MICROSCOPICAL THEORY, STRUCTURE OF SUPERFLUID COMPONENT AND ELEMENTARY EXCITATION SPECTRUM OF THE NON-RELATIVISTIC BOSE-LIQUID ^4He

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In frame of selfconsistent microscopical model of the superfluid non-relativistic Bose liquid ^4He with suppressed due to many-particle effects single-particle condensate and intensive pair coherent condensate the ab initio calculation of the spectrum $E(p)$, based on the realistic models for the pair interaction potentials $V(r)$, which has a finite oscillating Fourier-component, was presented. It was shown that oscillating character of the renormalized Fourier component of the fitting "hard sphere" and "semitransparent sphere" pair interacting potentials leads to non-monotonic behavior for the momentum dependence of the renormalized mass operators and, as a consequence, to emerging of the roton minimum in the quasiparticles spectrum $E(p)$, which exactly connected with the first largest negative minimum of the renormalized interaction Fourier-component of a pair of bosons.

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Investigation of a core insight of physical phenomena, namely the demonstration of the quantum laws on the macroscopical level, always was one of the primary interests for P.I. Fomin scientific activities. Strong scientific intuition, unique approach to analysis of experimental data, which were inherent to P.I. Fomin, allow him successfully use ideas and principles of quantum field theory for the creation of a relativistic theory of the jet quasar activity, applicable for quasars and radio galaxies; theoretical estimations for some parameters of the jets, which were done using this theory, gives a good correspondence with experimental data.

Superfluidity phenomenon in the Bose-liquid ^4He (discovered by P.L. Kapitsa in 1938) and Fermi-liquid ^3He (discovered by D. Osheroff, R. C. Richardson and D. M. Lee in 1972), together with superconductivity of electron Fermi-liquid in metals, also were a in the list of interest for P.I. Fomin. Along with the construction of the relativistic theory of superfluidity P.I. Fomin were extremely interested in the investigation of the structure of the Bose liquid ^4He superfluid component, which was reflected in our common with P.I. Fomin paper dedicated to this problem (Fomin P.I., Pashitskij E.A, Vilchynskyy S.I., On pair-wise character of superfluid condensate in helium-II. (1997) Low Temperature Physics, v.23, N12, p.12671271). Here, starting from the empirical data for the dynamics of ^3He atoms impurity in superfluid ^4He (what suggests an abnormal high effective mass of ^4He) was proposed that helium atoms form a pairs below the -point and the role of a paired condensate in ^4He were discussed. Further, for investigation of the superfluidity P.I. Fomin has been using methods and ideas of solid state physics and physics of crystals and E.P. together with S.V. developed the self-consistent model of the superfluid (SF) state of a Bose liquid with strong interaction between bosons and a weak single-particle Bose-Einstein condensate (BEC).

In series of papers renormalized perturbation theory was used, it was built on combined hydrodynamic variables with analytic normal and anomalous self-energy functions and a nonzero SF order parameter, proportional to the density of the SF component. On the base of this theory a closed system of nonlinear integral equations for the normal and anomalous self-energy parts were obtained. Unlike in the Bogoliubov theory of a quasi-ideal Bose gas, were small parameter is the ratio of the number of supracondensate excitations to the number of particles in an intensive BEC, the ratio of the BEC density to the total particle density of the Bose liquid was used as a small parameter of the model. Quasiparticle spectrum, obtained within this approach, is in a good agreement with experimental spectrum of elementary excitations in superfluid ^4He. And, as it was shown, the roton minimum in the spectrum is associated with negative minimum of the Fourier component of the pair interaction potential.

This article holds a brief discussion on current status in the microscopic theory of superfluidity of the

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non-relativistic Bose liquid and review of the results obtained by the authors during the last decade.

In the basis of the theory, which describes superfluidity and superconductivity phenomena, lays an appearance, as additional complex macroscopic order parameter, of the wave function of bosons or fermionic Cooper pairs, which stays in the same quantum state, in other words - appearance of the coherent condensate is the foundation of the superfluid and superconductivity phenomena. In Bose-systems such condensate appears because of direct accumulation of bosons in the ground state, but in the Fermi-systems due to the formation of the Cooper pairs of fermions.

Great amount of experimental and theoretical investigations performed over last 80 years allowed to achieve a high level of understanding of the properties of non-relativistic superfluid state in a Bose liquid 4He. Most important achievements in the theory of superfluidity reached at a phenomenological level, in particular, for the description of the properties of superfluid ^4He (so-called Ne-II) on the basis of two-fluid Landau hydrodynamics, according to which the superfluid helium can be divided into two components - the superfluid component with density ρ_s and velocity v_s that describes the non-dissipative motion of quantum fluid and normal component with density ρ_n and velocity v_n which describes dissipative flow of the gas of excitations (phonons and rotons). Numerical calculations of thermal conductivity and viscosity of superfluid helium, obtained according to this theory, are in good agreement with experiment. One of the greatest achievements of the phenomenological approach to the phenomenon of superfluidity is that Landau, basing on the temperature dependence of heat capacity of superfluid ^4He on the basis of his superfluidity criteria for the quantum liquids predicted the form of the superfluid helium elementary excitation spectrum with a linear (phonon) dispersion law $E(p) \approx pc$ for the small momentum $p \to 0$ and so called "roton" minimum for $p \neq 0$ Later this form of the spectrum was brilliantly confirmed in the experiments for the scattering of the slow neutrons in the liquid helium.

But in the frame of phenomenological theory of superfluidity exact calculation (from first principles) of the elementary excitations spectrum in superfluid Bose liquid with strong interaction between particles may not be possible. This is the task for the microscopic theory of superfluidity of Bose liquids. Theoretical study of the properties of superfluid Bose liquid ^4He at the microscopic level hampered by a number of fundamental problems associated with the strong interaction between bosons and complex quantum structure of effective coherent condensate, which is the main part of the superfluid component, unlike of almost ideal Bose gas, where this role is played by single-particle Bose Einstein condensate (BEC).

The first microscopic theory of superfluidity, based on a model of weakly non-ideal Bose gas, was proposed by N.N. Bogoliubov [1] more than 50 years ago. The main advantage of this theory is rejecting of the standard perturbation methods, which are based on series expansions on the weak interaction coupling constant. The insight of the theory is the idea of existence in the superfluid system the intensive BEC - namely that the number of particles in the single BEC is macroscopically large and close to the total number of particles (per unit volume). Consequently we can neglect the fact that operators of creation and annihilation of particles with zero momenta are not commute, diagonalize the initial Hamiltonian of the system and find an expression for the renormalized quasiparticle spectrum.

Bogoliubov theory has been improved further and in the most of these improved models of superfluidity by the selection of variational parameters one can achieve a good agreement between theoretical and experimental spectrum of elementary excitations in superfluid ^4He for a certain range of the momentum. But such agreement is more coincidental and is not true, because, as it was shown in the later experimental studies, a part of single-particle BEC in the superfluid ^4He is small and ranges from 2 to 10 percent, which is the opposite to a condition of weak non-ideality of the Bose-gas in the Bogoliubov theory. Therefore in order to give an adequate microscopic description of the Bose-liquid superfluid properties the most promising is the Greens function method, which for the first time was used in the papers of S.T. Beliaev and widely developed in the future.

Analysis of the modern experimental and theoretical works testifies to the fact that the study of unique phenomena of superfluidity of helium ^4He is far from the end. There are number of contradictions between theory and experiment, which are related to both the hydrodynamics of superfluid Bose liquid ^4He, and the form of quasi-particle spectra, which have not yet satisfactory explanation in frame of the microscopic theory. We shortly focus on the most significant contradictions between the fundamental principles and conclusions of a microscopic theory of superfluidity and experimental data (for more details see [2]).

Firstly, experiments on inelastic scattering of slow neutrons in liquid helium, which confirmed the form of proposed by Landau curve for the spectrum of elementary excitations, indicated a weak temperature dependence of the spectrum, as the consequence the size of the "roton" gap $\Delta_r = 8.65\ K$ at minimum of $E(p)$, that determines the critical velocity of normal fluid, changes only to the value of $\Delta_r = 5.24\ K$ near the temperature of the phase transition from superfluid to normal state, i.e. the so-called λ-point $T_\lambda = 2.17\ K$. At the same time, for the spectrum of quasiparticles in the electronic Fermi liquid in superconductors superfluidity criterion holds for the phase below critical temperature T_c, when there is a finite energy gap in the spectrum, and fail in the normal state at $T > T_c$ when gap in the spectrum becomes zero.

Moreover, obtaining of the quasiparticle spectrum from the "first principles" also fails. Using the latest calculation methods such as Monte-Carlo it is possible to obtain a good agreement between calculated and experimental data almost for all range of momentum, but remains unclear the physical reason for the roton minimum in the quasiparticle spectrum of the Bose-liquid.

Secondly, in the superfluid Bose-liquid, unlike the Bose gas, single-particle Bose-condensate (SPC) due to strong interaction between bosons must be significantly impoverished by the particles of zero energy and momentum (i.e. "depleted" condensate) even at temperature. Analysis of the experimental data from neutron scattering [3] and quantum evaporation of the helium atoms [4] shows that superfluid ^4He at low temperature in the Bose-condensate state contain less than 10% of the full density of liquid ^4He, whereas density of the superfluid component at the temperatures according to classical measurements of the viscosity of superfluid helium [5], almost equal to the liquid helium density. This means that against widespread conviction superfluidity of the Bose-liquid ^4He cannot be connected only with Bose-condensation phenomena. Microscopical structure of the superfluid component, as it was for the first time shown in [6], should have more complicated quantum nature in the form of effective coherent condensate.

These contradictions are somehow reduced to the question of the quantum structure of the superfluid component in liquid ^4He below λ-point. This is central question for the construction of the consistent microscopical theory of superfluidity of Bose-liquids.

Let us shortly focus on the results of investigation of the superfluid component microscopical quantum structure and the elementary excitation spectrum in the superfluid ^4He, which based on the conclusion that superfluid component at $T \to 0$ is the coherent superposition of weak BEC and intensive pair coherent condensate (PCC). This assumption is based on the fact that, as was already indicated, that according to numerous experimental data on inelastic scattering of neutrons and quantum evaporation of ^4He atoms the maximal density of single-particle BEC in the Bose liquid ^4He, even at low temperatures $T \ll T_\lambda$ does not exceed 10% of total density of liquid ^4He, although density of the superfluid component $\rho_s \to \rho$ when $T \to 0$. Such a low density of the BEC, appeared due to the strong interaction between atoms of ^4He, shows that quantum structure of the superfluid condensate in He-II with "excess" density ($\rho_s - \rho_0 \gg \rho_0$) requires a more detailed and deep investigation. Our approach, based on the microscopic model of superfluidity of the Bose-liquid with depleted BEC and intensive PCC, proposed by Pashitskij E.A. and Nepomnyashij Yu.A. [6]. Such PCC may occur at sufficiently high "effective attraction" between bosons in some ranges of momentum due to the effects of the quantum diffraction of Bosons in the process of their interaction, and is similar to attraction between fermions near the Fermi surface. The ratio of BEC density to the full density of the liquid was taken as a small parameter (i.e. $\rho_0/\rho \ll 1$), in contrast to the Bogoliubov [1] theory were small parameter is taken as ratio of the number of supracondensate excitations to the number of particles in the intensive BEC. Due to a small BEC density it is possible to derive a closed self-consistent system of integral equations for the normal and anomalous self-energy parts by cutting an infinite series of perturbation theory and keeping only a first order terms by small parameter ρ_0/ρ:

$$\sum_{11}(\varepsilon,p) = n_0\Lambda(\varepsilon,p)\,V(\varepsilon,p) + n_1 V(0) + \psi_{11}(\varepsilon,p),$$

$$\sum_{12}(\varepsilon,p) = n_0\Lambda(\varepsilon,p)\,V(\varepsilon,p) + \psi_{12}(\varepsilon,p),$$

here $V(p)$ – the Fourier component of the inoculating two-particle interaction potential of bosons; $V(\varepsilon,p) = V(p)\left[1 - V(p)\Pi(\varepsilon,p)\right]^{-1}$ – the renormalized ("shielded") due to many-particle collective effects Fourier component of non-local interaction; $\Pi(\varepsilon,p)$ – bosonic polarization operator; $n_1 = n - n_0$ – the number of "supra-condensate" particles. Functions $\psi_{11}(\varepsilon,p)$ and $\psi_{12}(\varepsilon,p)$, taking into account poles of single-particle Green functions, are defined by the following expressions:

$$\psi_{11}(\varepsilon,p) = -1/2\int d^3k\,(2\pi)^{-3}\,\Gamma(\varepsilon,p,k,E(k))\times$$
$$\times V(p-k,\varepsilon-E(k))\left[A(k,E(k))\,E^{-1}(k) - 1\right],$$

$$\psi_{12}(\varepsilon,p) = -1/2\int d^3k\,(2\pi)^{-3}\times$$
$$\times\Gamma(\varepsilon,p,k,E(k))\,E^{-1}(k)V(p-k,\varepsilon-E(k))\times$$
$$\times\left[n_0\Lambda(k,E(k))\,V(k,E(k)) + \psi\Lambda(k,E(k))\right],$$

here $\Gamma(\varepsilon,p,k,E(k))$ and $\Lambda(\varepsilon,p) = \Gamma(\varepsilon,p,0,0)$ – the vortex functions, which describes many-particle correlations; function $A(p,E(p))$ defined as

$$A(p,E(p)) = n_0\Lambda(p,E(p))\,V(p,E(p)) +$$
$$p^2/2m + \psi_{11}(p,E(p)) - \psi_{12}(0,0) + \psi 0,0,$$

and $E(p)$ – quasiparticle spectrum, which in our approximation have the following form

$$E(p) = \Big(A^2(p,E(p)) -$$
$$-\left[n_0\Lambda(p,E(p))\,V(p,E(p)) + \psi_{12}(p,E(p))\right]^2\Big)^{-1/2} +$$
$$+1/2\left(\psi_{11}(p,E(p)) - \psi_{12}(p,E(p))\right).$$

From the last expression, because of analiticity of the $\psi(\varepsilon,p)$ functions follows the fact that quasiparticle spectrum is acoustic at $p \to 0$ and its structure at $p \neq 0$ is strongly dependent from the properties of the renormalised two-particle interaction between bosons. In case when BEC is absent ($n_0 = 0$) integral equation for the $\psi_{12}(\varepsilon,p)$ function became homogeneous and degenerate over the phase of the function. Thus it is similar to the momentum space Bethe-Goldstone integral equation for a pair of particles with zero binding energy, which has a nontrivial solution $\psi \neq 0$ only in case of attraction $V(p) < 0$ in a sufficiently wide region of values of the transmitted momentum p. Follow this analogy, function $\psi_{12}(\varepsilon, p)$ can be considered as order parameter for bosonic PCC, which describe condensation of bosonic pairs in the momentum space (identically to the Cooper condensate of the fermion pairs).

Different fenomenological and semi-impirical potentials are used to describe interaction between helum atoms in a real space. Thay show strong repulsion at small distances and a weak Van der Vaals attraction at small distances. However, there are a few reasons why these potentials are inappropriate in case of microscopical field theory of superfluidity of the Bose-liquids, which formulated in terms of momentum space Green functions:

1. These are the fitting potentials and intended to agree experimental data with theoretical predictions using a large number of arbitrary parameters.

2. These potentials are divergent or characterized by the non-analytical exponential behavior of the radial dependence at $r \to 0$, which can be obtained on the base of the microscopical quantum theory of the atomic interactions. This non-analyticity can lead to uncontrolled nonphysical features in the behavior of the Fourier components of the potential.

3. Potentials, which describe interaction of two helium atoms, generally speaking, are not suitable for describing a pair interaction of the helium atoms in the quantum Bose-liquid, where the average interparticle distance is of the order or less then its De Broglie wave length at the temperatures $T < 1K$. In this case quantum diffraction and microscopic quantum coherence, as it is known, plays the main role. Therefore, effective pair interaction potential in the Bose-liquid (more precisely, pseudo-potential) may be very different from the interaction potential in the vacuum.

In this connection, pair interaction between bosons was chosen in the form of regularized repulsion potentials in models of "hard" and "semitransparent spheres", which Fourier components, due to diffraction of particles of one another, are oscillating and alternating functions of transmitted momentum and can be determined by the spherical Bessel functions of zero and first orders.

Many-particle collective effects in Bose liquid lead to a significant renormalization of the pair interaction, which determines the normal and anomalous self-energy parts (Fig. 1). An important feature of the renormalized interaction is that, as was shown in [7], in those areas of the phase volume (ε, p) , were the real part of the bosonic polarization operator $Re\Pi(p,\omega)$ is negative, repulsion become weaker (when $V(p) > 0$) and attraction effectively increased (when $V(p) < 0$) (see Fig. 1). The key point in the behavior of the renormalized (screened) potential $V(p, E(p))$, as it was shown by the numerical calculations in [8], is that for all values of momentum $p > 0$ the real part of the bosonic polarization operator is negative ($Re\Pi(p,\omega) < 0$, upper corner of Fig. 1) if quasiparticle spectrum is stable with respect to decay on a pairs of quasiparticles.

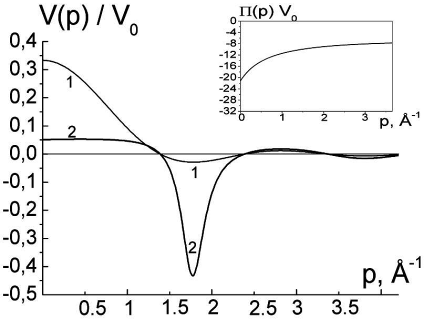

Fig. 1. *Fourier components of the non- renormalized (curve 1) and renormalized (curve 2) interactions in the model of "semitransparent spheres". In the upper corner depicted the momentum dependence of the real part of the bosonic polarization operator*

In the numerical calculations [8], based on the Fourier-component of the "hard spheres" potential $V(p) = V_0 \sin(pa)/pa$, were used a simplified model of the renormalized potential in the form

$$V(p,\omega) = V_0 \sin(pa)/(pa + \alpha \sin(pa)),$$

here $\alpha = -V_0\Pi = const$, and Π – the value of the polarization operator on the mass surface $\omega = E(p)$ averaged over the momentum p which was taken as negative constant value in case if quasiparticle spectrum not decay. Resulting spectrum qualitatively agrees with experimental spectrum E_{exp} in superfluid ^4He (Fig. 2), but numerical correspondence of the minimal and maximal values of quasiparticle energies are not satisfactory. In addition, calculated value of the speed of the first (hydrodynamical) sound $c = 2.08 \times 10^4\ cm/s$ appeared to be lower than experimental value $c = 2.36 \times 10^4\ cm/s$, and total concentration of particles is higher - $n = 2.57 \times 10^{22}\ cm^{-3}$; the value of the BEC particle concentration is lower $n_0 = 3\%$ then obtained from experiment.

Further calculations, carried in papers [9, 10], show that oscillating pseudo-potential in the model of "semitransparent spheres"

$$V(p) = V_0 (\sin(pa) - pa\cos(pa)) / (pa)^{-3}$$

is more appropriate than "hard sphere" potential both for the stability of the spectrum and its correspondence to the empirical spectrum in ^4He. In the "semitransparent spheres" model the explicit momentum dependence of the polarization operator $\Pi(p,\omega)$ is used.

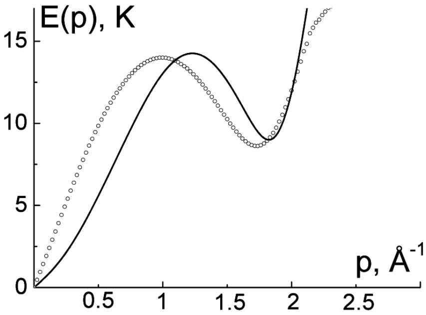

Fig.2. *Solid curve – spectrum obtained in the model of "hard spheres", dot curve – experimental data [11]-[14]*

Iterative numerical calculations of the self-energy and bosonic polarization operator, the two-particle order parameter and the quasiparticle spectrum at $T = 0$ allowed to find a conditions when theoretical spectrum $E(p)$ is in a good agreement with experimental spectrum of elementary excitations in ^4He. The roton minimum of the quasiparticle spectrum $E(p)$ in the Bose liquid, as it was shown in [10], clearly associated with the first negative minimum of the Fourier-component renormalized potential. The only fitting parameter in these calculations was the amplitude of the starting pseudo-potential with the parameter $a = 2.44\ A^{-1}$, which is equal to the twice of a quantum radius of the ^4He atom. For the calculations we take an experimental value of the BEC density $n_0 = 9\%\ n = 1.95 \times 10^{22}\ cm^{-3}$ [3, 4].

As a result, after numerical calculations it was possible to get a quite satisfactory agreement of the theoretical $E(p)$ and experimental $E_{exp}(p)$ spectrum under condition $p < 2.5\ A^{-1}$ (Fig. 3, curve 1). In calculations of $E(p)$ the only fitting parameter was taken in order to satisfy a condition that quasiparticle phase velocity $E(p)/p$ at $p \to 0$ coincide with hydrodynamical speed of sound $c_1 \cong 236\ m/sec$ in the liquid ^4He, which corresponds to the value $U_0 = V_0/4\pi a^3 = 1552\ K$ for the amplitude of the "semitransparent spheres" potential at $a = 2.44\ A^{-1}$. In the range $p < 2.5\ A^{-1}$ theoretical spectrum $E(p)$ lies slightly above $E_{exp}(p)$. This is, most likely, due to the fact that vertex function $\Gamma(k,p)$ which decay with increasing of the momentum p, was taken as constant $\Gamma = 1.5$. This value was obtained from the exact asymptotic of the polarization operator $\Pi(0,0) = n/mc^2$. In this connection vertex function on the interval $2.1\ A^{-1} < p < 3.8\ A^{-1}$ was approximated by the linear function, which slowly changed from $\Gamma = 1.5$ to $\Gamma = 1.1$. Spectrum, obtained within this approximation (see Fig. 3, curve 2), is in good agreement with experimental curve for all range of the momentum.

Self-consistency of the given model is confirmed by the value of the total particle density $n_{teor} = 2.12 \times 10^{22}\ cm^{-3}$, which is close to the experimental value of the particle density $n = 2.12 \times 10^{22}\ cm^{-3}$ of the liquid ^4He (when $n_0 = 9\%n$). On the other hand, independent calculation of the over-condensate density of particles n_1 for the given parameters gives a value of about $93\%n$, which also agrees with experiment under condition that BEC density is determined with accuracy of $\pm 1\%$.

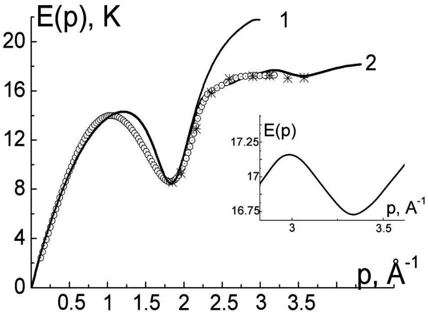

Fig.3. *Curve 1 – theoretical quasiparticle spectrum $E(p)$, obtained with a model of the "semitransparent spheres" for the constant value of the vertex function $\Gamma = 1.5$. Curve 2 – theoretical quasiparticle spectrum $E(p)$, obtained with a model of the "semitransparent spheres" for the weakly decaying (from $\Gamma = 1.5$ to $\Gamma = 1.1$) vertex function, on the interval $2.1\ A^{-1} < p < 3.8\ A^{-1}$. In both cases the value of the fitting parameter is $U_0 = V_0/4\pi a^3 = 1552\,K$. Circles show experimental spectrum, obtained by inelastic neutron scattering in the liquid ^4He [11]-[14], stars – show the results of the experiment [15] beyond the roton minimum $2\ A^{-1} < p < 3.6\ A^{-1}$. Inserted box shows weak oscillations of the spectrum with maximum $E_{max} = 17.2\,K$ at $p = 2.99\,A^{-1}$ and minimum $E_{max} = 16.7\,K$ at $p = 3.39\,A^{-1}$*

Thus, proposed in [6] model of superfluid Bose liquid with a suppressed BEC and intensive PCC, which is based on renormalized perturbation field theory with combined variables [16], allow to cut an infinite series by the low BEC density and obtain "truncated", closed system of nonlinear integral equations for the self-energy parts $\sum_{ij}(\varepsilon,p)\,,(j = 1,2)$. In this way it is possible to build a self-consistent microscopic theory of superfluid Bose liquid and calculate "from first principles" the spectrum of elementary excitations $E(p)$, based on realistic models for the pair interaction potential $V(r)$, with oscillating finite Fourier component. For a wide class of repulsion potentials, which characterized by the infinite ("hard sphere" model) or finite ("semitransparent spheres" model) jump or inflection point (jump of the first derivative) of the $V(p)$ Fourier components, and are the alternating functions of transmitted momentum p which form an effective attraction $V(p) < 0$ at $p \neq 0$ at certain areas of the momentum space, which is not associated with the presence of Van der Waals forces and have a quantum mechanical diffraction nature. This attraction significantly enhanced by the collective effects of renormalization ("screening") of the initial interaction, which are described by the bosonic polarization operator. Enhancement of the "attraction" occurs because of the negative value of the bosonic polarization operator on the "mass shell" $\omega = E(p)$ in the all range of the momentum where the quasiparticle spectrum $E(p)$ is stable against decaying processes. Rather strong "attraction" between a pairs of bosons that arise in the momentum space create an intensive PCC, which together with a weak BEC form an entire coherent condensate. Such condensate is the microscopical basis for the superfluid component of Bose liquid.

On the other hand, the oscillating character of the renormalized Fourier component of the potential leads to a non-monotonic behavior of the momentum dependencies of the mass operators $\sum_{ij}(\varepsilon, p), (j = 1, 2)$ and, as a result, to the appearance of the roton minimum in the spectrum of quasiparticles $E(p)$, which definitely connected with first deepest negative minimum of the renormalized Fourier component of the potential. Thus, for sufficiently large values of the amplitude of the initial potential V_0 quasiparticles excitation spectrum become unstable in some domain of momenta $p \neq 0$ where $E^2(p) < 0$.

For the conclusion it is necessary to emphasize that mentioned property of the polarization operator $\Pi(p, E(p)) < 0$ is typical only for Bose-systems, where single-particle and many-particle spectrum are coincide and have a common zero energy reference point, unlike Fermi-systems, where single-particle excitation spectrum due to the Pauli principle is counted from the Fermi energy. Therefore, corresponding effective increasing of the negative values of the polarization operator cannot take place for the Fermi-liquid ^3He, so formation of the Cooper pairs is possible only for non-zero values of the orbital momentum and a real Van der Waals attraction between fermions. Probably, this is the reason for the three order difference between critical temperatures of superfluid transition in ^4He and ^3He.

Finally, we briefly focus on the calculation of the temperature dependence of superfluid component density based on the model of the coherent structure of the condensate of the Bose liquid ^4He. Temperature dependence of the superfluid component density in this model is given by the following expression [10]

$$\frac{\rho_s(T)}{\rho} = \frac{\psi_0(T)}{\tilde{V}(0)n}\left[1 - \frac{\psi_s(T)}{\tilde{V}(0)n}\right]^{-1}, \quad \rho = mn,$$

here

$$\psi_0(T) = -\frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3}\tilde{V}(\mathbf{q})\times$$
$$\left[\frac{A_{12} - D_{12}}{c_1\mathbf{q}}\coth\left(\frac{c_1\mathbf{q}}{2T}\right) + \frac{B12}{c_2\mathbf{q}}\coth\left(\frac{c_2\mathbf{q}}{2T}\right)\right],$$

$$\psi_s(T) = -\frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3}\tilde{V}(\mathbf{q})\times$$
$$\left[\frac{D_{12}}{c_1\mathbf{q}}\coth\left(\frac{c_1\mathbf{q}}{2T}\right) + \frac{B12}{c_2\mathbf{q}}\coth\left(\frac{c_2\mathbf{q}}{2T}\right)\right].$$

In the last expression $\tilde{V}(\mathbf{q})$ - the Fourier component of the renormalized due to many-particle effects a pair interaction potential. For the calculation of the temperature dependences we use model potentials ("hard sphere" and "semitransparent sphere" potential), Aziz potential and expressions for the velocity of the first and second sounds in liquid ^4He and they temperature corrections, obtained in [18] from the solutions of a kinetic type equations:

$$E_1^2(\mathbf{q}) = c_1^2\mathbf{q}^2 = c_0^2\left(1 + \frac{29}{16}\frac{\rho_n}{\rho}\right)\mathbf{q}^2,$$

$$E_2^2(\mathbf{q}) = c_2^2\mathbf{q}^2 = \frac{c_0^2}{3}\left(1 - \frac{33}{8}\frac{\rho_n}{\rho}\right)\mathbf{q}^2.$$

Temperature dependence of the superfluid component, calculated in [10] and [19], is given on Fig. 3. For the comparison result obtained by Ginzburg V.L. [20], presented on the same figure.

So, field theoretical methods, which have been used in order to explain the maxon-roton spectru leads to a good agreement between theory and an experiment only for the case $T = 0$, but these attempts do not seem to be fully complete for case of nonzero temperature. This case requires futher investigations.

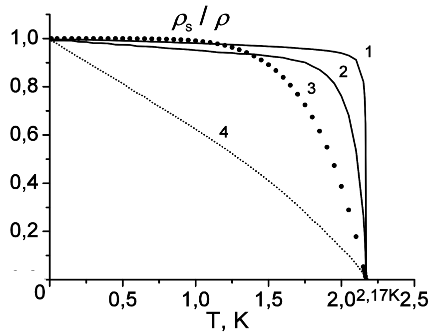

Fig.4. *Temperature dependencies of the Bose- liquid ^4He superfluid density obtained in [10] (curve 1); and in [19] (curve 2); experimental data obtained by Andronikashvilli E.L. [21] (curve 3); result of Ginzburg V.L. [20] (curve 4)*

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МИКРОСКОПИЧЕСКАЯ ТЕОРИЯ, СТРУКТУРА СВЕРХТЕКУЧЕЙ КОМПОНЕНТЫ И СПЕКТРА ЭЛЕМЕНТАРНЫХ ВОЗБУЖДЕНИЙ В БОЗЕ-ЖИДКОСТИ He-II

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В рамках самосогласованной микроскопической модели сверхтекучей нерелятивистской бозе-жидкости ^4He с подавленным за счет взаимодействия одночастичным и интенсивным парным конденсатами произведен расчет с первых принципов спектра элементарных возбуждений $E(p)$ исходя из реалистичных моделей для потенциалов парного взаимодействия $V(r)$, которые имеют конечную осциллирующую фурье-компоненту. Показано, что в рамках моделей "твердых"и "полупрозрачных сфер"осциллирующий характер перенормированой фурье-компоненты потенциала парного взаимодействия бозонов приводит к немонотонному поведению импульсных зависимостей перенормированных массовых операторов и, как следствие, к появлению в спектре квазичастиц $E(p)$ ротонного минимума, который однозначно связан с наиболее глубоким первым отрицательным минимумом фурье-компоненты перенормированного парного взаимодействия бозонов.

МІКРОСКОПІЧНА ТЕОРІЯ, СТРУКТУРА НАДПЛИННОЇ КОМПОНЕНТИ ТА СПЕКТРУ ЕЛЕМЕНТАРНИХ ЗБУДЖЕНЬ В БОЗЕ-РІДИНІ He-II

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В рамках самоузгодженої мікроскопічної моделі надплинної нерелятивістської бозе-рідини ^4He з подавленим за рахунок взаємодії одночастинковим і інтенсивним парним конденсатами обчислено з перших принципів спектр елементарних збуджень $E(p)$, виходячи з реалістичних моделей для потенціалів парної взаємодії $V(r)$, які мають скінчену осцилюючу фур'є-компоненту. Показано, що в рамках моделей "твердих"та "напівпрозорих сфер"осцилюючий характер перенормованої фур'є-компоненти потенціалу парної взаємодії бозонів приводить до немонотонної поведінки імпульсних залежностей перенормованих масових операторів і, як наслідок, до появи в спектрі квазічастинок $E(p)$ ротонного мінімуму, який однозначно пов'язаний з найбільш глибоким першим від'ємним мінімумом фур'є-компоненти перенормованої парної взаємодії бозонів.