The recent progress in electron cyclotron current drive calculations with the adjoint technique is reviewed. The main attention is focused on such points as parallel momentum conservation in the like-particle collisions and the relativistic effects which are especially important for high-temperature plasmas. For moderate-temperature plasmas, also the finite collisionality effects become to be important. The effectiveness and accuracy of the developed numerical models are demonstrated by ray-tracing calculations.

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### INTRODUCTION

The adjoint technique proposed in Refs. [1, 2] is an advanced and convenient method for calculation of the current drive (CD) in plasmas. Moreover, only this technique is directly applicable for stellarators while the bounce-averaged Fokker–Planck model explicitly assumes axisymmetric configurations. Formally, the applicability of the adjoint technique is limited by the natural condition that the plasma response to the rf source in electron cyclotron resonance heating (ECRH) remains linear, i.e., when the density of the rf power is sufficiently low in comparison with the rate of collisional thermalization, but practically, the standard ECRH/ECCD easily satisfy this condition.

The central idea of the adjoint technique is exploiting a self-adjoint properties of the linearized collision operator to express the current through the adjoint Green's function, which is proportional to the linear plasma response in presence of parallel electric field that is formally identical to a solution of the Spitzer-Härm problem [3, 4]. This technique was subsequently applied to determine the current generated by NBI [1] and RF sources [2], for the electron cyclotron current drive (ECCD) in toroidal plasmas [5], and ECCD generated by asymmetric reflecting wall (passive ECCD) in toroidal plasmas [6]. At present, the adjoint technique is commonly used for calculations of ECCD in different ray- and beam-tracing codes [7-9].

The key point of the adjoint technique is the choice of the approach. Formally, in toroidal plasmas, the adjoint 4D drift-kinetic equation (3D in tokamaks) must be solved while taking into account such a factors as the geometry, the small but finite collisionality, conservation of parallel momentum, relativity, etc. Due to toroidicity, the problem can be reduced to an easily solvable level only for two opposite limits, the highly collisional (not interesting for toroidal plasmas) and low collisionality (“collisionless”) limit, where trapped particles play essential role. In the last limit, the trapped particles do not contribute to the current drive, but produce a non-negligible drag on the passing particles. This model, which is accepted as most relevant to ECCD calculations in toroidal plasmas [5-9] usually tends to underestimate the current drive efficiency as it neglects all effects due to (barely) trapped electrons.

Historically, for calculation of ECCD the linearized collision operator was simplified by high-speed-limit (hlsl) approach [10-12], which does not conserve parallel momentum in like-particle collisions. This approach is rather marginal even for moderate temperature plasmas and surely not sufficient for high temperature plasmas. The collisionless Spitzer problem with parallel momentum conservation (pmc) was considered in [5] and with relativistic effects taken into account for different physical mechanism of CD generation in [6, 13, 14].

There are some specific scenarios when existence of the barely trapped electrons can also be important for generation ECCD. The effect of small but finite collisionality in ECCD was considered in Refs. [15, 16] (see also the references therein).

In the present paper, recent progress in ECCD calculations is reviewed. A comparative analysis of the different approaches and their applicabilities is presented. Considering the ITER scenarios, the role of parallel momentum conservation in like-particle collisions in high-temperature plasmas is illustrated. Also the role of finite collisionality effects is discussed.

### 1. ADJOINT TECHNIQUE

In the ray-tracing codes, the toroidal current driven on the elementary arc-length of the ray-trajectory can be calculated. The key value is the current drive efficiency, \( \eta = \langle j_b / (\langle p_{\text{abs}} \rangle) \rangle \), where \( \langle j_b \rangle = -\langle \nabla \cdot \mathbf{u} (\mathbf{v}_R \cdot \nabla f_b) \rangle \) is the density of current and \( \langle p_{\text{abs}} \rangle = m_0 c^2 \langle \mathbf{u} \cdot \mathbf{u} (\mathbf{v}_R - \mathbf{v}) \mathbf{Q} \mathbf{R} \rangle \) is the density of the absorbed RF power, which are the functions only of the flux-surface label (here, \( \langle \cdot \rangle \) denotes averaging over the magnetic surface). The current driven by the RF source can be formally calculated (in linear approach) by solving the drift kinetic equation (DKE), which describes the line-ar response of electrons to the RF source,

\[
\nabla \cdot \nabla \delta f_b - C^{\text{lin}}(\delta f_b) = -\nabla \cdot \mathbf{u} \cdot \Gamma_{RF}(f_{\text{Maxwellian}}),
\]

where \( \mathbf{u} = \partial / \partial l \) is the derivative along the field-line, \( \delta f_b \) is the distortion of the electron distribution function from the Maxwellian, \( f_{\text{Maxwellian}} \), and \( C^{\text{lin}} \) the linearized collision operator, \( \Gamma_{RF} \) the quasi-linear diffusion flux in \( \mathbf{u} \)-space, \( \nabla \mathbf{u} = \partial / \partial \mathbf{u} \), and \( \mathbf{u} = \mathbf{v}_R \) the momentum per unit rest mass. Exploiting self-adjoint properties of \( C^{\text{lin}} \), it’s possible to express the current drive through the convolution of the RF source with the adjoint Green's function \( \chi', \) which is solution of the adjoint kinetic equation,
\[ v \cdot \nabla (\chi_{\text{coll}}) + C_{\text{lin}}(\chi_{\text{coll}}) = -\nu_{\text{e}} \frac{b}{v_{\text{th}}} b|_{\text{b}}, \]  
(2)

where \( b = B/B_{\text{max}} \). Find that \( \chi(-b) \) is formally identical (apart from normalization) to the generalized Spitzer function in toroidal geometry. The final expression for ECCD efficiency is

\[ \eta = -\frac{\epsilon_{\text{e}} v_{\text{th}}(b)}{m_{\text{e}} v_{\text{th}}(b^2)} \left\{ \frac{1}{d} u \cdot v_{\text{th}}(b^2) \right\}. \]
(3)

Being quite convenient for numerical representation, this form is the usual basis for ECCD calculations in the ray-tracing codes.

Most important for a precise calculation of ECCD is the model chosen for the operator \( C_{\text{lin}} \). Historically, the high-speed-limit (hsl) approach [10-12] was commonly accepted as the standard tool for calculation of ECCD. This approach is based on the assumption that only the supra-thermal electrons with \( v \gg v_{\text{th}} \) are involved in the cyclotron interaction. Unfortunately, in high-temperature plasmas, the energy range of electrons which make the main contribution to ECCD is not so far from the bulk, and the hsl approach fails even for highly oblique launch.

### 2. ANALYTICAL LIMITS

In Eq. (2), different time-scales exist: while the first term is characterized by the transit time, \( \tau_\| \propto t_\|^{-1} \), the collision operator is characterized by the collision time \( C_{\text{lin}} \propto \tau_\|^3 \). For ordering Eq. (2), we take into account that the ratio \( \tau_\| / \tau_\perp = \nu^* = \nu_{\text{e}}(u) \gamma R / u \) can vary significantly. When collisionality is high, \( \nu^* \gg 1 \), i.e. \( \tau_\| \ll \tau_\perp \), Eq. (2) reduces to the local problem. In this case, only the 1st Legendre harmonic of \( \chi \) is necessary, i.e. \( \chi = \xi_\| \chi_\| \) and \( \chi_\| = \frac{1}{2} \int \chi \xi d\xi \) (here, \( \xi = \nu / \nu_{\text{th}} \)). Then, instead of Eq. (2), it becomes sufficient to solve the 1D integro-differential equation for \( \chi_\| \),

\[ C_{\text{lin}}(\chi_\|) = -\nu_{\text{e}} \frac{u}{\gamma v_{\text{th}}} \]  
(4)

Here, \( C_{\text{lin}} \) is the 1st Legendre harmonic of the linearized collision operator. This is the classical Spitzer problem for calculation of the plasma conductivity which has been thoroughly studied for both non-relativistic [3] and relativistic [4]. This solution being applied to Eq. (3) gives the upper limit for current drive efficiency, but is of no practical relevance for hot plasmas in toroidal devices.

In the opposite (low collisionality or “collisionless”) limit, \( \nu^* \ll 1 \), the impact of the trapped particles is important. In this case, the dimensionality of the problem can also be reduced to 2D since the spatial dependence appears only due to the coupling between the pitch, \( \xi = \sigma \sqrt{1 - \lambda b} \) with \( \sigma = \pm 1 \) and the local magnetic field, \( b(l) \), through the (normalized) magnetic moment, \( \lambda \). By averaging Eq. (2) over the magnetic surface, the Vlasov operator is annihilated and the problem is reduced to a 2D equation

\[ \langle \chi(\xi) \rangle = -\sigma v_{\text{e}} \frac{u}{\gamma v_{\text{th}}} (\xi^2). \]
(5)

This is the basic model for calculation of CD in the different ray- and beam-tracing codes. The form chosen for the collision operator is very important for the solution. Note also that in this approach the problem is antisymmetric with respect to \( \xi \), and, as a consequence, only the antisymmetric part of the quasilinear operator contributes in the current drive calculated by the convolution Eq. (3).

Representing the solution of Eq. (5) as series of the eigenfunctions \( \Phi_\lambda(\xi) \),

\[ \chi_\| = \sigma \sum_{\lambda = \text{odd}} f_\lambda(\xi) \Phi_\lambda(\xi) \]
(6)

one can obtain exact solution, where the coefficients \( f_\lambda(\xi) \) must be calculated as solutions of the set of 1D integro-differential equations. Following this line, the numerical solver SYNCH was developed in Ref. [6].

It is possible also to simplify Eq. (5) without significant loss of accuracy. Since the pitch-scattering of electrons is the dominating process, all terms in the collision operator apart from the Lorentz term can be approximated by only the first Legendre harmonic [5],

\[ \chi_{\text{lin}}(\xi) = \xi \chi_{\|}\chi_{\|} + f_{\text{c}} f_{\text{e}} \chi_{\|} \]
(7)

where \( L(\xi) \) is the Lorentz operator and \( \chi_{\text{lin}}(\xi) \) is the first Legendre harmonic of the linearized \( e/e \) collision operator. In this approximation, Eq. (5) can be solved analytically [6],

\[ \chi_{\text{lin}}(\xi) = \sigma H(\lambda) K(\xi) \]
(8)

\[ f_{\text{c}} = 1 - f_{\text{e}} = \frac{3}{4} (\xi^2) |(1/\lambda b)| \]

where \( f_{\text{c}} \) and \( f_{\text{e}} \) are the fractions of circulating and trapped particles, respectively, and \( H(\lambda) \) is the Heaviside function. A function \( K(\xi) = \frac{1}{2} \int_0^\infty \chi d\xi \), which is proportional to the Spitzer function, must be found as the solution of a 1D integro-differential relativistic equation,

\[ C_{\text{lin}}(K) - f_{\text{c}} f_{\text{e}} \chi_{\|} K(\xi) = -\nu_{\text{e}} \frac{u}{\gamma v_{\text{th}}}. \]
(9)

In this approach, only the anti-symmetrical part of the quasilinear operator can contribute in the convolution Eq. (3) which gives the driven current. Nevertheless, when applied only to the collisionless limit, this approach is sufficiently accurate.

Recently, a very fast and sufficiently accurate numerical model for calculating the ECCD efficiency was developed [14]. This model is based on the solution of the integro-differential equation, Eq. (9), where the Spitzer function, \( K(\xi) \), is calculated with parallel momentum conservation in the \( e/e \) collisions. In order to simplify and accelerate the numerical solution, relativistic effects are accounted through a power expansion in \( \mu = T_e / m_{\text{e}} c^2 \).
3. LOCAL ECCD EFFICIENCY IN TOKAMAKS

For comparison of the considered approaches, the local dimensionless ECCD efficiency, \( \zeta^* = \frac{e^4}{2\pi\epsilon_0} \frac{n_e}{T_e} \left( \frac{1}{\langle J \rangle} \right) \), is calculated for \( hsl \) model, second harmonic with the different values of \( N_0 = 0.34, 0.42, \) and 0.5. The calculations were performed for a circular tokamak with a magnetic field \( B = B_0/(1+c \cos \theta) \) and \( c = r/R_0 = 0.2 \) (here, \( \theta \) is the poloidal angle). The plasma parameters, \( n_e = 2 \times 10^{19} \text{ m}^{-3}, T_e = 5 \text{ keV}, \) and \( Z_{eff} = 1 \), are chosen in such a way that the main contribution in current drive is generated by electrons with \( u \sim 2v_{th} \), where parallel momentum conservation starts to be important.

The results of calculations are shown in Fig. 1, where the local ECCD efficiency, \( \zeta^* \), is plotted as a function of the normalized magnetic field, \( n_Y = n \omega_e/\omega_c \). The data are calculated with different approaches for circular tokamak (see [8]).

For comparison of the results obtained by TRAVIS with the \( pmc \) model and by CQL3D, one finds that these results also coincide well. It can be pointed out that the accuracy of the \( pmc \) models [6, 14] for solving Eq. (5) is well confirmed by Fokker–Planck calculations.

In a \textit{gedanken} experiment [7] with a smaller launch angle and absorption close to the axis, i.e., a smaller fraction of trapped electrons, a strong discrepancy (current drive differs more than in two times) between the codes with a parallel momentum conserving collision term and the widely used \( hsl \) model was obtained. All collision terms in the simple \( hsl \) do not conserve parallel momentum leading to a smaller ECCD at moderate electron velocities in the wave absorption; see Ref. [8] for more details.

4. COMPARISON OF THE MODELS

The practical importance of performing accurate calculations of the current drive can be illustrated for quite typical ECCD scenarios in ITER. In Fig. 2, the results of ray-tracing calculations for the ITER reference scenario-2 are presented [8], where the angle-scan for the equatorial launcher is depicted. Three different codes were applied: TORAY-GA [17] with \( hsl \) model, TRAVIS [18] with both \( hsl \) and \( pmc \) models, and the Fokker–Planck code CQL3D [19]. For calculations by TRAVIS, both “exact” and “approximate” numerical solvers were applied, which are based on the fully relativistic splining and the weakly relativistic polynomial fit, respectively. One can see in Fig. 2 that the results obtained by both TORAY-GA and TRAVIS with the same \( hsl \) approach applied are in perfect agreement. On the other hand, a comparison with the results obtained with the \( pmc \) model shows that the \( hsl \) model significantly underestimates the ECCD efficiency (especially for small and moderate angles) and the convergence with \( pmc \) results is observed only for very high obliqueness. For the angles which are of the main interest, i.e. 20°...40°, the discrepancy between the \( hsl \) and \( pmc \) models varies from 10 to 30 %.

Fig. 2. ECCD efficiency as a function of the toroidal launch angle for ITER, equatorial launcher, obtained by different codes using different approaches (see [8]).

5. FINITE COLLISIONALITY EFFECTS

The case, when the plasma parameters and magnetic equilibrium are such that the effects of finite collisionality can be important for generation of the current drive, is most complex. In this case, dimensionality of Eq. (2) cannot be reduced and Spitzer problem must be solved accounting the “mixing” of the variables in real and momentum spaces. The code NEO2 [20] solves this problem for arbitrary collisionality and the local generalized Spitzer function necessary for ECCD can be calculated [15], which, contrary to the collisionless model Eq. (5) where only the invariants of motion necessary, depends also from the local variables.
Apart from solution obtained by NEO2, the Spitzer function can be obtained also by momentum-correction technique [21]. To do it, the mono-energetic DKE Eq. (2) is solved (in 3D for stellarators and in 2D for axisymmetric tokamaks) by the drift-kinetic equation solver (DKES) [22]. In DKES, the distribution function is expanded in a Fourier series with respect to the poloidal and toroidal angles in Boozer coordinates and in Legendre polynomials with respect to the pitch, \( p = \frac{v_{||}}{v} \).

In Fig. 3, the pitch dependencies of the mono-energetic Spitzer function are shown. Calculations were performed for the circular tokamak for the poloidal angle \( \theta = 90^\circ \) at the magnetic surface \( \varepsilon = r/R_0 = 0.13 \). This point is interesting since the up-down asymmetry [23, 15] is well seen. Plasma parameters are chosen in such a way that the finite collisionality effects would be sufficiently pronounced. One can find also that these results are in good agreement with those obtained by NEO2 (see [15]).

However, both NEO2 and DKES are too “expensive” and cannot be used as permanent tool ECCD calculations. Instead, the “off-set” approximation which significantly simplifies calculations of ECCD with finite collisionality was developed. Following Ref. [21], the “effective” circulating particle fraction can be expressed through the mono-energetic longitudinal conductivity, \( D_{\nu}(\nu^*) \),

\[
f_c^{\text{eff}}(\nu^*(u)) = \frac{3v_{\nu}(u)}{2u} D_{\nu}(\nu^*(u)) \langle B^2 \rangle ,
\]

which is equivalent to the conductivity normalized to its Pfirsch-Schlütter value. In the highly collisional limit, \( f_c^{\text{eff}}(\nu^* \to \infty) = 1 \), and \( f_c^{\text{eff}}(\nu^* \to 0) = f_c \) is the circulating particle fraction for collisionless limit, considered before. The generalized Spitzer function given by Eq. (7) can be reformulated then as [16]

\[
\chi^{\text{eff}}(u, \lambda; \sigma) = \sigma H^{\text{eff}}(\lambda; \nu^*(u)) K^{\text{eff}}(u) .
\]

Here, the equation for \( K^{\text{eff}}(u) \) coincides formally with Eq. (8) where \( f_c \) and \( f_{tr} \) must be replaced by \( f_c^{\text{eff}}(u) \) and \( f_{tr}^{\text{eff}}(u) = 1 - f_c^{\text{eff}}(u) \), respectively. Then

\[
H^{\text{eff}}(\lambda; \nu^*(u)) = \frac{f_c^{\text{eff}}(\nu^*)}{f_c^{\text{eff}}(\nu^*)} H(\lambda) + \frac{2}{3} h(1 - \lambda) \frac{\delta f_c^{\text{eff}}(\nu^*)}{f_c^{\text{eff}}(\nu^*)} ,
\]

with \( \delta f_c^{\text{eff}} = f_c^{\text{eff}} - f_c \) and \( H(\lambda) \) given by Eq. (8). This formulation being supported by pre-calculated the mono-energetic transport coefficient data-base has been in TRAVIS code implemented.

In Fig. 4, the results of ray-tracing calculations for O2-mode in stellarator W7-X are shown. From the absorption rate is seen that contributions from both passing and trapped electrons are important. Contrary to this, in ECCD only the passing electrons contribute. If the finite collisionality effects are not negligible, the barely trapped electrons, due to their diffusion into the passing domain, create an enhancement of the distribution function for the passing particles. As consequence, driving the current is changed for those electrons which are in resonance close to the boundary passing/trapped particles that is seen in Fig. 4 (right).
CONCLUSIONS

In this paper, the different approaches necessary for calculations of the electron cyclotron current drive in plasmas with low and finite collisionalities and recent progress in numerical modelling has been reviewed. All the formulations based on the adjoint technique are oriented for usage in ray- and beam-tracing codes, which at the present time are the main tools for numerical studies of ECRH and ECCD physics. The main attention was focused on parallel momentum conservation in the like-particle collisions which is of high importance in ECCD problem with parallel momentum conservation in like-particle collisions is of high importance in ECCD physics and may give a significant effect.

A models for ECCD calculations with small yet finite collisionalities has also been described. In particular, there is considered the numerical model which adds to the collisionless solution of the drift-kinetic equation for the parallel conductivity a simple “off-set” contribution only in the passing particle domain. The basic information needed is the effective “off-set” contribution only in the passing particle kinetic equation for the parallel conductivity a simple “off-set” contribution only in the passing particle domain. The basic information needed is the effective circulating particle fraction which is equivalent to the mono-energetic parallel conductivity coefficient normalised to the Pfirsch-Schlüter value; these values can be simply interpolated from databases of mono-energetic transport coefficients calculated, e.g., with the DKES code. Both for tokamaks and stellarators, this approach is very fast and can be directly implemented in ray-tracing codes (the test-version of the “off-set” model in already implemented in the code TRAVIS).

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РАЗВИТИЕ МОДЕЛЕЙ ДЛЯ ВЫЧИСЛЕНИЯ ЭЛЕКТРОННО-ЦИКЛОТРОННОГО ТОКА
УВЛЕЧЕНИЯ

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Сделан обзор недавних достижений в методах вычисления электронно-циклotronного тока увлечения. Основное внимание направлено на учёт сохранения продольного импульса в операторе столкновений, а также учёт релятивистских эффектов, существенных в высокотемпературной плазме. В плазме относительно небольшой температуры также эффекты конечной столкновительности становятся существенными. Эффективность и точность развитых численных моделей продемонстрированы результатами, полученными с помощью метода лучевых траекторий.

РОЗВИТКО МОДЕЛЕЙ ДЛЯ ОБЧИСЛЕНИЯ ЕЛЕКТРОННО-ЦИКЛОТРОННОГО СТРУМУ
ЗАХОПЛЕННЯ

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Зроблено огляд недавніх досягнень в методах обчислення електронно-циклотронного струму захоплення. Головна увага спрямована на облік збереження продольного імпульсу в операторі зіткнень, а також на облік релятивістських ефектів, значних у високотемпературній плазмі. У плазмі відносно невеликої температури ефекти кінцевої зіткненості також стають істотними. Ефективність та точність розвинутих моделей продемонстрировано результатами, які були отримані за допомогою методу променевих траекторій.