POLARIZATION EFFECTS IN ELASTIC PROTON-ELECTRON SCATTERING

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Proton elastic scattering from electrons at rest is calculated in the Born approximation. The interest of this reaction is related to the possibility of polarizing high energy antiproton beam and to high energy proton polarimetry. The differential cross section and polarization observables have been derived assuming one photon exchange. Numerical estimates are given for the cross section and the spin correlation coefficients in a wide kinematical range.

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1. INTRODUCTION

The polarized and unpolarized scattering of electrons by protons has been widely studied, as it is considered the simplest way to access information on proton structure. The expressions which relate the polarization observables to the proton electromagnetic form factors were firstly derived in Ref. [1], assuming that the interaction occurs through the exchange of a virtual photon. In the scattering of proton from electrons at rest (inverse kinematics) approximations such as neglecting the electron mass no longer hold. Liquid hydrogen targets are considered as proton targets, but any reaction with such targets also involves reactions with atomic electrons, which we will assume to be at rest.

A large interest in inverse kinematics (proton projectile on electron target) has been aroused due to two possible applications: the possibility to build beam polarimeters, for high-energy polarized proton beams, in the relativistic heavy-ion collider (RHIC) energy range [2] and the possibility to build polarized antiprotons beams [3], which would open a wide domain of polarization studies at the GSI facility for Antiproton and Ion Research (FAIR) at Darmstadt (Germany).

Concerning the polarimetry of high-energy proton beams, in Ref. [2] analyzing powers corresponding to polarized proton beam and electron target were numerically calculated for elastic proton-electron scattering.

The possibility of polarizing a proton beam in a storage ring by circulating through a polarized hydrogen target would be extremely interesting. Such polarization was indeed observed [4]. Possible explanations of the polarizing mechanisms were published in a number of papers (see [5] and references therein). Two mechanism could be responsible for the polarization: 'spin-filtering', where the proton-proton interaction would scatter preferentially at large angles protons with one spin component which would be lost from recirculating in the beam, and 'spin-flip' where the reaction proton-electron at very small scattering angles would have very large analyzing powers. The second explanation is extremely interesting as one could polarize antiproton beams without losses of particles.

We calculated the cross section and the polarization observables for proton electron elastic scattering, in a relativistic approach assuming the Born approximation with particular attention to the kinematics which is very specific for this reaction. Three types of polarization effects were studied: – the spin correlation, due to the polarization of the proton beam and of the electron target, – the polarization transfer from the polarized electron target to the scattered proton, – and the depolarization coefficients which describe the polarization of the scattered proton which depends on the polarization of the proton beam. Numerical estimations of the polarization observables have been performed over a wide range of proton-beam energy and for different values of scattering angle. Our results show that polarization effects are sizable in the high energy domain.

2. GENERAL FORMALISM

Let us consider the reaction $p(p_1) + e(k_1) \rightarrow p(p_2) + e(k_2)$, where particle momenta are indicated in paren-

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theses, and $k = k_1 - k_2 = p_2 - p_1$ is the four-momentum of the virtual photon.

A general characteristic of all reactions of elastic and inelastic hadron scattering by atomic electrons (which can be considered at rest) is the small value of the transfer momentum squared, even for relatively large energies of colliding hadrons. Let us first give details of the order of magnitude and the range which is accessible to the kinematic variables, as they are very specific for this reaction, and then derive the spin structure of the matrix element and the unpolarized and polarized observables.

Kinematics

The following formulas can be partly found in Ref. [6]. One can show [6] that, for a given energy of the proton beam, the maximum value of the four-momentum transfer squared, in the scattering on the electron at rest, is: (Fig. 1):

$$(-k^2)_{max} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}, \qquad (1)$$

where $m(M)$ is the electron (proton) mass. Being proportional to the electron mass squared, the four momentum transfer squared is restricted to very small values.

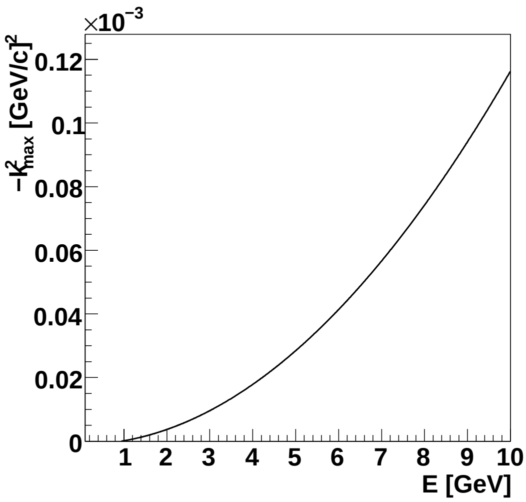

Fig. 1. *Maximum four-momentum squared as a function of the proton beam energy*

In order to have the same total energy s in proton-electron and electron-proton scattering the proton energy should be 2000 times the electron energy. The electron can never be scattered backward. For one proton angle there may be two values of the proton energy, (and two corresponding values for the recoil-electron energy and angle, and for the transferred momentum k^2). The two solutions coincide when the angle between the initial and final hadron takes its maximum value, $\sin\theta_{h,max} = m/M$. Protons are scattered from atomic electrons at very small angles.

In the one-photon-exchange approximation, the matrix element $\mathcal{M}$ of reaction $p + e \to p + e$ can be written as:

$$\mathcal{M} = \frac{e^2}{k^2} j_\mu J_\mu, \qquad (2)$$

where $j_\mu (J_\mu)$ is the leptonic (hadronic) electromagnetic current:

$$\begin{aligned} j_\mu &= \bar{u}(k_2)\gamma_\mu u(k_1), \\ J_\mu &= \bar{u}(p_2)\left[F_1(k^2)\gamma_\mu - \frac{1}{2M}F_2(k^2)\sigma_{\mu\nu}k_\nu\right]u(p_1) \\ &= \bar{u}(p_2)\left[G_M(k^2)\gamma_\mu - F_2(k^2)P_\mu\right]u(p_1). \end{aligned} \qquad (3)$$

Here $F_1(k^2)$ and $F_2(k^2)$ are the Dirac and Pauli proton electromagnetic form factors (FFs), $G_M(k^2) = F_1(k^2) + F_2(k^2)$ is the Sachs proton magnetic FF, and $P_\mu = (p_1 + p_2)_\mu/(2M)$.

The matrix element squared is:

$$|\mathcal{M}|^2 = 16\pi^2 \frac{\alpha^2}{k^4} L_{\mu\nu} W_{\mu\nu}, \qquad (4)$$

with $L_{\mu\nu} = j_\mu j_\nu^*$, $W_{\mu\nu} = J_\mu J_\nu^*$, where $\alpha = 1/137$ is the electromagnetic fine structure constant. The leptonic tensor, $L_{\mu\nu}^{(0)}$, for unpolarized initial and final electrons (averaging over the initial electron spin) has the form:

$$L_{\mu\nu}^{(0)} = k^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}). \qquad (5)$$

The contribution to the electron tensor corresponding to a polarized electron target is

$$L_{\mu\nu}^{(p)} = 2im\epsilon_{\mu\nu\alpha\beta}k_\alpha S_\beta, \qquad (6)$$

where S_β is the initial electron polarization four-vector.

The hadronic tensor, $W_{\mu\nu}^{(0)}$, for unpolarized initial and final protons can be written in the standard form, through two unpolarized structure functions:

$$W_{\mu\nu}^{(0)} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) W_1(k^2) + P_\mu P_\nu W_2(k^2). \ (7)$$

Averaging over the initial proton spin, the structure functions W_i, $i = 1, 2$, can be expressed in terms of the nucleon electromagnetic FFs as:

$$\begin{aligned} W_1(k^2) &= -k^2 G_M^2(k^2), \\ W_2(k^2) &= 4M^2 \frac{G_E^2(k^2) + \tau G_M^2(k^2)}{1+\tau}, \end{aligned} \qquad (8)$$

where $G_E(k^2) = F_1(k^2) - \tau F_2(k^2)$ is the proton electric FF and $\tau = -k^2/(4M^2)$.

The differential cross section over the solid angle can be written as:

$$\frac{d\sigma}{d\Omega_e} = \frac{1}{32\pi^2} \frac{1}{m|\vec{p}|} \frac{\vec{k}_2^3}{(-k^2)} \frac{\overline{|\mathcal{M}|^2}}{E+m}, \qquad (9)$$

where $d\Omega_e = 2\pi d\cos\theta_e$ (due to azimuthal symmetry),$\vec{p}$ is the three-momentum of the proton beam. The expression of the differential cross section for unpolarized proton-electron scattering, in the coordinate system where the electron is at rest, can be written as:

$$\frac{d\sigma}{dk^2} = \frac{\pi\alpha^2}{2m^2\vec{p}_1^2}\frac{\mathcal{D}}{k^4}, \tag{10}$$
$$\mathcal{D} = k^2(k^2+2m^2)G_M^2(k^2) + 2\left[k^2M^2 + 2mE\left(2mE+k^2\right)\right]\left[F_1^2(k^2)+\tau F_2^2(k^2)\right].$$

The differential cross section diverges as k^4 when $k^2 \to 0$. This is a well known result, which is a consequence of the one photon exchange mechanism.

The proton structure is taken into account through the parametrization of FFs. However, due to the small maximum value of k^2 which can be achieved in inverse kinematics any FFs parametrization, and even of constant FFs, where the constants correspond to the static values would give the same results.

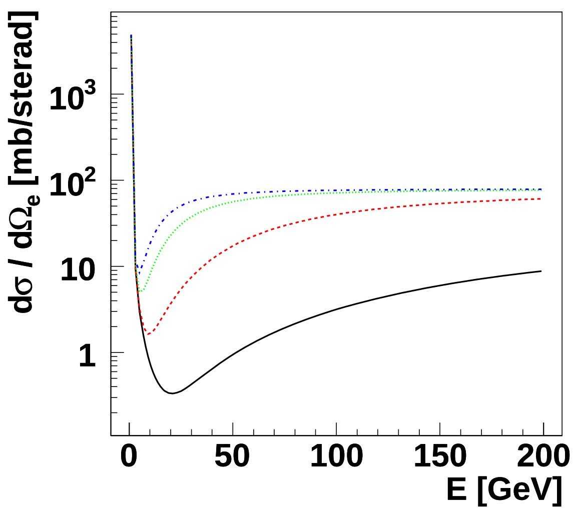

Fig. 2. *Differential cross section as a function of incident energy E for different angles: $\theta_e = 0$ (solid line), 10 mrad (dashed line), 30 mrad (dotted line), 50 mrad (dash-dotted line)*

The energy dependence of the cross section for different angles: $\theta_e = 0$ (solid line), 10 mrad (dashed line), 30 mrad (dotted line), 50 mrad (dash-dotted line) is given in Fig. 2. The unpolarized differential cross section is divergent at small values of energy; it has an angle dependent minimum and then increases smoothly up to large energies.

3. POLARIZATION OBSERVABLES

Let us focus here on three types of polarization observables, for elastic proton-electron scattering:

1. The polarization transfer coefficients which describe the polarization transfer from the polarized electron target to the scattered proton, $p + \vec{e} \to \vec{p} + e$.
2. The spin correlation coefficients when both initial particles have arbitrary polarization, $\vec{p} + \vec{e} \to p + e$.
3. The depolarization coefficients which define the dependence of the scattered proton polarization on the polarization of the proton beam, $\vec{p} + e \to \vec{p} + e$. In our knowledge, this case was not previously considered in the literature.

The first case is the object of a number of recent papers [3] in connection with the possibility to polarize proton (antiproton) beams. The second case was considered in Ref. [2], in view of using polarized proton-electron scattering to measure the longitudinal and transverse polarizations of high-energy proton beams.

The explicit expressions of the polarization observables can be found in Ref. [5]. At high energy, the polarization transfer coefficients depend essentially on the direction of the scattered proton polarization.

Let us give, for illustration, the correlation coefficients when the incident proton and the target electron are polarized.

The contraction of the spin-dependent leptonic $L^{(p)}_{\mu\nu}$ and hadronic $W_{\mu\nu}(\eta_1)$ tensors, in an arbitrary reference frame, gives:

$$\mathcal{D}C(S,\eta_1) = 8mMG_M(k^2)[(k\cdot Sk\cdot\eta_1 - k^2S\cdot\eta_1) \times G_E(k^2) + \tau k\cdot\eta_1(k\cdot S + 2p_1\cdot S)F_2(k^2)].$$

All spin correlation coefficients for the elastic $\vec{p}\vec{e}$ collisions can be obtained from this expression and are, therefore, proportional to the proton magnetic FF. This is a well known fact for $\vec{e}\vec{p}$ scattering [6].

In the considered frame, where the target electron is at rest, the four-vector of the proton beam polarization has the following components:

$$\eta_1 = \left(\frac{\vec{p}\cdot\vec{S}_1}{M}, \vec{S}_1 + \frac{\vec{p}(\vec{p}\cdot\vec{S}_1)}{M(E+M)}\right), \tag{11}$$

where $\vec{S}_1$ is the unit vector describing the polarization of the initial proton in its rest system.

Applying the P-invariance of the hadron electromagnetic interaction, one can write the following expression for the dependence of the differential cross section on the polarization of the initial particles:

$$\frac{d\sigma}{dk^2}(\vec{\xi},\vec{S}_1) = \left(\frac{d\sigma}{dk^2}\right)_{un}[1 + C_{\ell\ell}\xi_\ell S_{1\ell} + C_{tt}\xi_t S_{1t} + C_{nn}\xi_n S_{1n} + C_{\ell t}\xi_\ell S_{1t} + C_{t\ell}\xi_t S_{1\ell}],$$

where C_{ik}, $i,k = \ell,t,n$ are the corresponding spin correlation coefficients which characterize $\vec{p}\vec{e}$ scattering. Small coefficients (in absolute value) are expected for the transverse component of the beam polarization at high energies. This can be seen from the expression of the components of the proton-beam-polarization four-vector at large energies, $E \gg M$:

$$\eta_{1\mu} = (0,\vec{S}_{1t}) + S_{1\ell}\left(\frac{|\vec{p}|}{M}, \frac{\vec{p}}{M}\frac{E}{p}\right) \sim S_{1\ell}\frac{p_{1\mu}}{M}. \tag{12}$$

The effect of the transverse beam polarization appears to be smaller by a factor $1/\gamma$, $\gamma = E/M \gg 1$. This is a consequence of the relativistic description of the spin of the fermion at large energies.

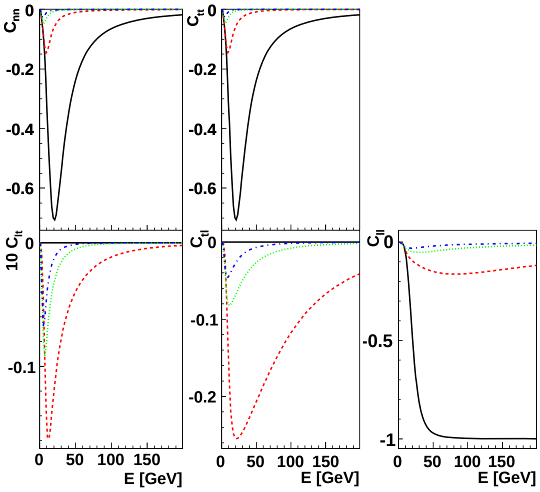

Fig. 3. *Spin correlation coefficients as a function of E for different angles. Notations are the same as in Fig. 2*

The spin correlation coefficients are shown in Fig. 3.

In collinear kinematics, in general, either polarization observables take the maximal values or they vanish. An interesting kinematic region appears at $E = 20$ GeV, where a structure is present in various observables.

It appears that polarization coefficients are in general quite large, except at low energy. Proton electron scattering can be used, in principle, to measure the polarization of high-energy beams. Let us calculate the figure of merit, for measuring the polarization of a secondary proton beam, after scattering from atomic electrons.

The differential figure of merit is defined as

$$\mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A_{ij}^2(\theta_p),$$

where A_{ij} stands for a generic polarization coefficient and $\epsilon(\theta_p) = N_f(\theta_p)/N_i$ is the number of useful events over the number of the incident events in an interval $\Delta\theta_p$ around θ_p. Because it is related to the inverse of the statistical error on the polarization measurement, this quantity is useful for a proton with degree of polarization P:

$$\left(\frac{\Delta P(\theta_p)}{P}\right)^2 = \frac{2}{N_i(\theta_p)\mathcal{F}^2(\theta_p)P^2} = \frac{2}{Lt_m(d\sigma/d\Omega)d\Omega A_{ij}^2(\theta_p)P^2}, \tag{13}$$

where t_m is the time of measurement.

The integrated quantity, calculated for a transverse polarized proton beam scattering from a longitudinally polarized electron target ($\vec{p} + \vec{e} \to p + e$)

$$F^2 = \int \frac{d\sigma}{dk^2} C_{\ell t}^2(k^2)\mathrm{d}k^2 \tag{14}$$

is shown in Fig. 4 as a function of the incident energy. In Refs. [7] it was suggested to use the scattering of a transverse polarized proton beam from a longitudinally polarized electron target. From Fig. 4, one can see that F^2 takes its maximum value for $T \simeq 10$ GeV. Assuming a luminosity of 10^{32} cm^{-2} s^{-1} for an ideal detector with an acceptance and efficiency of 100%, one could measure the beam polarization with an error of 1% in a time interval of 3 min.

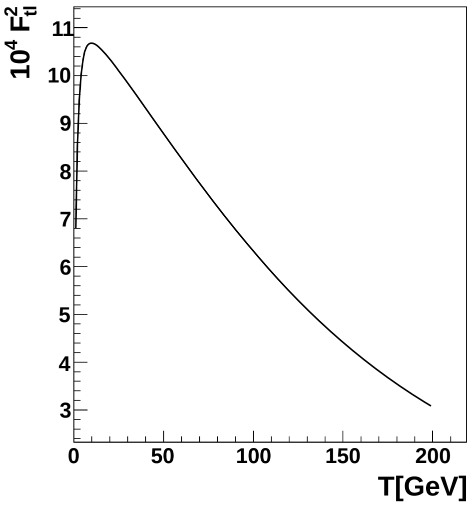

Fig. 4. *Variation of the quantity F^2 [a.u.] as a function of proton kinetic energy T for a transverse polarized proton beam scattering from a longitudinally polarized electron target ($\vec{p} + \vec{e} \to p + e$)*

4. CONCLUSIONS

The elastic scattering of protons from electrons at rest was investigated in a relativistic approach in the one-photon-exchange (Born) approximation. This reaction, where the target is three orders of magnitude lighter than the projectile, has specific kinematical features due to the "inverse kinematics" (i.e., the projectile is heavier than the target). For example, the proton is scattered at very small angles and the allowed momentum transfer does not exceed the MeV^2 scale, even when the proton incident energy is of the order of GeV. The differential cross section and various double spin polarization observables have been calculated in terms of the nucleon electromagnetic FFs. However, for the values of transferred momentum involved, any parametrization of FFs is equivalent and is very near to the static values. The spin transfer coefficients to a polarized scattered proton were calculated when the proton beam is polarized or when the electron target is polarized. The correlation spin coefficients when the proton beam and the electron target are both polarized were also calculated. Numerical estimates showed that polarization effects may be sizable in the GeV range, and that the polarization transfer coefficients for $\vec{p} + e \to \vec{p} + e$ could be used to measure the polarization of high energy proton beams. The calculated values of the scattered proton polarization for the reaction $p + \vec{e} \rightarrow \vec{p} + e$ at proton-beam energies lower then a few tens of MeV show that it is not possible to obtain sizable polarization of the antiproton beam in an experimental setup where antiprotons and electrons collide with small relative velocities. The present results confirm that the polarization of the scattered proton has large values at high proton-beam energies. Thus, one could consider an experimental setup where high-energy protons collide with a polarized electron target at rest. The low values of momentum transfer which are involved ensure that the cross section is sizable.

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ПОЛЯРИЗАЦИОННЫЕ ЭФФЕКТЫ В УПРУГОМ ПРОТОН-ЭЛЕКТРОННОМ РАССЕЯНИИ

Г.И. Гах, А. Дбейси, Д. Маршанд, В.В. Бытев, Э. Томаси-Густафсон

В Борновском приближении вычислено упругое рассеяние протонов на покоящихся электронах. Интерес к этой реакции обусловлен возможностью поляризации высокоэнергетического антипротонного пучка и поляриметрией протонов высоких энергий. В предположении однофотонного обмена получены выражения для дифференциального сечения и поляризационных наблюдаемых. Выполнены численные оценки сечения и коэффициентов корреляции спина в широкой кинематической области.

ПОЛЯРИЗАЦІЙНІ ЕФЕКТИ В ПРУЖНОМУ ЕЛЕКТРОН-ПРОТОННОМУ РОЗСІЮВАННІ

Г.І. Гах, А. Дбейсі, Д. Маршанд, В.В. Битєв, Е. Томасі-Густафсон

У Борнівському наближенні обчислено пружне розсіювання протонів на електронах, які знаходяться у стані спокою. Інтерес до цієї реакції обумовлено можливістю поляризації високоенергетичного антипротонного пучка та поляриметрією протонів високих енергій. У передбаченні однофотонного обміну одержані вирази для диференційного перерізу та поляризаційних спостережуваних. Виконані чисельні оцінки перерізу та коефіцієнтів кореляції спінів у широкій кінематичній області.