TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN DIELECTRIC RESONATOR BY SEQUENCE OF RECTANGULAR ELECTRON BUNCHES WITH LINEAR GROWTH OF CHARGE

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Transformation ratio has been derived in the case of the wakefield excitation in a dielectric resonator by sequence of rectangular in longitudinal direction (the same charge along the bunch) electron bunches, whose charge is profiled according to linear dependence. Long periodic sequence of electron bunches has been built to increase the number of accelerated electrons. In this sequence, the short trains of rectangular in longitudinal direction profiled bunches – drivers alternate by accelerated "high-current" bunches. This long sequence provides a large transformation ratio and the same for all bunches decelerating wakefield. The coupling of transformation ratio with reduction rate of field after witness and coupling of transformation ratio with witness charge and driver charge in this sequence have been derived.

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INTRODUCTION

The maximum energy to which electrons can be accelerated at some energy of the electron driver-bunches of sequence, which excite wakefield in dielectric resonator, is determined by the transformation ratio [1, 2]. The transformation ratio, defined as ratio $R=\frac{E_2}{E_1}$ of the wakefield E_2, which is excited in dielectric resonator accelerator by sequence of the electron bunches, to the field E_1, in which an electron bunch is decelerated, is considered with charge shaping of rectangular in longitudinal direction (equal charge along bunch length) bunches according to linear law along sequence [3, 4], so that ratio of charges of bunches of sequence equals 1:3:5: … [3, 4]. The bunch length equals to half of wave-length $\Delta\xi_b=\lambda/2$. The choice of such length of bunches is determined by the necessity to provide not only large R but high gradient wakefield too excited by sequence of N bunches. The porosity between bunches is multiple of wave-length $\delta\xi=p\lambda$, p=1, 2, ... A next bunch is injected in the resonator, when the back wavefront of wakefield pulse, excited by previous bunches, is on the injection boundary ($z=0$). A next bunch leaves the resonator, when the first wavefront of wakefield pulse, excited by previous bunches, is on the end of the resonator. Then wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized. For all major bunches the decelerating wakefield is small, identical, but inhomogeneous along their length. Then one can provide a large transformation ratio R. But several conditions should be satisfied for this purpose. The wakefield and transformation ratio have been derived after N-th bunch.

Also long periodical sequence, composed of accelerated bunches – witnesses and short trains of electron bunches – drivers, exciting wakefield, has been derived. Each bunch of these short trains is rectangular in longitudinal direction (equal charge along bunch length) and the charges of consecutive bunches grow. The connection of transformation ratio with the reduction rate of the wakefield after witness and connection of the transformation ratio with the witness charge and driver charge in this periodical sequence have been derived analytically.

1. TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN DIELECTRIC RESONATOR BY SEQUENCE OF RECTANGULAR ELECTRON BUNCHES WITH LINEAR GROWTH OF CHARGE

In this paper the transformation ratio R is investigated theoretically. In many cases transformation ratio can be concluded to the ratio of maximum accelerating wakefield, experienced by witness bunch, to the maximum slowing down wakefield, experienced by driver bunches. In [5] the expression for the wakefield, excited in a dielectric resonator by the sequence of electron bunches, each of which is a infinitely thin ring, has been derived. We consider injection of bunches with length $\Delta\xi_b$, equal to the half of wavelength $\Delta\xi_b=\lambda/2$, the charge of which is profiled according to linear law along the sequence of bunches, in the dielectric resonator of length L. The choice of such length of bunches is determined by the necessity to provide not only large R but high gradient excited wakefield too.

So the charge density of sequence of rectangular bunches is distributed according to Fig. 1.

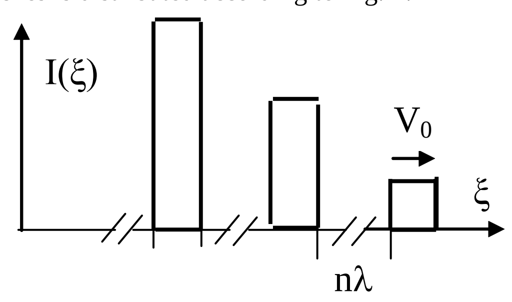

Fig. 1. The current distribution of sequence of rectangular bunches, charge of which is shaped

$$n_b(z,t)=n_{b0}(2N-1),\ N\geq 1,\ 0<V_0\left(t-T\left(N-1\right)\right)-z<\Delta\xi_b,$$

$$T(N\text{-}1)<t<T(N\text{-}1)+\frac{(L+\Delta\xi_b)}{V_0}. \quad (1)$$

Then the ratio of charges Q_N of consecutive bunches equals to known values 1:3:5 … .

A next (N+1)-th bunch is injected in the resonator, when the back wavefront of wakefield pulse, excited by previous N bunches, is on the injection boundary ($z=0$) (Fig. 2).

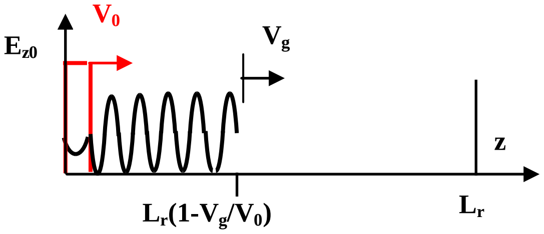

Fig. 2. A schematic of the wakefield pulse, excited by previous N bunches, when (N+1)-th bunch is injected in the resonator

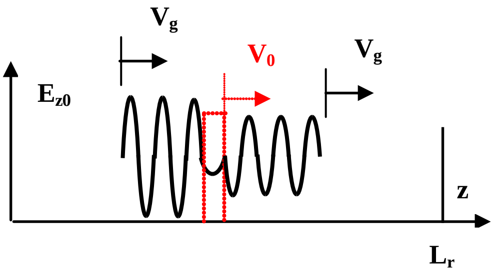

Fig. 3. An approximate view of the wakefield pulse, excited by previous N bunches and excited by (N+1)-th bunch, when (N+1)-th bunch is in the middle of the resonator

An approximate view of the wakefield pulse, excited by previous N bunches and excited by (N+1)-th bunch, when (N+1)-th bunch is in the middle of the resonator, is shown in Fig. 3. Excited longitudinal decelerating wakefield E_z is small and identical for all bunches but non-uniform along them. Then one can provide a large transformation ratio R.

A next (N+1)-th bunch leaves the resonator, when the first wavefront of wakefield pulse, excited by (N+1) bunches, is on the end of the resonator ($z=L$) (Fig. 4).

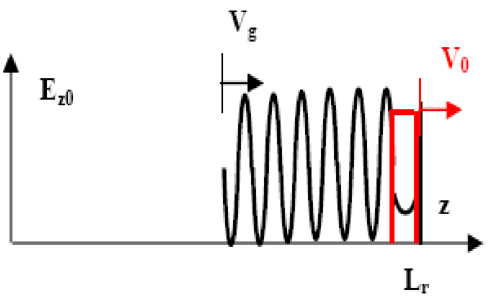

Fig. 4. A schematic of the wakefield pulse, excited by (N+1) bunches, when (N+1)-th bunch leaves the resonator

For achieving a large transformation ratio R several conditions should be satisfied. Namely, we choose the length of the resonator L, the group velocity V_g, the bunch repetition frequency ω_m and the wave frequency, which satisfy the following equalities

$$T=\frac{2L_r}{V_g}=\frac{2\pi}{\omega_m}=\frac{\pi q}{\omega_0},\ q=1,3,\ldots,\ \frac{V_g}{V_0}=\frac{4L_r}{q\lambda}. \quad (2)$$

Then for the selected length of the resonator and q, equal $\frac{L}{\lambda}=4$ and $q=20$ group velocity should be equal $\frac{V_g}{V_0}=0.8$. V_0 is the beam velocity. For $\frac{L}{\lambda}=5$ and $q=32$ group velocity should be equal $\frac{V_g}{V_0}=0.625$.

Thus, all the next bunches after the first one begin to be injected in the resonator (on the boundary $z=0$), when the trailing edge of the wakefield pulse, created by the previous bunches, is located at the point $z=0$. At this moment the leading edge of the wakefield pulse, located at the distance from the injection boundary, equal to $L\left(1-\frac{V_g}{V_0}\right)+\Delta\xi_b$ (see Fig. 2), is located at the distance $L\left(\frac{V_g}{V_0}\right)\text{-}\Delta\xi_b$ from the end of the resonator ($z=L$).

Again injected bunch reaches the end of the resonator together with the leading edge of the wakefield pulse, created by the previous bunches. Then wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized.

At wakefield pulse excitation by the 1-st bunch the wakefield in the whole resonator (one can derive, using [2, 5]) within the time $0<t<\frac{L_r+\Delta\xi_b}{V_0}$ is proportional to

$$Z_{\parallel}(z,t)=\left(\frac{1}{k}\right)[\theta(V_0t\text{-}z)\text{-}\theta(V_0t\text{-}\Delta\xi_b\text{-}z)]\sin\left[k\left(V_0t\text{-}z\right)\right]+$$
$$+\left(\frac{2}{k}\right)\left[\theta(V_0t\text{-}\Delta\xi_b\text{-}z)\text{-}\theta(V_gt\text{-}z)\right]\sin\left[k\left(V_0t\text{-}z\right)\right]. \quad (3)$$

The 1-st term is the field inside of the 1-st bunch, the 2nd term is the wakefield after the 1st bunch. Thus, after 1-st bunch T_E=2.

Inside the 2-nd bunch $0<\xi=V_0(t\text{-}T)\text{-}z<\Delta\xi_b$ the wakefield on the times $T<t<T+\frac{L_r+\Delta\xi_b}{V_0}$ is proportional to

$$Z_{\parallel}(z,t)= \quad (4)$$
$$\left[\theta(V_0(t\text{-}T)\text{-}z)\text{-}\theta(V_0(t\text{-}T)\text{-}\Delta\xi_b\text{-}z)\right]k^{-1}\sin\left[k\left(V_0(t\text{-}T)\text{-}z\right)\right].$$

The decelerating field into the 2-nd bunch equals to decelerating field into the 1-st bunch.

After the 2-nd bunch $\xi=V_0(t\text{-}T)\text{-}z>\Delta\xi_b$ on the times $T<t<T+(L_r+\Delta\xi_b)/V_0$ it excites wakefield, which is proportional to

$$Z_{\|}(z,t)=\left[\theta(V_0\left(t-T\right)-\Delta\xi_b-z)-\theta(V_g\left(t-T\right)-z)\right]\times$$
$$\times 4k^{-1}\sin\left[k\left(V_0\left(t-T\right)-z\right)\right]. \tag{5}$$

Thus, after 2-nd bunch T_E=4.

At wakefield pulse excitation by the N-th bunch the wakefield in whole resonator within the time $T\left(N-1\right)\le t\le T\left(N-1\right)+\frac{\left(L+\Delta\xi_b\right)}{V_0}$, $T=\frac{2L}{V_g}$, is proportional to

$$Z_{\|}(z,t)=$$
$$\left(\frac{1}{k}\right)\left[\theta(V_0(t-T(N-1))-z)-\theta(V_0(t-T(N-1))-\Delta\xi_b-z)\right]\sin\left(k\xi\right)$$
$$\left[\theta(V_0(t-T(N-1))-\Delta\xi_b-z)-\theta(V_g(t-T(N-1))-z)\right]\times$$
$$\times\left(\frac{2N}{k}\right)\sin\left(k\xi\right)+$$
$$+\left[\theta(V_g(t-T(N-1))+L_r\left(1-\frac{V_g}{V_0}\right)-z)-\theta(V_0(t-T(N-1))-z)\right] \tag{6}$$
$$\times\left(\frac{2}{k}\right)\left(N-1\right)\sin\left(k\xi\right).$$

Here $\xi\equiv V_0t-z$, the 1-st term is the decelerating field inside the N-th bunch, the second term is the wakefield after the N-th bunch, the 3-rd term is the field before the N-th bunch, excited by (N-1) bunches. Thus, after the N-th bunch the transformation ratio is equal to T_E=2N, similar to [3, 6, 7]. The decelerating field inside the N-th bunch is equal to the decelerating field inside the 1-st bunch.

2. LONG SEQUENCE OF SHORT TRAINS OF PROFILED DRIVERS, INTERCHANGED BY "HIGH-CURRENT" WITNESSES

For the increase of number of accelerated electrons we consider the case, when after N profiled bunches the sequence continues as periodical long sequence of interchanging short (K bunches-drives) trains of the profiled drivers and separate "high-current" witnesses (Fig. 5).

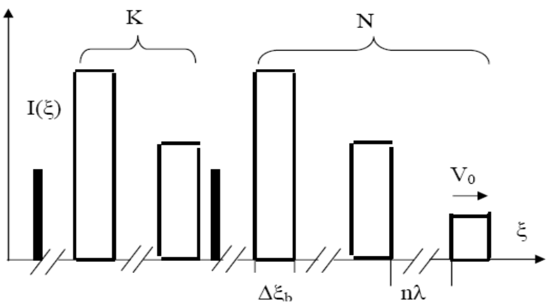

Fig. 5. The current distribution of infinite periodical sequence of short trains of profiled bunches-drives and of witnesses

Then after every K-th bunch-driver a "high-current" witness follows, so that it can take away considerable energy. Thus after witness the amplitude of the wakefield decreases from $E_{z0}=NE_{11}$ to $\chi E_{z0}=(N-K)E_{11}$, $\chi=\frac{(N-K)}{N}<1$, $K=N(1-\chi)$. Here E_{11} is the wakefield after the 1-st bunch. In this case the transformation ratio, at the use of the averaged over accelerating time of accelerating field, equals

$$R^*\approx\frac{[NE_{11}+(N-K)E_{11}]}{2E_{sl}}=(N-\frac{K}{2})\frac{E_{11}}{E_{sl}}. \tag{7}$$

We use that the transformation ratio R=2N known at K=0. Then $\frac{E_{11}}{E_{sl}}=2$ and we obtain the connection of R* with χ and with number of bunches N of the sequence, after which the periodic quasi-stationary asymptotic wakefield is set,

$$R^*\approx N(1+\chi). \tag{8}$$

Here E_{sl} is the maximum decelerating wakefield in the region of being of bunch-drives.

We specify that the words "high-current" witness mean. From balance of energies one can derive

$$q_W R^*=q_W(2N-K)=(2/\pi)\sum_{i=1}^{K} q_{dr\,i}=(2/\pi)q_1K(2N-K). \tag{9}$$

Here $q_{dr\,i}$ is the charge of i-th driver bunch of the sequence. Then the ratio of charge of witness bunch q_W to charge of first driver bunch of sequence q_1 equals

$$q_W/q_1=(2/\pi)K. \tag{10}$$

One can see that $q_W\ge q_1$, however for $q_K=(2K-1)q_1$,

$$q_W/q_K=1/\pi(1-1/2K),$$

and for $q_N=(2N-1)q_1$,

$$q_W/q_N=K/\pi(N-1/2).$$

The maximal ratio q_W/q_N equals $q_W/q_N=1/\pi(1-1/2N)$ at K=N, i.e. at χ=0. But

$$q_W/q_N\ge 1/\pi(N-1/2),$$

because K≥1. Thus R*=N for infinite sequence. From here one can derive coupling of decrease rate χ of wakefield (from E_{z0} to χE_{z0}) after witness bunch with ratio of witness charge to driver charge q_W/q_{dr}

$$q_W/q_1=(2/\pi)N(1-\chi).$$

The transformation ratio equals to

$$R^*=2N-(\pi/2)(q_W/q_1). \tag{11}$$

The maximal transformation ratio equals to

$$R^*=2N-1.$$

CONCLUSIONS

So it has been shown that in the case of wakefield excitation in dielectric resonator by sequence of rectangular electron bunches, the charge of which is shaped according to linear law, the transformation ratio can achieve large value.

For the increase of number of accelerated electrons the long periodical sequence of electron bunches has been derived. In this infinite periodical sequence the short trains of shaped bunches-drivers are interchanged by "high-current" witnesses. This long periodical sequence provides the large transformation ratio and identical decelerating wakefield for all bunches-drivers. The coupling of transformation ratio with the reduction rate of the wakefield after witness and coupling of the transformation ratio with the witness charge and driver charge in this long periodical sequence have been derived.

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КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ПРЯМОУГОЛЬНЫХ ЭЛЕКТРОННЫХ СГУСТКОВ С ЛИНЕЙНО НАРАСТАЮЩИМ ЗАРЯДОМ

В.И. Маслов, И.Н. Онищенко

Получен коэффициент трансформации в случае возбуждения кильватерного поля в диэлектрическом резонаторе последовательностью прямоугольных в продольном направлении (одинаковый заряд вдоль сгустка) электронных сгустков, заряд которых профилирован по линейному закону. Для увеличения числа ускоренных электронов построена длинная периодическая последовательность электронных сгустков. В этой последовательности короткие цепочки прямоугольных в продольном направлении профилированных сгустков-драйверов чередуются с ускоряемыми “сильноточными” сгустками. Эта длинная последовательность обеспечивает большой коэффициент трансформации и одинаковое для всех сгустков тормозящее кильватерное поле. Определена связь коэффициента трансформации с коэффициентом уменьшения поля после витнеса и связь коэффициента трансформации с зарядом витнеса и зарядом драйвера в этой последовательности.

КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ В ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ ПОСЛІДОВНІСТЮ ПРЯМОКУТНИХ ЕЛЕКТРОННИХ ЗГУСТКІВ З ЛІНІЙНО ЗРОСТАЮЧИМ ЗАРЯДОМ

В.І. Маслов, І.М. Онищенко

Отримано коефіцієнт трансформації в разі збудження кільватерного поля в діелектричному резонаторі послідовністю прямокутних у поздовжньому напрямку (однаковий заряд уздовж згустка) електронних згустків, заряд яких профільований за лінійним законом. Для збільшення числа прискорених електронів побудована довга періодична послідовність електронних згустків. У цій нескінченній послідовності короткі ланцюжки прямокутних у поздовжньому напрямку профільованих згустків-драйверів чергуються з прискорюваними "сильнострумовими" згустками. Ця довга послідовність забезпечує великий коефіцієнт трансформації і однакове для усіх згустків кільватерне поле. Визначено зв'язок коефіцієнта трансформації з коефіцієнтом зменшення поля після вітнеса і зв'язок коефіцієнта трансформації із зарядом вітнеса і зарядом драйвера в цій послідовності.