GENERAL PROPERTIES OF INTERACTION CURRENTS OF HIGHER SPIN FERMIONS AND THEIR CONSEQUENCES FOR $\pi N$-SCATTERING

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The model for the currents of interactions of higher spin fermions with the 0- and 1/2-spin particles is proposed. These currents obey to the theorem on currents and fields as well as the theorems on current asymptotics. The comparison of the proposed model with partial wave analysis of the $\pi N$-scattering in $\Delta(1232)$-region shows the validity of the theorem on currents and fields. It is shown that in consequence of the theorems on current asymptotics the contributions of higher spin nucleon resonances to $\pi N$-scattering amplitudes must decrease at high energy at least as $s^{-6}$.

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1. INTRODUCTION

At present a lot of higher spin particles (the spin $J \geq 1$) is known. It is known that the higher spin particles as well as the nucleons, the pions and nuclei are not the elementary particles. But the approximation of elementary particles gives rather good description of reactions at low and intermediate energies. Therefore we can assume that the higher spin particles may be considered approximately as elementary particles also, similarly to the nuclei and the pions. The non-elementarity of the particles can be taken into account by means of the form factors in the interaction currents. As a rule the Rarita-Schwinger formalism for the amplitude of any interaction of higher spin particles is used in the calculations of the reaction amplitudes. We can write for the amplification of any interaction of higher spin fermion (HSF)

$$ V = U(p)_{\mu_1...\mu_l} \eta(p)_{\mu_1...\mu_l}, $$

where $\eta(p)_{\mu_1...\mu_l}$ is the HSF interaction current. The $U(p)_{\mu_1...\mu_l} = U(p)_\mu^l$ is the spin-tensor of HSF with the spin $J = l + 1/2$ and the momentum $p$. This spin-tensor is symmetric and traceless and its convolutions with momenta $p$ and $\gamma$-matrices vanish. As usual we assume that the HSF interactions are described by the system of the non-homogeneous Dirac equations. The field spin-tensors $U(x)_{\mu_1...\mu_l}^l$ and $U(x)_{\mu_1...\mu_l}^l$ have got $2J + 1 = 2l + 2$ independent components. As a rule for the current-spin-tensors $\eta(x)_{\mu_1...\mu_l}$ and $\eta(x)_{\mu_1...\mu_l}$ it is assumed that they are the symmetric one. Therefore they have got $N_l = 4 \cdot 4 \cdot 5 \cdot \ldots \cdot (l + 3)! / l! = 2(l + 1)(l + 2)(l + 3)/3$ independent components. We name the approaches with such currents as the common (or conventional) ones. Unfortunately the common approaches have got some shortcomings [3]: 1) the inconsistencies of equation systems (as $N_l > 2J + 1$); 2) power divergences due to the higher spin particle propagators and the interaction currents; 3) the ambiguities of the vertex functions; 4) contradictions to the experimental data in wide energy regions. Therefore we conclude that the common approaches must be modified. As the shortcomings of common approaches exist for different higher spin particles we may propose that the interaction currents for higher spin particles must obey some general properties in addition to the symmetry property. In Refs. [3-6] it is shown that the interaction currents for higher spin particles must obey the theorem on currents and fields as well as the theorem on current asymptotics. In Refs. [7, 8] the model for the interaction of higher spin boson with two spinless particles is proposed in the agreement with these theorems. The calculation of the virtual higher spin boson contributions to the self-energy operator of the spinless particle shows that these contributions are finite in the one-loop approximation [3]. These finite values must be compared with the logarithmic divergences for two spinless particle contribution to the self-energy operator.

In present paper we propose the model for the vertex of the HSF interaction with 0- and 1/2-spin particles (e.g., $\pi N \leftrightarrow N^*$), which obeys the theorem on currents and fields as well as the theorem on current asymptotics. We study the application of this model to the elastic $\pi N$-scattering.

2. CONSEQUENCES OF THEOREM ON CURRENTS AND FIELDS

In accordance with the theorem on currents and fields [6] the system of the algebraic equations for...
the Fourier components is consistent only in the case when the current spin-tensors have got the same properties as the field spin-tensors. We name the current which obeys the theorem on currents and fields as the physical one and denote \( j(p)_{\mu} = j(p)_{\mu_1 \ldots \mu_\ell} \). Thus for physical currents we have
\[
\begin{align*}
    j(p)_{\mu_1 \ldots \mu_\ell} p_{\mu_\ell} &= 0, \\
    j(p)_{\mu_1 \ldots \mu_\ell} g_{\mu_\ell \mu_1} &= 0, \\
    j(p)_{\mu_1 \ldots \mu_\ell} g_{\mu_\ell \mu_1} &= 0,
\end{align*}
\]
where \( i, k, l = 1, 2, \ldots, l \). The physical currents have got the current conservation (2) and the condition (4) the power divergences due to the HSF propagator disappear, so the one source of the power divergences to eliminate the possible contributions of \( \Delta(1232) \) to these amplitudes correspond to isospin \( I = 3/2 \) and the total angular momentum \( J_{\ell} = 1/2, 1/2, 3/2 \), respectively (where \( W \) is the total energy in the c.m.s.).

3. CONSISTENT MODEL FOR INTERACTION CURRENTS OF HIGHER SPIN FERMIONS

We consider the simplest HSF interactions determined by one amplitude. Using the definition(5) we may write for the \( J(p) \rightarrow O(q_2) + 1/2(p_2) \) transition:
\[
j(p)_{\mu} = g_F F(p,q)(p^2) \varphi^+(q_2) \pi(p_2),
\]
where \( q' = q_2 - p_2, g_F \) is the coupling constant; \( F(p,q) \) is the form factor providing the necessary asymptotic decrease in agreement with the theorem on current asymptotics. In particular for \( J = 3/2 \) we have
\[
j(p, q)_{\mu} = g_F F_1(p,q) \varphi^+(q_2) \pi(p_2),
\]
where \( q'_\mu = -d_{\mu \nu} p_2^2 q_\nu - p_\mu (p q') \). The 1 and \( i \gamma_5 \) matrices in the currents (7), (8) correspond to different sets of the particle parities. Note that different common currents lead to the physical currents (5) with the same momentum dependencies, as the convolutions of the projection operators with the \( p \) momentum and the \( \gamma \)-matrices vanish in our approach. Therefore the physical currents corresponding to different common currents in Eq. (5) differ by the coupling constant only. Thus the ambiguities of vertex functions do not appear in our approach.

4. TEST OF THEOREM ON CURRENTS AND FIELDS IN \( \Delta(1232) \)-REGION

HSF in the \( 0 + 1/2 \equiv J(p) \) -transitions can be \( N^*(J) \) resonances in the \( s \)-channel of the elastic \( \pi N \)-scattering (\( \pi N \rightarrow N^*(J) \rightarrow \pi N \)). In elastic \( \pi N \)-scattering the \( S_{31}(W) \), \( P_{31}(W) \), \( P_{33}(W) \)-amplitudes correspond to isospin \( I = 3/2 \) and the total angular momentum \( J_{\ell} = 1/2, 1/2, 3/2 \), respectively (where \( W \) is the total energy in the c.m.s.). The \( \Delta(1232) \) on the mass shell (at \( W = M_{\Delta} \)), \( M_{\Delta} \) is the \( \Delta(1232) \) mass contributes to the amplitude \( P_{33} \) only. But at \( W \neq M_{\Delta} \) the \( \Delta(1232) \) contribute to other amplitudes of the \( \pi N \)-scattering too. These contribution are different in our and common approaches.

In common approach \( \Delta(1232) \) contribute to the amplitudes \( S_{31}, P_{31}, P_{33}, \) and \( D_{33} \) at \( W \neq M_{\Delta} \). We denote the possible contributions of \( \Delta(1232) \) to these amplitudes as \( S_{31}^{-}, P_{31}^{-}, P_{33}^{-}, \) and \( D_{33}^{-} \), respectively. To derive these contributions in common approach we use common propagator of the 3/2-spin particle [2] and the Breit-Wigner formula [13]. The calculations of the \( S_{31}^{-}, P_{31}^{-}, P_{33}^{-}, D_{33}^{-} \) -amplitudes show that among these amplitudes \( S_{31}^{-} \) achieves largest values and \( D_{33}^{-} \) have smallest values for \( M_{\Delta} - \Gamma_2 / 2 \leq W \leq M_{\Delta} + \Gamma / 2 \) (where \( \Gamma = 112 MeV \) is the total width of \( \Delta(1232) \) [13]). In common approach energy dependences are fairly sharp for \( S_{31}(W) \).

In our approach \( S_{31}^{-} = P_{31}^{-} = 0 \), as consequence of (2), (4). From comparison with the partial wave analysis [13, 14] we may conclude that \( S_{31}^{-} = 0 \) in agreement with the consequence of the theorem on current and fields.

5. CONSEQUENCES OF THEOREMS ON CURRENT ASYMPTOTICS

We consider HSF which moves along the z-axis (i.e., \( p = (p_0, 0, 0, p_3) \)). Then the physical currents in momentum representation \( j(p)_{\mu} \) depend on \( p_0 \) and \( p_3 \), whereas \( J(x)_{\mu} \) depend on \( x_0 \) and \( x_3 \). As the components of \( j(p)_{\mu} \) are the Fourier components of \( j(x)_{\mu} \).
these currents in coordinate representation are the improper integrals depending on the parameters \( x_0 \) and \( x_3 \). In Ref [6] it is shown, that the physical currents \( j(x)_\mu \) and some their derivatives must be continuous functions. We use the Weierstrass test to study the continuity of the currents. Therefore, we consider the integrals

\[
\int_{-\infty}^{+\infty} dp_0 \int_{-\infty}^{+\infty} dp_3 \left| j(p)_\mu \right|^m |p_0|^n |p_3|^{m_3}, \tag{9}
\]

where \( m_0, m_3, m(j) \) are integer non-negative numbers \((m_0 + m_3 = 0, 1, 2, \ldots, m(j))\). The theorem on current asymptotics may be formulated as:

If the currents \( J(x)_\mu \) and their partial derivatives of upper degree \( m(j) \) are continuous functions then their Fourier components \( j(p)_\mu \) must decrease at \( |p_\nu| \to \infty \) to provide the convergence of all integrals (9).

Note that the integrals (9) must be convergent in all kinematic regions. The powers of the decrease for \( j(p)_\mu \) are determined by the number \( m(j) \). In Ref. [6] it is derived that \( m(j) = 2 \), as consequence of the condition that \( 2J + 1 \) equations must be for \( U(x)_\mu \).

Now in addition we demand that the double Fourier transformations for the function of two variables converges to the value of this function in any space-time point. But it is possible if this function, first derivative and its mixed derivative of the second degree are continuous [15]. It allows to derive \( m(j) = 4 \).

We propose that the form factors \( F_l(p, q) \) in the currents (7) have a form:

\[
\hat{F}_l(p, q) = (pq)^{2n_1} \left[ (p^2 - M^2)^{2n_2} + a^{4n_2} \right]^{-1} \left[ (pq)^{2n_3} + b^{4n_3} \right]^{-1}, \tag{10}
\]

where \( n_1, n_2, n_3 \) are integer non-negative numbers, \( a \) and \( b \) are positive constants. Using the method of Refs. [7, 8] we derive at \( n_1 = 1: n_2 \geq m_1(\eta)/4 + 1, n_3 \geq m_1(\eta)/2 + 2 \), where even number \( m_1(\eta) = m(j) + 2l \) for even \( m(j) \) and \( m_1(\eta) = m(j) + 2l + 1 \) for odd \( m(j) \). For \( m(j) = 4 \) we have the restrictions \( n_2 \geq l/2 + 2, n_3 \geq l + 4 \).

Now we consider the convergence of the integrals (9) for the contributions of \( N^*(J) \) to \( s \)-channel amplitudes of the \( \pi N \)-scattering. In c. m. s. we have \( p = (W, 0, 0, 0) \), \( p^2 = W^2 = s, p \cdot q = m^2 - \hat{M}^2 \), \( q = (q_0 - E, 2q_3) \), where \( q_0, E \), and \( 2q_3 \) are the pion energy, the nucleon energy, and the 3-momentum of the final pion, respectively. The form factor (10) may be written as \( F(p, q) = A/ \left( W^{4n_2} + a^{4n_2} \right) \), where \( A \) is the constant. The asymptotic behavior of the current (7) is given by \( j(p)_\mu \sim C_j W^2 \hat{F}_l(p, q) \), where \( C_j \) is some constant.

The \( p_3 \)-dependence of the current (7) in c. m. s. is determined by the factor \( \delta(p_3) \). Then the integrals (9) converge in the case of the convergence of the integral

\[
\int_{-\infty}^{+\infty} \frac{W^{3l+4dW}}{W^{4n_2} + a^{4n_2}}. \tag{11}
\]

This integral converges at \( n_2 \geq 3/4 \cdot l + 3/2 \) Thus we derive two restrictions. For better convergence of the integrals we must choose larger \( n_2 \). Both restrictions give the same values of the number \( n_2 \): \( n_2 = 3, 4 \) for \( J = 3/2, 5/2, 7/2 \), respectively. At \( l \geq 4 \) second restriction gives larger integer number \( n_2 \). It is of interest to study the asymptotic behaviour of the \( N^*(J) \) contributions to the \( \pi N \)-scattering amplitudes including the physical currents (7) with the form factors (10)

\[
T(\pi N \to N^*(J) \to \pi N) = \frac{A_j}{W^2 - M^2 + iMT} \left[ \frac{W^4}{W^{4n_2} + a^{4n_2}} \right]^2, \tag{12}
\]

where \( A_j \) are the constants. In general we derive that the asymptotic contributions of \( N^*(J) \) to invariant amplitudes decrease at least as \( s^{-6} \). But we can derive stronger decrease at integer \( n_2 \). Indeed, for \( J = 3/2, 5/2, 7/2 \) we have \( n_2 = 3, 4 \) and

\[
T(\pi N \to N^*(J) \to \pi N)/A_j \leq s^{-6}, s^{-6}, s^{-7} \text{ respectively.}
\]

6. EQUATIONS FOR GENERATIONS OF HIGHER SPIN FERMIONS

In Ref[16] it is shown that the integrals corresponding to the Green functions of the Klein-Gordon and Dirac equations diverge. To derive the convergent integrals for the Green functions we may study the partial differential equations of higher degrees. We consider some sets (kinds, families, dynasties) of particles, which have different masses but the same values of the electric charge, the spin, and the parities. The members of such kinds belong to generations. We can consider the electron kind \((e_1 = e, e_2 = \mu, e_3 = \tau, \ldots)\), the neutrino kind \((\nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau, \ldots)\), three colored up-quark kinds \((u_1 = u, u_2 = c, u_3 = t, \ldots) \) and three colored down-quark kinds \((d_1 = d, d_2 = s, d_3 = b, \ldots) \).

We propose the equations for the 1/2-spin particles as the generalization of Dirac equation:

\[
\left( M_1 - i\hat{\sigma} \right) \left( M_2 - i\hat{\sigma} \right) \ldots \left( M_N - i\hat{\sigma} \right) = \chi(x), \tag{13}
\]

where \( M_1, M_2, \ldots, M_N \) are the particle masses \((M_1 < M_2 < M_3 < \ldots < M_N)\). The number \( N \) is the degree of the differential equation, which is equal to the number of the generations in the kind. It follows from the convergence of the integrals for the Green function of Eq. (13) that \( N \geq 5 \). We denote the minimal \( N \) for the elementary fermions (the leptons and the quarks) as \( N_f \). The numbers of the generations for composite particles are larger then ones for elementary particles. For example, the minimal numbers for the proton and the neutron kinds are equal to 75, as we may derive \( N(\text{proton kind})_{\min} = N(\text{neutron kind})_{\min} = N_f^2(N_f + 1)/2 \) [16]. The proton kind includes \( \sum^\dagger (1189), \Lambda_3^+ (2285) \) [17]. The neutron kind includes \( n, A^0(1115), \sum^0(1193), \Xi^0(1315) \).
We propose that the generalization of the equation system for the HSF generations may be written as
\[
(-\square)^{\ell} \Pi(x)_{\mu_1 \ldots \mu_l \nu_1 \ldots \nu_l} (-i\partial + M_1) (-i\partial + M_2) \ldots (-i\partial + M_N) U(x)_{\nu_1 \ldots \nu_l} \gamma = \ell^{(x)_{\mu_1 \ldots \mu_l}},
\]
where the physical currents \( j(x)_{\mu_1 \ldots \mu_l} \) must obey the conditions (2)-(4). We may rewrite Eq. (14) in the form:
\[
(-\square)^{\ell} \Pi(x)_{\mu_1 \ldots \mu_l \nu_1 \ldots \nu_l} (\partial + M_1^2)(\partial + M_2^2) \ldots (\partial + M_N^2) U(x)_{\nu_1 \ldots \nu_l} = (-i\partial + M_1) (-i\partial + M_2) \ldots (-i\partial + M_N) j(x)_{\mu_1 \ldots \mu_l}.
\]
We see that the physical currents \( j(x)_{\mu_1 \ldots \mu_l} \) must have the continuous derivatives of the degree \( N + 3 \), (i.e., \( m(j) = N + 3 \)). Then for the number \( n_2 \) in the form factor (10) we derive \( n_2 \geq (2l + N + 7)/4 \).

For these \( n_2 \) we have
\[
T(\pi N \to N^*(J) \to \pi N) \leq \overline{A}_l/s^{N+7-l},
\]
where \( \overline{A}_l \) is constant. We may find \( N_{min} \) for \( \Delta \)-isobar kinds. We assume that the \( \Delta^{++} \) kind includes \( \Delta^{++}(1232) \) and different three-quark systems with \( J^p = 3/2^+ \) consist of \( u-, c-, f- \) quarks. The \( \Delta^- \) kind includes \( \Delta^-(1232), \sum_\delta(1385), \Xi_\delta(1531), \Omega^- (1672) \) and different three-quark systems with \( J^p = 3/2^+ \) consist of \( d-, s-, b- \) quarks. We may derive that
\[
N(\Delta^{++} - \text{kind})_{min} = N(\Delta^- - \text{kind})_{min} = N_f(\mathtt{N}_f^2 + 1)(\mathtt{N}_f + 2)/6 \geq 35.
\]
We may expect that \( N(\Delta^+ - \text{kind})_{min} = N(\Delta^0 - \text{kind})_{min} = N(\text{proton kind})_{min} = N(\text{neutron kind})_{min} \geq 75 \).

From Eq. (16) we can derive
\[
T(\pi p \to \Delta^{++} \to \pi^+ p) \leq \overline{A}_l/s^{41}.
\]

7. CONCLUSIONS
We proposed the model for the currents of HSF interactions with the 0- and 1/2-spin particles. These physical currents obey the general properties formulated in Ref. [6] for the currents of the HSF interactions. All physical currents must obey the theorem on currents and fields as well as the theorem on current asymptotics. We consider the nucleon resonances \( N^*(J) \) as example of HSF. In consequence of the theorem on currents and fields the virtual HSF can change the parity but they do not contribute to the amplitudes corresponding to the values of the angular momentum less than \( J \), whereas in common approaches such contributions exist. We have tested the predictions of our and common approaches for the virtual \( \Delta(1232) \) in the elastic \( \pi N \)-scattering. The calculations performed in common isobar model show sharp energy dependence of the \( \Delta(1232) \)-contributions to the \( S_{31} \) and \( P_{31} \)-amplitudes at \( W \approx M_\Delta \). It turned out that the \( S_{31} \)-amplitude is the most sensitive. According to the partial wave analyses the energy dependences of the amplitudes are approximately linear in the \( \Delta(1232) \) region, i.e., they differ from the predictions of common isobar model. It means that the predictions of our approach are valid. Thus we have examined the validity of the conditions (2), (4). To examine the validity of the conditions (2)-(4) we must consider HSF with \( J > 3/2 \).

For example, it is of interest to study the contributions of \( F_{15}(1680) \) [17] to the \( S_{31} -, P_{31} -, P_{33} -, D_{13} -, F_{15}^- \) amplitudes, \( F_{15}(1905) \) to the \( S_{31} -, P_{31} -, P_{33} -, D_{33} -, F_{35}^- \) amplitudes, \( F_{27}(1950) \) to the \( S_{31} -, P_{31} -, P_{33} -, D_{33} -, F_{35}^- \) amplitudes.

We propose the form-factor (10), which allows to obey the theorem on current asymptotics. The restrictions for the integer number \( n_2 \) lead to the high-energy decrease of the HSF contributions to the \( \pi N \)-scattering amplitudes. This decrease explains the absence of the \( N^*(J) \) contributions at high energies.

In the cases for the interactions of several higher spin particles (e.g., the \( \rho \Delta N^*(J) \)-interaction) the theorems on currents and fields as well as the theorem on current asymptotics must be valid for the interaction currents of each higher spin particle. The theorem on current and fields can be satisfied using the projection operators. The theorem on current asymptotics can be satisfied by consideration of the product of the form factors (such as (10) or from Refs. [7,8]) for each higher spin particle. Therefore we may expect that the contributions of the vertex functions for several higher spin particle interaction to the amplitudes ought to decrease at high energies, in comparison with similar vertex functions for the 0- and 1/2-spin particle interactions.

References

АВИЧЕ СВОЙСТВА ТОКОВ ВЗАИМОДЕЙСТВИЙ ВЫСОКОСПИНОВЫХ ФЕРМИОНОВ И ИХ СЛЕДСТВИЯ ДЛЯ $\pi N$-РАССЕЯНИЯ

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Предложена модель для токов взаимодействий высокоспиновых фермионов с частицами, обладающими спином 0 и 1/2. Эти токи удовлетворяют теореме о токах в полях и теореме об асимптотике токов. Сравнение предложенной модели с парциальными волновыми анализами $\pi N$-рассеяния в области $\Delta(1232)$ показывает справедливость теоремы о токах в полях. Показано, что в области высоких энергий $\pi N$-рассеяния и в области $\Delta(1232)$ теорема об асимптотике токов вклады высокоспиновых нуклонных резонансов в амплитуду $\pi N$-рассеяния должны убывать наименее быстрее при $s^{-6}$.

ЗАГАЛЬНІ ВЛАСНИСТІ СТРУМІВ ВЗАЄМОДІЙ ВИСОКОСПІНОВИХ ФЕРМИОНІВ І ЇХ НАСЛІДКИ ДЛЯ $\pi N$-РОЗСЯГАННЯ

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Запропоновано модель для струмів взаємодій високоспинових фермионів з частинками, які мають спин 0 та 1/2. Ці струми задовольняють теореми про струми та поля та теореми про асимптотику струмів. Порівняння запропонованої моделі з парциальними волновими аналізами $\pi N$-розсіяння в області $\Delta(1232)$ показує справедливість теореми про струми і поля. Показано, що внаслідок теореми про асимптотику струмів високоспинових нуклонних резонансів в амплітуду $\pi N$-розсіяння повинні спадати при високих енергіях по мінійшій мірі як $s^{-6}$.