NUMERICAL SIMULATIONS OF QUASI-STATIONARY STREAMER PROPAGATION

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The numerical simulations are carried out for the quasi-stationary stage of the cathode-directed axially symmetric streamer propagation and for the linear stage of its azimuthal perturbation development. The dependences of the streamer velocity on the average electric field strength are obtained for different gas mixtures. It is found the instability of the azimuthal perturbations, and it is shown that this instability is typical for streamers. The connection of the instability with the streamer branching is discussed.

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1. INTRODUCTION

Streamer is the phenomenon, which may arise in gas or semiconductor in presence of electric field when an ionization wave propagating between electrodes leaves behind the ionized channel with relatively small transverse dimension. Streamer discharge until now is widely used and processes in it are investigated [1]. Together with tendency to transverse localization, streamers have tendency to branching when the difference in the rates of ionization wave expansion at the different parts of wave front leads to appearance of the gap between the spaces of intensive ionization and to developing of full-value streamers from these spaces. And further each new branch of the streamer may to form several branches again. The picture of branching may be somewhat similar to one arising with ionization developing along the traces of the particles formed in nuclear reactions, but the conditions, in which discharge has the streamer form (in particular, comparatively large gas pressure), are different from the discharge conditions, for example, in Geiger tube. There are experimental and theoretical works, devoted to the problem of streamer branching. In the paper [2] the streamer branching is connected with the excess of streamer transverse dimension over characteristic length of absorption of the photons, which energy is sufficient for photoionization. In the paper [3], in particular, the experimentally obtained value of ratio of the transverse dimension and the absorption length characteristic for streamer branching is presented. In the paper [4], to explain experimental results at different pressures, photoionization is considered more precisely with account of quenching. In the paper [5], in the consideration of the processes, which have an effect on the branching, the considerable attention is paid to the processes of seed electron production different from direct photoionization of molecule in ground state. In the paper [6], the branching is considered as instability of thin space charge layer. In the paper [7], the experimental results for streamer branching at different gas pressures are presented. In the mentioned papers, the numerical simulations are carried out under assumption of axial symmetry, and the formation of concave streamer front is interpreted as the branching. But the axially symmetric branching is only one of the possible ways of the process development. Moreover, it almost cannot be realised through its instability with respect to the azimuthal perturbations. Though the streamer branching description demands three-dimensional consideration, an initial stage of the branching may be considered in the linear approximation, as development of the azimuthal perturbations of the state close to stationary one. Of course, in the laboratory frame a streamer cannot be considered as stationary state. But if the process of streamer propagation is considered in the frame of reference moving with the streamer head at the stage of streamer development when the head is far from electrodes and the time variation of the process characteristics in the space near the head in the moving frame is almost absent then the state of process is close to stationary one, and the streamer branching may be considered as consequence of instability of this state with respect to some perturbations. In this approach, together with nonlinear simulations of the axially-symmetric process evolution (the process relaxation to stationary one), at the same two-dimensional mesh, but with linearized equations, it is simulated the evolution of the small perturbations, which characteristics are dependent on the azimuthal angle φ through the factor $\cos(m\varphi)$ (with m inte-

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*corresponding author E-mail address: ostroushko-v@kipt.kharkov.ua ger). Thereby, the initial stage of three-dimensional process is considered through two-dimensional simulations, which require much less time. Such approach was realized in the work [9]. It was directed on the study of fast streamers, and in the model used there, the ions were considered as immobilized. In the present work the ion motion is taken into account, in particular, with the aim to consider the streamer under the small excess of voltage over one minimum necessary for quasi-stationary streamer propagation.

2. SIMULATION MODEL

In the simulations, electron and ion drift and diffusion, and the processes of impact ionization, attachment, electron-ion and ion-ion recombination, and photoionization are taken into account. Time evolution of the densities is determined with the following equations:

$$\partial_t N_e = M_e + R_e,$$

$$\partial_t N_p = M_p + R_p, \quad \partial_t N_n = M_n + R_n,$$

$$\varepsilon_0 \nabla^2 \Phi = -q\left(N_p - N_e - N_n\right), \quad \vec{E} = -\nabla\Phi.$$

Here ∂_t is time derivative,

$$M_e = \operatorname{div}\left(D_e \nabla N_e + N_e \mu_e \vec{E}\right),$$

$$R_e = S_{ph} + \left(\nu_i - \nu_a\right) N_e - \beta_{ep} N_e N_p,$$

$$M_p = \operatorname{div}\left(D_p \nabla N_p - N_p \mu_p \vec{E}\right),$$

$$R_p = S_{ph} + \nu_i N_e - \beta_{ep} N_e N_p - \beta_{np} N_n N_p,$$

$$M_n = \operatorname{div}\left(D_n \nabla N_n + N_n \mu_n \vec{E}\right),$$

$$R_n = \nu_a N_e - \beta_{np} N_n N_p,$$

q is elementary charge, ε_0 is electric constant; N, μ, and D are density, mobility, and diffusion coefficient of electrons or of ions, positive or negative (indexes e, p or n); ν_i and ν_a are ionization and attachment frequencies; β_{ep} and β_{en} are relevant recombination coefficients.

$$S_{ph}(\vec{r}) = \int dV' N_e(\vec{r}\,') \nu_{ph}(|\vec{E}(\vec{r}\,')|) \kappa g(|\vec{r} - \vec{r}\,'|),$$

where κ is photon absorption coefficient, $\nu_{ph}(E)$ is the frequency of the photoionization acts made in the unbounded space by the photons radiated from the states exited by one electron moving in the field with the strength E, $g(r) = \exp(-\kappa r)/(4\pi r^2)$. The quantities μ, ν, and β, with indexes, are taken dependent on the electric field strength value, and D are constant. As it is known from the streamer simulations, the streamer characteristics almost does not change when photoionization is replaced with small initial electron density. Moreover, the characteristic value of electron energy necessary for generation of the photons, which energy is sufficient for direct photoionization of molecule in ground state, is even somewhat greater than one necessary for impact ionization of the molecule. As a result, in the case when the conditions of the streamer propagation are close to minimum necessary ones (overvoltage is not large), the role of another processes (different from direct photoionization) in seed electron supplying may be relatively large. One of such processes is associative ionization (association of the exited particle and other one with emission of electron), but it lags behind in time from the act of relevant photon radiation and needs the model for its description different from one stated above. The considerable part of the present simulations is carried out with replacing of photoionization with the initial electron density 10^6 cm^{-3}. The calculations with the initial electron density 10^3 cm^{-3} give only somewhat greater voltage value required for the same streamer velocity. The used mesh is homogeneous along the coordinate z of the cylindrical coordinate system (ρ, φ, z). At the boundaries $z = const$ the conditions $\Phi = const$ are imposed with the difference $\Delta\Phi$, corresponding to the average applied electric field strength, $\bar{E} = \Delta\Phi/L$, where L is distance between the boundaries. Potential distribution is calculated with Fourier transformation along the coordinate z. The boundary condition for potential at the maximum radius of simulation domain, $\rho_{\max}$, is imposed on each Fourier component, and it corresponds to potential distribution in the space $\rho > \rho_{\max}$ free of the charged particles. The average value of z coordinate of electrons in the simulation domain is kept constant, through shift of mesh on the part of cell at each time step, with relevant particle redistribution between the cells. If L is sufficiently large (in comparing with characteristic transverse streamer dimensions) and the streamer front is situated far from the boundaries then the electric field strength near the front is not very different from one near the front of the half-infinite streamer propagating in the unbounded space with the same $\bar{E}$. For the coordinate system moving in z direction with velocity u relative to laboratory frame, introduction of the variables (ζ, τ) connected with the variables (z, t) through the equalities $\zeta = z - ut$ and $\tau = t$ leads to the equations

$$\begin{aligned} \partial_\tau N_e - u\partial_\zeta N_e &= M_e + R_e\,, \\ \partial_\tau N_p - u\partial_\zeta N_p &= M_p + R_p\,, \\ \partial_\tau N_n - u\partial_\zeta N_n &= M_n + R_n\,. \end{aligned} \qquad (1)$$

The equations for perturbations may be obtained with linearization of the equations (1). Some linear relationships used in calculations for the perturbations dependent as $\cos(m\varphi)$ $(m = 1, 2, 3, \ldots)$ on the azimuthal angle φ are described in the papers [8] and [9]. For example, in the linear approximation, when $\Phi = \Phi_0 + \Phi_1 \cos(m\varphi)$, $N = N_0 + N_1 \cos(m\varphi)$ (the indexes *epn* are omitted here), the field strength absolute value is equal to $E_0 + E_1 \cos(m\varphi)$, where $E_0 = |\nabla\Phi_0|$, $E_1 = -\vec{e}_0 \nabla\Phi_1$, $\vec{e}_0 = -\nabla\Phi_0/E_0$, and the perturbation of the quantity $\operatorname{div}(N_e \mu_e \vec{E})$ is equal to

$$\operatorname{div}(N_{e1}\mu_{e0}\vec{E}_0 + N_{e0}\mu_{e1}\vec{E}_0) +$$
$$+ N_{e0}\mu_{e0} Q_1 - \nabla\Phi_1 \nabla(N_{e0}\mu_{e0}),$$

where $Q_1 = q\epsilon_0^{-1}(N_{p1} - N_{e1} - N_{n1})$, $\mu_{e1} = \mu_e^{(1)} E_1$, $\mu_e^{(1)}$ is derivative of the function $\mu_e(E)$. The simulations are carried out for gases at atmospheric pressure in the simulation domain with width 1 mm. Their results may be, also, used for discharges at other pressure with taking into account the similarity of discharge processes in the conditions with the same value of the product of gas pressure and characteristic dimension.

3. NUMERICAL RESULTS

In the Fig. 1, the obtained dependences of streamer velocity on the average applied electric field strength are shown for different gas mixtures, namely, for pure N_2, for the mixtures $N_2 : O_2$ in the proportions $4 : 1$ and $3 : 2$, and for pure O_2, from left to right. More intensive attachment for greater O_2 percentage leads to lower electron density, weaker ionization intensity, and slower streamer propagation.

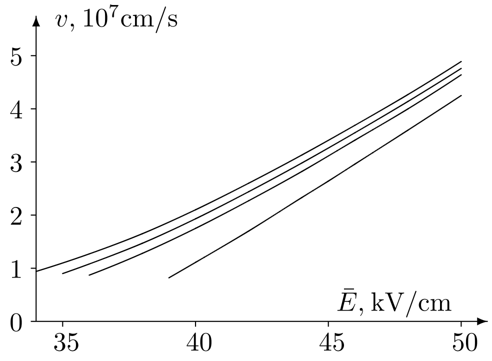

Fig.1. *Streamer velocity for* N_2, $N_2 : O_2 = 4 : 1$, $N_2 : O_2 = 3 : 2$, O_2, *from left to right*

In the calculations for the points of curves corresponding to minimum streamer velocity the characteristics of the process come to their stationary values very slowly. Moreover, in connection with the rules accepted for movement of the simulation domain in laboratory frame, the simulations for the smaller values of average field strength give the artificial nonsteady regimes, which cannot arise in real experimental conditions. Let us briefly describe such regime.

If the average external field strength value is close to minimum one sufficient for streamer propagation then the conditions for ionization wave development differ very much in the different areas in front of streamer (due to great sensitivity of ionization coefficient to the field strength value in relatively weak field). Near the axis the wave develops faster, and a new streamer, plasma channel of which has smaller transverse dimension, begins to form there. With it formation the electric field in the space near head and channel of new streamer becomes stronger, the ionization coefficient there increases, the multiplication of electrons in their way near the channel of new streamer to the main streamer front intensifies, and the conditions are created for transverse ionization wave development near the main streamer front. In this wave, the role of streamer head is played by the annular positively charged space, which propagates along the main streamer front, and the initial electron density there has been increased due to multiplication of electrons during their drift near the channel of new streamer. In the numerical simulations the described process develops slower than in reality, through the movement of the simulation domain in laboratory frame, for the purpose to keep constant the average coordinate of electrons in the simulation domain. However, before the transverse ionization wave development, the simulation domain moves slower than the head of new streamer (due to lesser electron number per unit length in smaller channel), and so, the head moves to the cathode boundary of the simulation domain. But then, the transverse ionization wave creates great amount of electrons, and to keep the average electron coordinate in the simulation domain constant the simulation domain is moved, and the velocity of this movement is greater than the velocity of new streamer head propagation. As a result, the head moves away from the cathode end of the simulation domain, and the field near the head becomes weaker, which does not correspond to real conditions of experiment, as the head does not move away from the real cathode. In the simulation, the repetition of the described stages gives the pulsed regime of streamer propagation, which does not realize in experiment. And the simulation model accepted here is not fit for study of non-steady processes, at all.

In the Fig. 2, the values of the perturbation growth increments for several azimuthal harmonics are shown for the case of average strength 40 kV/cm in air. The value of the increment for $m = 0$ (axially symmetric perturbation) was obtained formally, in calculation under the rules accepted for another m, according to which, in particular, the velocity of the moving frame does not depend on the perturbation development. All obtained values of the perturbation growth increments are positive, which corresponds to the perturbation instability. In particular, for $m \geq 2$, the instability is connected with the initial stage of streamer branching.

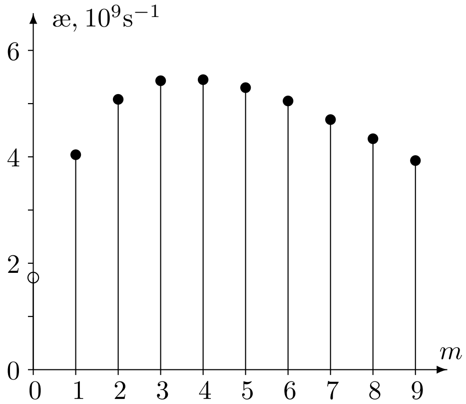

Fig.2. *Perturbation growth increments for azimuthal harmonics*

But experimental facts witness, in particular, practical absence of cathode-directed streamer branching near anode. The questions arises, how to explain the instability in the frames of the model and how to connect real streamer branching with the linear instability of azimuthal harmonics.

4. INSTABILITY DISCUSSION

As it follows from the arguments adduced below, the azimuthal perturbations are unstable in the wide range of the stationary streamer propagation modes. Let us consider the case of stationary propagation of the axially symmetric streamer, assuming its stability with respect to axially symmetric perturbations.

The axially symmetric solutions of the equations for perturbations are characterized by relevant time increments. As the considered streamer propagation is stable, these increments cannot have positive real part. On the other hand, there are axially symmetric perturbations having negative real part of time increment with very small absolute value (for example, the long-wave, in z direction, perturbations of the streamer channel side). So, in the case of the stationary stable axially symmetric streamer propagation, the maximum real part of time increment of the axially symmetric ($m = 0$) perturbation should be equal to zero. The perturbations with $m \geq 1$ should have greater increments, in connection with the additional curvature of their equipotential surfaces in the space of ionization front, and with relevant additional field enhancement. So, for $m \geq 1$, the unstable perturbation (with increment having positive real part) should exist.

The positive value of the increment for the case $m = 0$ formally obtained in the calculations does not contradict to stability of stationary propagation with respect to axially symmetric perturbations, because this value was obtained in conditions when the velocity of moving frame does not depend on the perturbation development. For the perturbations with $m \geq 1$ it is natural, because such perturbations do not change the average value of z coordinate of electrons in the simulation domain, whereas the axially symmetric perturbations may be accompanied with such change. In particular, if ionization enhancement with the increase of space charge density takes place in front of streamer, nearer to cathode, then it leads to field enhancement near the streamer head. This field enhancement may be interpreted as the field of the space charge image in the mirror-cathode, as the image of perturbation is nearer to streamer than the image of the unperturbed streamer itself. So, positive value of the perturbation growth increment for the case $m = 0$ is connected with formal allowing to develop such perturbation without demand to keep constant the average value of z coordinate of electrons in the simulation domain. On the other hand, the described mechanism of ionization enhancement (through the field of image) is applicable for perturbations with $m \geq 1$, too (though account of contribution of the sectors corresponding to the different signs of perturbation somewhat diminishes the enhancement). And so, all obtained increment values partially are connected with not very large distance between streamer and cathode, and the solution of the problem for streamer in the unbounded space with the same average applied electric field strength should give somewhat smaller increment values.

As the azimuthal perturbation linear instability exists at any excess of the average field strength value over one minimum necessary for the streamer stationary propagation possibility, the question remains, why the streamer branching is not observed in some experimental conditions.

One possible cause may be connected with the combined development of the azimuthal perturbations with different m, in the case when their increments have the same order of magnitude, and so, relevant perturbations develop during approximately the same time. The perturbation with $m = 1$, as a rule, strengthens or weakens the perturbation with another m to the different extent in the different radial directions. As a result, the sum of perturbations may to develop in such a way that in some sector the perturbation comes to nonlinear stage faster, the perturbation development in other sectors is slowing down, and instead of streamer branching the small spatial variation of the streamer propagation way takes place.

It should be also taken into account the great role of fluctuations in perturbation development. In particular, electron density 10^6 cm^{-3}, at the beginning of avalanche multiplication, corresponds to a few electrons in the volume with strong electric field near the streamer head, and the question arise about validity of the continuous medium model for description of phenomena. If a phenomenon is determined mainly by the sum of processes (as it is for the stationary mode) then application of statistical averaging (which is used to get the equations of the continuous medium model) is natural. But instability development is connected with fluctuations, and, to a great extent, it is determined with the difference of avalanche development rates in the different parts of the space near the streamer head. As this difference is random quantity, the instability development simulations in the continuous medium approximation may give the process characteristics, essentially different from the experimentally observed ones.

5. CONCLUSIONS

The numerical simulations are carried out for the quasi-stationary stage of the process of development of an axially symmetric streamer, which can propagate in gas or semiconductor under sufficiently strong electric field. The dependences of the streamer velocity on the average electric field strength are studied for the different gas mixtures. On the background of the axially symmetric process, which development is determined with the usual nonlinear equations for evolution of the charged particle densities, the linear stage of development of the azimuthal harmonics of perturbation is studied. When the space distribution of the particle densities for the axially symmetric process approaches to a stationary one (in the frame of reference moving together with the streamer) the relative distributions for the azimuthal harmonics also approaches to the certain ones, but the amplitudes of the several first harmonics increase with time approximately exponentially. For the second and higher harmonics such behavior corresponds to the beginning of the streamer branching with formation of relevant number of new streamers from it. The physical causes are pointed out, due to which such instability is typical for streamers.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ КВАЗИСТАЦИОНАРНОГО РАСПРОСТРАНЕНИЯ СТРИМЕРА

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Выполнено численное моделирование квазистационарной стадии распространения направленного к катоду аксиально-симметричного стримера и линейной стадии развития его азимутального возмущения. Зависимости скорости стримера от средней напряжённости электрического поля получены для разного состава газа. Выявлена неустойчивость азимутальных возмущений и показано, что она типична для стримеров. Обсуждается её связь с ветвлением стримера.

ЧИСЛОВЕ МОДЕЛЮВАННЯ КВАЗІСТАЦІОНАРНОГО ПОШИРЕННЯ СТРИМЕРУ

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Виконане числове моделювання квазістаціонарної стадії поширення спрямованого до катоду аксіально-симетричного стримеру та лінійної стадії розвитку його азимутального збурення. Залежності швидкості стримеру від середньої напруженості електричного поля отримані для різного складу газу. Виявлено нестійкість азимутальних збурень та показано, що вона є типовою для стримерів. Обговорюється її зв'язок з галуженням стримеру.