We obtain an analytical estimate of space-charge limiting (SCL) current of relativistic charged-particle beam propagating in infinitely long grounded coaxial drift tube with dielectric inserts in the strong magnetic field approximation. The received analytical estimate is compared with numerical calculations of SCL current.

PACS: 84.30.Jc

INTRODUCTION

Recently, there has been some activity in studying space-charge limiting (SCL) current of a relativistic charged-particle beam in infinite coaxial geometry with dielectric inserts lining the outer and inner conductors of the drift tube [1–12].

In our paper in the approximation of strong magnetic field, we consider the charged-particle beam propagating in the grounded infinitely long coaxial drift tube with the dielectric insert of permittivity 1 lining the inner conductor and that of permittivity 2 lining the outer conductor. We find an analytical estimate of the SCL current for such a beam and compare it with results of numerical calculations.

MAIN PART

In approximation of the strong magnetic field the scalar potential created by the charged-particle beam in the infinitely long grounded coaxial drift tube with dielectric inserts lining the inner and outer conductors is described by the Poisson equation [1–3]:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \varphi}{\partial r} \right) = \begin{cases} 0, & r_i \leq r < r_{d1}, r_{d1} \leq r < r_i, \\ r_o \leq r < r_{d2}, & r_{d2} \leq r \leq r_2, \\ -\frac{4\pi I_0}{c\beta_1(r_o^2 - r_i^2)}, & r_i \leq r < r_o, \end{cases}
\]

with the boundary conditions

\[
\begin{align*}
\varphi(r_i) &= 0 = \varphi(r_2) = 0, \\
\varphi(r_i - 0) &= \varphi(r_i + 0), \\
\frac{\partial \varphi}{\partial r}(r_i - 0) &= \frac{\partial \varphi}{\partial r}(r_i + 0), \\
\varphi(r_{d1} - 0) &= \varphi(r_{d1} + 0), \\
\frac{\partial \varphi}{\partial r}(r_{d1} - 0) &= \frac{\partial \varphi}{\partial r}(r_{d1} + 0), \\
\varphi(r_{d2} - 0) &= \varphi(r_{d2} + 0), \\
\frac{\partial \varphi}{\partial r}(r_{d2} - 0) &= \frac{\partial \varphi}{\partial r}(r_{d2} + 0),
\end{align*}
\]

where \( r_i \), \( r_2 \), \( r_i \), \( r_o \), \( r_{d1} \), and \( r_{d2} \) are the radii of the inner and outer tube coaxial conductors, inner and outer radii of charged-particle beam, outer radius of inner dielectric insert, and inner radius of outer dielectric insert, respectively; \( I_0 \) is the injection current;

\[
\beta_1 = \sqrt{\frac{1 - \gamma_0^2}{\gamma_0^2 - q^2 \varphi / m_q^2}},
\]

is the longitudinal dimensionless velocity of the beam in the units of speed of light \( c \) in vacuum; \( \gamma_0 \) and \( \gamma_0 = \gamma_0 (1 + \gamma_0^2 \beta_1^2)^{-1/2} \) are the initial relativistic factor and dimensionless longitudinal beam kinetic energy; \( \beta_{1,0} \) is the transversal dimensionless initial velocity of the beam; \( m_q \) and \( q \) are the mass and charge of charged-particles, respectively.

We can found analytically the solution to Eq. (1) with boundary conditions (2) under the assumption that the longitudinal beam velocities, \( \beta_i \), are equal to their respective injection values, \( \beta_{i,0} \), i.e. in the approximation of constant longitudinal velocities [10]

\[
\begin{align*}
\varphi_1(r), & \quad r_i \leq r < r_{d1}, \\
\varphi_2(r), & \quad r_{d1} \leq r < r_i, \\
\varphi_3(r), & \quad r_i \leq r < r_o, \\
\varphi_4(r), & \quad r_o < r < r_{d2}, \\
\varphi_5(r), & \quad r_{d2} \leq r \leq r_2,
\end{align*}
\]

where

\[
\begin{align*}
\varphi_1(r) &= \frac{I_0 \ln(r / r_i) G_{d1}}{\varepsilon_c \beta_{i,0} Z}, \\
\varphi_2(r) &= \frac{I_0 G_{d2}}{c\beta_{i,0} Z} \left( \ln(r / r_i) - \frac{\varepsilon_c - 1}{\varepsilon_c} \ln(r_{d1} / r_i) \right), \\
\varphi_3(r) &= \frac{I_0 G_{d2}}{Z} \left( \ln(r / r_i) - \frac{\varepsilon_c - 1}{\varepsilon_c} \ln(r_{d1} / r_i) \right) \\
&\quad - \frac{r^2 - r_i^2}{r_o^2} + \frac{2r^2 - 2r_i^2 r_o}{r_2 - r_i^2}, \\
\varphi_4(r) &= \frac{I_0 G_{d2} - 2Z}{c\beta_{i,0} Z} \left( \ln(r / r_o) + \frac{\varepsilon_c - 1}{\varepsilon_c} \ln(r_2 / r_{d2}) \right), \\
\varphi_5(r) &= \frac{I_0 G_{d2} - 2Z}{\varepsilon_c \beta_{i,0} Z} \ln(r / r_o).
\end{align*}
\]
\[ Z = \ln\left(\frac{r_2}{r_1}\right) - \frac{\varepsilon_1 - 1}{\varepsilon_1} \ln\left(\frac{r_{d1}}{r_1}\right) - \frac{\varepsilon_2 - 1}{\varepsilon_2} \ln\left(\frac{r_2}{r_{d2}}\right), \]
\[ G_{d2} = G - 2 \frac{\varepsilon_2 - 1}{\varepsilon_2} \ln\left(\frac{r_2}{r_{d2}}\right), \]
\[ G = 1 + 2 \ln\left(\frac{r_2}{r_1}\right) + \frac{2r_1^2}{r_{d1}^2 - r_1^2} \ln\left(\frac{r_1}{r_o}\right). \]

We can easily find the radial position
\[ r_{ext} = r_1 \left(1 + G_{d2} \frac{r_{d1}^2 - r_1^2}{2r_1^2 Z}\right)^{1/2}, \tag{5} \]

at which the dimensionless potential \( f(r) = q\varphi(r)/(m_q c^2) \) has the extremal value
\[ f_{ext} = q\varphi(r_{ext})/(m_q c^2). \]

In Fig. 1 dimensionless distributions of normalized scalar potential \( f(r) \) are shown for different values of permittivities of the inner, \( \varepsilon_1 \), and outer, \( \varepsilon_2 \), dielectric linings. Results of (linear) analytical calculations and nonlinear numerical simulations presented in Fig. 1 show that greater values of the permittivity of dielectric inserts lead to the reduction of extremal value of the dimensionless scalar potential \( f(r_{ext}) \). Also one can see that for symmetric geometry this influence is more significant than for an asymmetric one.

We can easily obtain the analytical estimate of SCL current:
\[ I_{coax}^{\text{ext}} = I_d \frac{\gamma_0}{\gamma_0 W_d}, \tag{6} \]

where \( I_d = m_q c^3 / q \) is the Alfvén current (\( I_d \approx 17.05 \) kA for the electrons),
\[ W_d = \frac{G_{d2}}{Z} \left[ \ln\left(\frac{r_{ext}}{r_1}\right) - \frac{1}{2} \right] + \frac{2r_1^2}{r_{d1}^2 - r_1^2} \ln\left(\frac{r_1}{r_{ext}}\right). \]

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**REFERENCES**


Article received 26.10.12