can be larger then $g\tilde{B}$.

The high temperature limit of the fermion contribution looks as follows:

$$V_{fermion}=-\frac{\alpha}{\pi}\sum_f\frac{1}{6}q_f^2\tilde{B}^2\log\frac{T}{\tau}, \quad (5)$$

where the sum is extended to all leptons and quarks, and q_f is the fermion electric charge in positron units. We observed the stable vacuum state in the lattice simulations [7]. Therefore, we believe that this problem has a positive solution.

3. MAGNETIC FIELD STRENGTH AT T_{ew}

Let us now show that the spontaneous vacuum magnetization does not happen for non-small values of $\phi\neq 0$. To do that we notice that the magnetization is produced by the gauge field contribution, given in Eq. (1). We consider the limit of $\frac{gB}{T^2}\ll 1$ and $\phi^2>h$. For this case we use the asymptotic expansion of $K_1(z)$,

$$K_1(z)\sim\sqrt{\frac{\pi}{2z}}e^{-z}\big(1+\frac{3}{8z}-\frac{15}{128z^2}+\cdots\big), \quad (6)$$

where $z=n\beta(\phi^2\pm h)^{1/2}$. Now, we investigate the limit of $\beta\to\infty,\frac{T}{\phi}\ll 1$. We can also substitute $(\phi^2\pm h)^{1/2}=\phi(1\pm\frac{h}{2\phi^2})$. In this approximation, the sum of the tree level energy and (1) reads

$$V=\frac{h^2}{2}-\frac{h^2}{\pi^{3/2}}\frac{T^{1/2}}{\phi^{1/2}}\big(1-\frac{T}{2\phi}\big)e^{-\frac{\phi}{T}}. \quad (7)$$

The stationary equation $\frac{\partial V}{\partial h}=0$ has the solution $h=0$. Hence we conclude, after symmetry breaking the spontaneous vacuum magnetization does not happen.

To estimate the magnetic field strength in the restored phase at the EWPT temperature the total EP deduced in the previous section must be used. This can be best done numerically. To explain the procedure, we consider here a part of this potential accounting for the one-loop W-boson contributions given in Eq. (2). The value of the spontaneously generated magnetic weak isospin field is calculated from Eqs. (2) and (3):

$$\tilde{B}(T)=\frac{1}{16}\frac{g^3}{\pi^2}\frac{T^2}{(1+\frac{5}{12}\frac{g^2}{\pi^2}\log\frac{T}{\tau})^2}. \quad (8)$$

We relate this expression with the intergalactic magnetic field B_0.

Let us introduce the standard parameters and definitions, $\frac{g^2}{4\pi}=\alpha_w,\alpha=\alpha_w\sin\theta_w^2,\frac{(g')^2}{4\pi}=\alpha_Y$ and $\tan^2\theta_w(T)=\frac{\alpha_Y(T)}{\alpha_w(T)}$, where α is the fine structure constant. Here, instead to find the temperature dependence of the Weinberg angle, we, for a rough estimate, substitute the zero temperature number: $\sin^2\theta_w(T)=\sin^2\theta_w(0)=0.23$.

Other point – re-scaling – must be taken into account in the expanding Universe. As is well known, the temperature dependence $B(T_0)=B(T)(\frac{a(T)}{a(T_0)})^2$ takes place, where $a(t)$ is a metric scale factor. At the same time, for magnetic fields after symmetry breaking (as for relic photons) the scaling behavior $T(t)\sim 1/a(t)$ is usually assumed. That results in the temperature dependence of $B\sim(T/T_0)^2$. Hence, the possibility to relate B_0 with $B(T_{ew})$ is in order.

For the given temperature of the EWPT, T_{ew}, the magnetic field is

$$B(T_{ew}) = B_0 \frac{T_{ew}^2}{T_0^2} = \sin\theta_w(T_{ew})\tilde{B}(T_{ew}). \quad (9)$$

Assuming $T_{ew} = 100$ GeV $= 10^{11}$ eV and $T_0 = 2.7K = 2.3267 \cdot 10^{-4}$ eV, we obtain

$$B(T_{ew}) \sim 1.85\ 10^{14}\ \text{G}. \quad (10)$$

To take into consideration the fermion contribution Eq.(5) we have to substitute the expression $\frac{5}{12}\frac{g^2}{\pi^2}\log\frac{T}{\tau}$ in Eq.(8) by the value

$$(\frac{5}{3} - \sum_f \frac{1}{6} q_f^2)\frac{\alpha_s}{\pi}\log\frac{T}{\tau}. \quad (11)$$

In the above estimate, we have taken into account the one-loop part of the EP of order $\sim g^2$ in the coupling constant. The ring diagrams have order $\sim g^3$ and give a small numeric correction to this result in the high temperature approximation. Note, had we taken into account all the terms listed in the previous section, the results not changed essentially as compared to given in Eq.(10).

4. DISCUSSION

We here summarize our main results. In the problem under investigation, the key point is the spontaneous vacuum magnetization, which eliminates the magnetic flux conservation principle at high temperature. Vacuum polarization is responsible for the value of the field strength $B(T)$ at each temperature and serves as a source of it. We also have shown that, at finite temperature and after symmetry breaking, a scalar field condensate suppresses the magnetization. At T_{ew} the magnetization is stopped and the frozen in of the magnetic field lines into the plasma happens. Due to this property the field strengths at different temperatures can be estimated and related to B_0 in various models.

Hence it follows that the actual nature of the extended model is not essential at sufficiently low temperatures when the decoupling of heavy gauge fields has happened. In particular, from this one can conclude that the vacuum polarization “washed out” the relics of the magnetic fields generated at very high temperature or at inflation.

The present value of the intergalactic magnetic field is related in our model with the field strengths at high temperatures in the restored phase. Because of the zero magnetic mass for Abelian magnetic fields, as discovered recently [8], there is no problem in the generation of fields having a large coherence length. In fact, we have assumed that it is of the order of the horizon scale, $\lambda_B \sim R_{H(T)}$, in our estimate. This is reasonable because at a given temperature the field $B(T) = $ const, generated due to vacuum polarization, occupies all space. In this scenario, a large scale domain structure is also permissible that requires an addition consideration. Knowing the particular properties of the extended model it is possible to estimate the field strengths at any temperature. This can be done for different schemes of spontaneous symmetry breaking (restoration) by taking into account the fact that, after the decoupling of some massive gauge fields, the corresponding magnetic fields are screened. Thus, the higher the temperature is the larger number of strong long range magnetic fields of different types will be generated in the early Universe.

One of us (VS) was supported by the ESF CASIMIR Networking program. This work has been also partly supported by MICINN (Spain), projects FIS2006-02842 and FIS2010-15640, by the CPAN Consolider Ingenio Project, and by AGAUR (Generalitat de Catalunya), contract 2009SGR-994.

References

1. S. Ando, A. Kusenko. Evidence for Gamma-Ray Halos Around Active Galactic Nuclei and the First Measurement of Intergalactic Magnetic Fields // *Astrophys. J. Lett.* 2010, v. 722, p. L39-L44.

2. A. Neronov, E. Vovk. Evidence for Strong Extragalactic Magnetic Fields from Fermi Observations of TeV Blazars // *Science.* 2010, v. 328, p. 73-75.

3. W. Essey, S. Ando, A. Kusenko. Determination of intergalactic magnetic fields from gamma ray data // *Astropart. Phys..* 2011, v. 35, p. 135-139.

4. D. Grasso and H.R. Rubinstein. Magnetic Fields In The Early Universe // *Phys. Rept.* 2001, v. 348, p. 163-266.

5. A.O. Starinets, A.S. Vshivtsev and V.C. Zhukovsky. Color ferromagnetic state in SU(2) gauge theory at finite temperature // *Phys. Lett.* 1994, v. B322, p. 403.

6. V. Skalozub and M. Bordag. Once more on a colour ferromagnetic vacuum state at finite temperature // *Nucl. Phys.* 2000, v. B576, p. 430.

7. V.I. Demchik and V.V. Skalozub. On the Spontaneous Creation of Chromomagnetic Fields at High Temperature // *Phys. Atom. Nucl.* 2008, v. 71, p. 180.

8. S. Antropov, M. Bordag, V. Demchik and V.V. Skalozub. Long range chromomagnetic fields at high temperature // *arXiv*: 1011.314/v1 [hep-ph], 13 Nov. 2010.

9. G.K. Savvidy. Infrared Instability of the Vacuum State of Gauge Theories and Asymptotic Freedom // *Phys. Lett.* 1977, v. B71, p. 133.

10. V. Skalozub and V. Demchik. Electroweak Phase Transition in Strong Magnetic Fields in the Standard Model of Elementary Particles // *arXiv*: hep-th/9912071, 8 Dec. 1999.

11. M. Bordag and V. Skalozub. Neutral gluon polarization tensor in color magnetic background at finite temperature // *Phys. Rev.* 2007, v. D75, 125003.

12. M. Bordag, V.V. Skalozub. Polarization tensor of charged gluons in color magnetic background field at finite temperature // *Phys. Rev.* 2008, v. D77, 105013.

ГРАНИЦА СНИЗУ НА НАПРЯЖЕННОСТЬ МАГНИТНОГО ПОЛЯ В ГОРЯЧЕЙ ВСЕЛЕННОЙ

Э. Элизалде, В. Скалозуб

Высказано предположение о том, что межгалактические магнитные поля произошли в ранней вселенной при высокой температуре T вследствие спонтанного намагничения вакуума неабелевых калибровочных полей. Развита процедура, позволяющая оценить напряжённости поля $B(T)$ при различных T, и получено значение напряжённости поля $B(T_ew) \sim 10^{14}$ G при температуре электрослабого фазового перехода, принимая значение существующего в настоящее время межгалактического магнитного поля $B_0 \sim 10^{-15}$ G.

ГРАНИЦЯ ЗНИЗУ НА НАПРУЖЕНІСТЬ МАГНІТНОГО ПОЛЯ В ГАРЯЧОМУ ВСЕСВІТІ

Е. Елізалде, В. Скалозуб

Висловлено припущення про те, що міжгалактичні магнітні поля виникають у ранньому всесвіті при високій температурі T внаслідок спонтанного намагнічування вакууму неабелевих калібрувальних полів. Розроблено процедуру, яка дає змогу оцінити напруженість поля $B(T)$ при різних T, і обчислено значення напруженості поля $B(T_ew) \sim 10^{14}$ G при температурах електрослабкого фазового переходу, приймаючи значення існуючого в наш час міжгалактичного магнітного поля порядку $B_0 \sim 10^{-15}$ G.