PLASMA WAKEFIELD EXCITATION PROVIDING HOMOGENEOUS FOCUSING OF ELECTRON BUNCHES

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Wakefield plasma lens, which focuses all relativistic electron bunches of the sequence identically and uniformly, are investigated analytically and by numerical simulation. The necessary conditions of such lens operation are the followings: length of the bunches is \(\xi_0=q\lambda/2\), \(q=1, 2, \ldots\) (\(\lambda\) is the plasma wavelength), the distance between bunches is \(\Delta x=p\lambda, p=1, 2, \ldots\) All bunches have the same charge and the 1-st bunch has a half of this charge. It is shown that with the exception of 1-st bunch all other bunches are occurred in zero longitudinal wakefield and in uniform along bunch length focusing radial wakefield. In the case of inhomogeneous longitudinal distribution of electron bunch density the middle of bunches are focused slower in comparison with their edges.

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INTRODUCTION

The focusing of relativistic electron bunches by wakefield, excited in plasma, is very interesting and important (see, for example, [1-2]). The focusing of bunches by wakefield, excited in plasma by resonant sequence of relativistic electron bunches, is inhomogeneous. In [3] the mechanism of focusing by plasma wakefield, in which all bunches of sequence are focused identically and uniformly, has been proposed and numerically investigated. We analytically and numerically investigate by 2.5D code lcode [4] the longitudinal distribution of radial wake-force, excited by sequence of lengthy electron bunches in inhomogeneous plasma.

1. RESULTS OF SIMULATION

At first the wakefield excitation by resonant sequence of bunches is considered. We find the longitudinal \(E_z\), radial \(E_r\) fields and focusing force \(F_r\) in the areas of location of bunches. For this purpose we use the theory developed in [1]. For the bunch of permanent density, the length of which equals \(\xi_0=\lambda/2\), we get that \(E_z\) and \(E_r\) are proportional to \(Z_{II}(\xi)\) and \(Z_{II}(\xi)\) where

\[ Z_{II}(\xi)=(2/k)\sin(k\xi), \quad Z_{II}(\xi)=-(2/k)\cos(k\xi). \]

(1)

\(E_z\) in 1-st half of bunch equals

\[ Z_{II}(\xi)=\left(\frac{1}{k}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\xi \cos(k\xi-x_0)\cos(\xi)=1, \quad \xi=0, \quad \xi=\pi. \]

(2)

One can see it is, as well as observed, in 2 times less than amplitude of the wakefield after 1-st bunch.

Now we derive the fields into the 2nd resonant bunch,

\[ Z_{II}(\xi)=\left(\frac{1}{k}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\xi \cos(k\xi-x_0)\cos(\xi) \]

(3)

\(Z_{II}(\xi)\) changes from \(Z_{II}(\xi)=0\) to \(Z_{II}(\xi)=2.5k\) and then again \(Z_{II}(\xi)=3k\). Thus \(Z_{II}(\xi)\) changes from \(Z_{II}(\xi)=0\) to \(Z_{II}(\xi)=6k\), reaching zero in the 1-st half of bunch, where \(\cos(x_0)=1/2, \quad x_0=2\pi n=3\pi/2\). I.e. longer (in \(\pi-x_0\)) appearance (see Fig. 1).

In resonant case 1-st bunch only focuses, and for other bunches (if length of bunch equals \(\xi_0=\lambda/2\)) back fronts (more long) are in larger focusing field \(E_r\), than (more short) 1-st fronts are in defocusing \(E_r\). I.e. this lens is inhomogeneous.

In [3] wakefield plasma lens has been numerically simulated with homogeneous focusing force for the sequence of bunches, lengths of which equal \(\xi_0=\lambda/2\), with the 1-st bunch, charge of which in two times less than charges of other bunches \(Q_1=Q/2, i=2, 3, \ldots\) distance between bunches equals \(1.5\lambda\). From Fig. 1 one can see that in this case dips of electron plasma density, in which bunches are localized, wide and shallow, and humps are narrow and high. From Fig. 1 one can see that \(E_z=0\) in the areas of location of bunches.

\[
\begin{align*}
\text{Fig. 1. Longitudinal distribution of density } n_b & \\
& \text{of sequence of bunches (yellow), longitudinal wakefield } E_z & \\
& \text{of bunches (red), } \delta n (\text{grey}) \text{ and coupling factor } <E_z> & \\
& \text{of bunches with } E_z & \\
& \text{excited by sequence of 10 bunches. } t & \\
& \text{is normalized on } \omega_d.
\end{align*}
\]

From Fig. 2 one can see that in the areas of bunches location \(F_r\) does not approximately depend on a longitudinal coordinate.

\[
\begin{align*}
\text{Fig. 2. } n_b & (\text{yellow}), \text{radiuses } r_b (\text{blue}), E_z (\text{red}), \text{radial force } F_r (\text{dark blue}), \text{magnetic-field } H_0 (\text{pale blue})
\end{align*}
\]

In the areas of location of bunches the radial field of their volume charge is compensated by radial field, appearing as a result of shift of several number of plasma electrons from the areas of bunches location. \(E_z=0\) in the areas of bunches location, except 1-st one (see Fig. 2).

Thus screening of bunches takes place due to that wide \((\approx \lambda)\) dips of plasma electron density \(\delta n<0\) and narrow \((\approx \lambda/2)\) humps of \(\delta n>0\) appear (see Fig. 1). I.e.
plasma in the vicinity of bunches is positively charged. Due to inertness of electrons screening of 1st bunch is realized only in its end, where dip of density \( \delta n_e < 0 \) is not flat, as for 2nd bunch, but approximately three-cornered.

For the bunch of permanent density and \( \xi_b = \lambda/2 \) longitudinal \( Z_{\lambda/2}^{0,2}(\xi) \) and transversal \( Z_{\lambda/2}^{0,2}(\xi) \) fields are equal (1). Now we derive the wakefield into 2nd bunch \( Z_{\lambda/2}^{0,2}(\xi) = (2/k) \sin(k \xi) + 2 \int_0^\xi d\xi_0 \cos[k(\xi - \xi_0) + 3\pi] = 0 \) (4)

As well as in numeral simulation, \( E_z = 0 \) got in the areas of bunches location.

Now we consider wakefield plasma lenses for the sequence of bunches for three cases of their lengths at the interbunch gap equal to \( \Delta \xi = \lambda \). The bunch-precursor of half-charge is used. We connect it with next (1st) bunch. 1st case: \( \xi_b = \lambda \) (Fig. 3), 2nd case: \( \xi_b = 1.5 \lambda \) (Figs. 4, 5), 3rd case: \( \xi_b = 2 \lambda \) (Figs. 6–8). From Figs. 3–5, and 7, 8 one can see that in the areas of bunches location \( E_z = 0 \), \( F_r \approx \text{const.} \)

Now we consider the wake plasma lens for the bunches of identical charge with lengths equal to \( \lambda/2 \) and the interbunch gap equal to \( \lambda \) (Figs. 9–11).

If all bunches are identical and they are placed through \( 1.5 \lambda \), all bunches are focused identically, but inhomogeneously along bunches (see Figs. 10, 11). Odd number bunches are decelerated, and the even number bunches are accelerated (see Fig. 9).

We consider the wake plasma lens for the bunches of identical charges with lengths and interbunch gap equal to \( \lambda \) (Figs. 12, 13). Both longitudinal and radial fields between bunches are equal to zero, and the fields in every bunch are equal to

\[
Z_{\lambda/2}^{0,2}(\xi) = (1/k) \sin(k \xi), Z_{\lambda/2}^{0,2}(\xi) = (2/k) \sin^2(k \xi/2).
\]

The radial field has one sign along bunch. Therefore bunches are focused, however on fronts of bunches \( F_r = 0 \). The longitudinal field is oscillated between \( E_0 \) in the forehand of bunch and \( -E_0 \) in the back-end of bunch. Therefore the front half of bunch is decelerated, and the back is accelerated.

The results of numeral simulation for the case of length of bunches, equal to \( 2 \lambda \), are presented in Fig. 14.

We consider the case of electron density distribution along bunch according to \( \sin^2(k \xi) \) (Figs. 15, 16). Bunches are placed through \( 1.5 \lambda \). The charge of 1st bunch is less than charges of other bunches in 2 times. From Fig. 16 one can see that 1st front of bunch is accelerated, and the back is decelerated.
It has been shown that all bunches of the sequence can be focused identically and uniformly under the conditions: all bunches lengths are equal to \( q \lambda /2 \), \( q=1, 2, 3, \ldots \), the distance between them equals \( p \lambda \), \( p=1, 2, 3, \ldots \), the charge of 1-st bunch equals a half of the charges of the other bunches. It has been shown that only 1-st bunch is in finite \( E_z \neq 0 \). Other bunches are in zero longitudinal electrical wakefield \( E_z=0 \). Hence the 1-st bunch loss the energy for excitation of wakefield, which amplitude is constant along the sequence. Radial wake force \( F_r \) in regions, occupied by bunches, is constant along bunches length.

In the case of inhomogeneous longitudinal distribution of electron bunch density the middle of bunches are focused slower than edges.

If all bunches are identical and they are placed over 1.5\( \lambda \), then bunches are focused identically, but not uniformly along bunch length.

REFERENCES

3. K. V. Lotov, V. I. Maslov, I. N. Onishchenko, O. M. Svistun. Homogeneous focusing of electron bunch sequence: all bunches are identical and they are placed over 1.5\( \lambda \), then bunches are focused identically, but not uniformly along bunch length.

CONCLUSIONS

It has been shown that all bunches of the sequence can be focused identically and uniformly under the conditions: all bunches lengths are equal to \( q \lambda /2 \), \( q=1, 2, 3, \ldots \), the distance between them equals \( p \lambda \), \( p=1, 2, 3, \ldots \), the charge of 1-st bunch equals a half of the charges of the other bunches. It has been shown that only 1-st bunch is in finite \( E_z \neq 0 \). Other bunches are in zero longitudinal electrical wakefield \( E_z=0 \). Hence the 1-st bunch loss the energy for excitation of wakefield, which amplitude is constant along the sequence. Radial wake force \( F_r \) in regions, occupied by bunches, is constant along bunches length.

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