Space Plasma

ALFVEN WAVE INSTABILITY BASED ON TEMPERATURE ANISOTROPY WITH NON-MAXWELLIAN DISTRIBUTION FUNCTION

M.N.S. Qureshi, Sundas Saeed, H.A. Shah

Department of Physics, GC University, Lahore 54000, Pakistan

Space observations by numerous satellites reveal that the distributions often possess non-Maxwellian characteristics such as high energy tails or flat top (broad shoulders) in the profile of distribution functions. Distributions with high energy tails are well modelled by family of kappa type distribution. However, when distributions contain flat tops with or without high energy particles, generalized \((r,q)\) distribution function is the best choice. In general the spectral index \(r\) corresponds to the flat part of the distribution and \(q\) to the high energy tail in the profile of the distribution function. By following the kinetic theory, we employ this distribution function to study the Alfvén waves in anisotropic plasma and found that Alfvén wave can grow when there is temperature anisotropy in plasma. Instability conditions are then studied for different temperature ratios by using the plasma parameters observed downstream the bow shock by CLUSTER.

PACS: 52.27Aj; 52.35Hr; 52.35Qz

INTRODUCTION

Both in astrophysical and space plasmas Alfvén waves are found to play major role in the dynamics of many planetary processes like in the dynamics of Earth’s magnetosphere such as in the plasmasphere\([1]\), in central plasma sheet\([2]\), in tail lobes\([3]\), in plasma sheet boundary layer\(\text{PSBL}\)\([4-6]\), in magnetotail lobe region\([7]\), in magnetotail reconnection region\([8]\), and in solar corona\([9]\). In the solar wind Alfvén waves accelerate the charged particles like protons, electrons and helium by transferring its energy to these particles through resonant interactions. Being feasible source of parallel electric fields Alfvén waves are also believed to accelerate the electrons in auroral region which further generates fascinating auroras\([8,9]\). Investigating the heating mechanism of solar atmosphere through Alfvén waves, efforts are made to produce controlled fusion reactions in laboratory plasmas.

Alexandrova et al.\([10]\) observed Alfvén waves downstream of a quasi-perpendicular shock by Cluster and showed that anisotropies in the proton and alpha particles temperature can destabilize the Alfvén waves. Similar observations have also been reported by Anderson et al.\([11]\) and Gary et al.\([12]\). In this paper, we employed a non-Maxwellian distribution such as generalized \((r,q)\) distribution function and study the instability criteria for Alfvén waves based on the temperature anisotropy observed downstream of the Earth’s bow shock.

1. DISTRIBUTION FUNCTION AND DISPERSION RELATION

The generalized \((r,q)\) distribution function has the form:

\[
\begin{aligned}
\psi_s &= \frac{\Gamma}{m} \left[ \frac{3}{5} \frac{(q-1)^{-\frac{3}{5}r+\frac{3}{5}q}}{\Gamma(q-1)} + \frac{3}{5} \frac{(q-1)^{-\frac{3}{5}r+\frac{3}{5}q}}{\Gamma(q-1)} \right] \times \\
\psi_\perp &= \frac{\Gamma}{m} \left[ \frac{3}{5} \frac{(q-1)^{-\frac{3}{5}r+\frac{3}{5}q}}{\Gamma(q-1)} + \frac{3}{5} \frac{(q-1)^{-\frac{3}{5}r+\frac{3}{5}q}}{\Gamma(q-1)} \right] 
\end{aligned}
\]

(2)

(3)

Where \(T||\) and \(T\perp\) are parallel and perpendicular temperatures, \(\Gamma\) is the usual Gamma function, \(m\) is the mass of the particle, \(r\) and \(q\) are the spectral indices satisfying the conditions \(q>1\) and \((1+r)>5/2\). The distribution \(f_{(r,q)}\) is the generalization of Maxwellian and Lorentzian distributions and in the limiting cases when \(r=0\) and \(q\rightarrow\kappa+1\), \(f_{(r,q)}\) reduces to generalized Lorentzian and when \(r=0\) and \(q\rightarrow\infty\) the distribution \(f_{(r,q)}\) reduces to the Maxwellian distribution\([13]\). In general, high energy tail in the profile of distribution \(f_{(r,q)}\) increases as the spectral index \(q\) decreases and flat top (or shoulders) in the profile of distribution increases as spectral index \(r\) increases. Distribution functions with flat tops are often observed in space plasma\([14]\) and these are closely resembles the distribution function given in Eq. (1) for appropriate values of \(r\) and \(q\).

By following kinetic theory and standard text book procedure we derive the following relations for left hand and right hand circularly polarized waves, as

\[
L = 1 - \sum \frac{\partial^2}{\partial\psi_\perp} f_{(r,q)} \left( \frac{\partial}{\partial\psi_\perp} + \frac{\partial}{\partial\psi_s} \right) \frac{\partial^2}{\partial\psi_\perp} 
\]

(4)

\[
R = 1 - \sum \frac{\partial^2}{\partial\psi_\perp} f_{(r,q)} \left( \frac{\partial}{\partial\psi_\perp} + \frac{\partial}{\partial\psi_s} \right) \frac{\partial^2}{\partial\psi_\perp} 
\]

(5)

where \(L\) and \(R\) correspond to left and right hand circularly polarized waves, respectively, and the operator \(\bar{G}\) is

\[
\bar{G} = \left( 1 - \frac{k_1}{k_0} \right) \frac{\partial}{\partial\psi_\perp} + \frac{k_1}{k_0} \frac{\partial}{\partial\psi_s} 
\]

(6)

Now we consider that plasma consists of electrons and ions and using the distribution (1) in Eqs. (4) and (5), we obtained the general dispersion relation of \(L\) and \(R\) waves. Then by applying the condition for low frequency waves and considering high beta plasma, i.e. \(\omega<<\Omega_e<<\Omega_i\) and \(\beta_i>1\), respectively the argument of plasma dispersion functions \(\bar{G}\), becomes
We derive the dispersion relation of Alfvén waves in anisotropic plasma modeled by generalized \((r,q)\) distribution function, as

\[
\omega^2 (\nabla l - A l_{2}) - k_i^2 v_A^2 \pm \omega \frac{(\nabla l - A l_{2})(k_l v_\parallel)^2}{\nabla l} = 0
\]

where

\[
\left(\begin{array}{c}
\xi_l \approx \frac{\Omega_l}{k_l v_\parallel} \\
\xi_l \approx \frac{\Omega_l}{k_l v_\parallel} \gg 1
\end{array}\right)
\]

We have used the condition \(v_A^2 \ll c^2\) and neglect the electron terms as electrons do not take part in damping/growth due to their high frequency.

2. NUMERICAL RESULTS AND CONCLUSION

We solve the dispersion relation (8) for real and imaginary parts of frequencies and found that L-wave grows when there is temperature anisotropy, however, R-wave shows damping. In Fig. 1 we plot the growth rates of L-wave for the observed temperature anisotropies \(T_l/T_{||} = 2.5\), \(T_l/T_{||} = 0.5\) and \(T_l/T_{||} = 0.55\) from the downstream of a quasi-perpendicular shock[10]. In Fig. 1 (upper panel), we can see that as \(q\) increases for a fix value of \(r\), the growth rate increases and approaches to Maxwellian growth but remains lower than the Maxwellian (shown in dotted line). So, the high energy tail corresponds to small values of \(q\) reduces the temperature anisotropy effect and thus the number of resonant particles. In Fig. 1 (lower panel), we can see that as the value of \(r\) increases for a fix value of \(q\) damping rate decreases. The increase in the value of \(r\) which corresponds to the increment in the flat top in the distribution function reduces the temperature anisotropy effect and thus the number of resonant particles. Fig. 2 (upper panel) is plotted for different values of temperature ratio \(T_l/T_{||}\) when the other ratios are \(T_d/T_{||} = 0.5\) and \(T_e/T_{||} = 0.55\). We can see that as the perpendicular ion temperature increases the growth rate increases. In Fig. 2 (lower panel), growth rate is plotted for different temperature ratio \(T_e/T_{||}\) when the other anisotropies are \(T_d/T_{||} = 0.5\) and \(T_e/T_{||} = 0.55\). It can be noted that as the perpendicular electron temperature increases the growth rate decreases.

In this paper, growth of Left handed circularly polarized Alfvén wave is studied which is introduced by proton and electron temperature anisotropies in high plasma \(\beta\) (in this study we have taken \(\beta = 2\)). It is observed that the anisotropy in the ion temperatures
increases the growth rate whereas the anisotropy in the electron to ion temperatures decreases the growth rate. Therefore, the growth in the Alfven wave is due to the ion temperature anisotropies as observed in high beta plasmas at the earth’s bow shock[10,15]. Moreover, in the real situations where we use non-Maxwellian distributions, the growth rate is suppressed as compared to the Maxwellian growth rate.

Acknowledgement: This research was supported by the HEC grant no. 20-1886/R&D/10.

REFERENCES


Article received 25.10.12