# ONE HYPOTHESIS ON THE NUCLEAR STRUCTURE

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The work is devoted to connection between theory of polyhedrons and theory of nuclear structure, especially of regular polyhedrons and light nuclei. For confirmation of expressing hypothesis we compare the experimental values of nuclear radius and theoretical values, which calculated with the help of the hypothesis. Then we investigate the agreement of the hypothesis with the theory of chemical element origin.

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## 1. INTRODUCTION

Some geometrical characteristics present at the theory of nuclei. Here one suppose the existence of the Euclidean space and its 3-dimension. The sizes of nuclei and nucleons are well known together with width and depth of potential pit. Majority of heavy nuclei have almost spherical form, and consequently it gave the solution of geometrical isoperimetrical problem. In this article we introduce other notions from Geometry.

## 2. CONSIDERATION OF LIGHT NUCLEI

At the theory of nucleus there exist few models concerning of nuclear structure. More known models are model of liquid drops and model of shells. At the model of shells the following set of numbers is well known

$$2, 8, 20, 28, 50, 82, 126,$$

which have name " magic" numbers. It is the number $Z$ of protons or the number $N$ of neutrons at shell of nucleus, which are very stable (the number 126 has relation only to neutrons). Remark, that it has sense add to this set the number 4, because $\alpha$-particle is very stable. These "magic" numbers arose on graphs for spreading of chemical elements in Universe. The explanation for beginnings of "magic" numbers was been given at Physics with the help of Schrödinger equation under special form of potential function.

But we want to give another explanation with geometrical point of view. The principal idea is following. *If nuclear shell consists $l$ nucleons and this number is "magic", for example, 8 or 20, then these nucleons lie on the shell non by chaotic way, but at determined order, just these nucleons lie at the vertexes of some regular polyhedron. If $l$ represents as a sum of vertex numbers, for example, 28=8+20, 50=4+6+8+12+20, then such regular polyhedrons will be at a few numbers, which must be included each other at determined order and with some distances between spheres, keeping these vertexes.*

This construction could be at conformity with principal non excited state of nucleus.

If the number $l$ is large, then it possible to apply another polyhedrons, for example, semi-simple polyhedrons. At any case, at every moment of time some polyhedron exists. It is the convex cover of nucleon center set, that is the intersection of all half spaces, which contain given finite discrete set of points.

But here we consider only light nuclei for which $l$ is small.

It is strange, but in Physics one non give enough attention to disposition of nucleons in the space. Maybe, this circumstance connects with difficulties of correspondent experiments. At the end of the article we indicate on works with $\alpha$-particle hypothesis, which have relation to this question.

Remark, that the number 4 is the number of vertexes of a tetrahedron, the number 8 is the number of vertexes of a cube and 20 is the number of vertexes for a dodecahedron. Late

$$50 = 4 + 6 + 8 + 12 + 20$$

is the sum of vertex numbers of all 5 regular polyhedrons. Correspondent nucleus or nuclear shell, maybe, have the following structure

*tetrahedron, octahedron, cube, icosahedron, dodecahedron,*

which are enclosed each other with common center. Hence, we have 4 surprising coincidences. Also we have:

$$82 = 12 + 20 + 50.$$

Although the regular polyhedrons there exist even at school textbooks, but it is masterpiece of ancient Greek mathematics. Very often one use it in different mathematical constructions. It possible to meet they at Chemistry. For example, the molecule of $CH_4$ has form of tetrahedron with atoms of $H$ at vertexes and atom of $C$ at it's centrum.

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The regular polyhedron has equal edges and equal sides. All it's vertexes lie on some sphere.These polyhedrons are symmetric and have some group of symmetry. Therefore, the resulting force of attraction, which acts on a nucleon by others nucleons, is directed to the center of the polyhedron. One can say that only the force of this center acts on the nucleon. It means that it possible to use one-particle description, which with success was used earlier.

Well known physicist D.D. Ivanenko in the article "The principal stages of shell nuclear model development ", which was introductory article to the book by M.Goeppert-Mayer and J.H.D.Jensen [1], wrote: "First of all it is surprising as manner, in spite of strong interaction between nucleons, that one-particle model is good approach. Exhaustive answer on this question does not given until".

*The answer for this question is symmetry of regular polyhedrons.*

The edges of considering polyhedrons are only mathematical notion. We always can join the centers of nucleons nearest each other by imaginary segments of straight lines. But these segments have some physical sense, because along its is going interaction between nucleons. For construction considering polyhedrons we can take a convex hull of nucleon center set at some fixed moment of time.

Also at above mention article D. D. Ivanenko say: "The principal question, which directly connects with structural nuclear models, is the space distribution of protons and neutrons in a nucleus. Meanwhile to last time under consideration of different nuclear models this side of nuclear structure turn out of view point."

Let us give some arguments in side of our hypothesis by using the comparison of radius $R_1$ of a sphere, which is described around vertexes of regular polyhedrons, with nuclear radius $R_2$. We remark, that at the book by V. G. Soloviev [2] on the p.25 is given: "If nuclear radius $R_2$ is the distance from nuclear center to a surface, where the density of nuclear matter is decreasing at 2 times, then approximately one can write

$$R_2 = r_0 A^{\frac{1}{3}},$$

where $r_0 = 1,1fm$ and $A = Z + N$ is the sum of proton and neutron numbers". The same formula is given at the book by L.Landau and E.Lifschic [3] on the p.559. This formula is founded on experiments with scattering of electrons on nuclei. We will use it lower at the case $^{20}Ne$. But for lightest nuclei the value of $r_0$ is less. Here we use the table from the book by L. Elton [4] on the p. 45 with nuclear parameters , which were obtained from experiments. But in the book [4] in the capacity of nuclear radius $R_2$ at the sense of book [2] the parameter $c$ is given. At the Elton's book one wrote: " Such distribution can be characterized by radius of half of density, when density falls to the half of its maximal value..." (L.Elton uses the letter $R$ for another parameter.) Taking into attention this remark, we put $R_2 = c$ and write some data from Elton's table

$$^4He : R_2 = 1.10\,fm, \quad ^6Li : R_2 = 1.56\,fm,$$

$$^9Be : R_2 = 1.80\,fm, \quad ^{12}C : R_2 = 2.30\,fm,$$

$$^{16}O : R_2 = 2.60\,fm, \quad ^{24}Mg : R_2 = 2.93\,fm.$$

It is known the mean value of distance between two neighboring nucleons , which we denote by $a$ , see , for example, [10], p. 88

$$a = 1.8 \cdot 10^{-13} cm.$$

Supposing that at the case $^4He$ nucleons lie at the vertexes of regular tetrahedron, we calculate the radius $R_1$

$$R_1 = \frac{a\sqrt{3}}{2\sqrt{2}} = 1.8 \cdot 0.612 \cdot 10^{-13} cm = 1.102 \cdot 10^{-13}\, cm.$$

Then, by using the Elton's table we have

$$R_2 - R_1 = -0.002 fm\,.$$

It means that the radiuses $R_1$ and $R_2$ are nearly coinciding.

At the case of $^6Li$ we suppose that nucleons lie at vertexes of octahedron with length of edges equal to $a$. We have

$$R_1 = \frac{a}{\sqrt{2}} = 1.27\,fm\,.$$

Comparison with Elton's table give to us

$$R_2 - R_1 = (1.56 - 1.27)\,fm = 0.29\,fm\,.$$

For nucleus $^9Be$ one can correspond a cube+1 neutron at the center of this cube. We suppose that this neutrons lies on the distance $a = 1.8fm$ from other nucleions, which lie at the vertexes of the cube. Then, $R_1 = 1.8\,fm$ . Comparison with Elton's table gives

$$R_2 - R_1 = 0.$$

We remark, that element $^8Be$ is unstable. Maybe, the stability of nucleus of $^9Be$ is ensuring due to position of ninth nucleon at the center of the cube, because this position gives symmetry.

For $^{12}C$ we suppose that nucleons lie at the vertexes of a icosahedron. For this polyhedron the radius of describing sphere $R_1$ can be calculated by the formula

$$R_1 = \frac{a}{4}\sqrt{2(5+\sqrt{5})} = a \cdot 0.951\,.$$

Taking into consideration the value of number $a$, we have

$$R_1 = 1.712\,fm\,.$$

Consequently, we obtain

$$R_2 - R_1 = 0.6\,fm\,.$$

At last, let us consider the element $^{20}Ne$. At the Elton's table the date about this element are absent. Therefore, we use the formula $R_2 = 1.1A^{\frac{1}{3}}$. We suppose that 20 nucleons lie at vertexes of a dodecahedron. Then we can calculate the radius of described sphere of this polyhedron in terms of length $a$. By the formula from Pogorelov's book [5], p.283 (answers to problems 10,11) we have

$$R_1 = \frac{a}{2\sin\frac{\pi}{5}}\sqrt{1+\cos^2\frac{\pi}{5}tg^2\gamma}\,,$$

where $\gamma$ is the dihedral angle under edge and

$$\cos\gamma = \frac{\cos\frac{2\pi}{5}}{2\sin^2\frac{\pi}{5}}\,.$$

Transform the expression under radical

$$1+\cos^2\frac{\pi}{5}tg^2\gamma = 1+\cos^2\frac{\pi}{5}(\frac{1}{\cos^2\gamma}-1) =$$

$$=\sin^2\frac{\pi}{5}+\frac{4\cos^2\frac{\pi}{5}\sin^4\frac{\pi}{5}}{\cos^2\frac{2\pi}{5}} = \frac{\sin^2\frac{\pi}{5}}{\cos^2\frac{2\pi}{5}}\,.$$

After substitution of this expression into formula for $R_1$ we obtain

$$R_1 = \frac{a}{2\cos\frac{2\pi}{5}} = a\cdot 1.618\,.$$

Consequently,

$$R_1 = 2.916\cdot 10^{-13}\,cm\,.$$

At the considering case the number $A=20$. We have $R_2 = 2,98\,fm$. We see that $R_2$ and $R_1$ are different very little

$$R_2 - R_1 = 0.06\,fm\,.$$

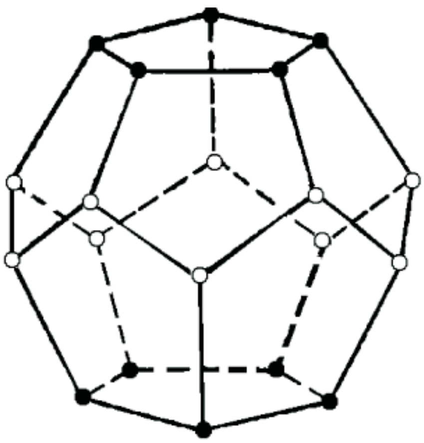

***Fig.1.*** *Light vertexes represent possible positions of protons and dark vertexes correspond to neutrons*

Thus, the hypothesis that nucleons lie at vertexes of a dodecahedron gives near correct value of nuclear radius. Only information on distance between two neighboring nucleons is insufficient to determine the radius of described sphere. For its calculation we essentially use the supposition about dodecahedron (Fig.1).

Of course, these vertexes do not fixed. All figure can be moving (revolving) as the whole. Besides, the distances between nucleons can be slightly change. One can be independent oscillations of vertexes. Remark, that in this case density of nuclear matter,which calculated by formula $\frac{20}{\frac{4}{3}\pi R_1^3} = 0.19\frac{nucleon}{fm^3}$, is close to standard value $0.17\frac{nucleon}{fm^3}$.

Let us give the possible construction of nuclei of $^4He$, $^{16}O$, $^{23}Na$, $^{24}Mg$.

Remark, that nucleus of $^4He$ has name $\alpha$-particle. Its two protons and two neutrons could have disposition at vertexes of quadrat, or at vertexes of a tetrahedron, or at vertexes of two equilateral triangles with common side. This triangles non necessarily have common plane. We have

$$^{16}O:\ Z=8,\ N=8.\quad 16=4+12\,.$$

The possible structure is: icosahedron, which contains $\alpha$-particle.

$$^{23}Na:\ Z=11,\ N=12.\quad 23=1+4+6+12\,.$$

The corresponding set of polyhedrons is :

1 nucleon at center+tetrahedron+octahedron +icosahedron.

$$^{24}Mg:\ Z=12,\ N=12.\quad 24=4+20\,.$$

The possible structure: $\alpha$-particle at center + dodecahedron.

If consider the shell structure, which is given with the help of wave function and Schrödinger equation, then can see that near all numbers of nucleons at decomposition shells on subshells are numbers of vertexes of regular polyhedrons. For example, the shell V at the book [6], p.193 has the following representation

$$8+6+4+2+12=32\,.$$

It is difficult to imaginer such organization of nuclei. Maybe, possible cause for this organization consists at the action of maximal volume principle for the volume inside polyhedron under constant and equal length of edges.

Natural question arises why the numbers 6 and 12 are not "magic" ? Take into attention, that from every vertex of octahedron four edges go out, and from every vertex of a icosahedron five edges go out.

In the book by Preston "The nuclear theory" there exists after some analysis the following conclusion: "...the groupings consisting more than three nucleons, evidently, do not play important role at forming of nuclear properties".

One can also express other supposition connected with inner structure of nucleon. Now in Theoretical Physics supposed that every nucleon consists of three quarks. Maybe, these particles determine forces which collect nucleons at nucleus.On this, maybe, every quark answers for attraction only one nucleon. Therefore every nucleon have no more than three neighbors with interaction.

## 3. AGREEMENT WITH THE THEORY OF CHEMICAL ELEMENT ORIGIN

Proposed hypothesis does not have good consent with the theory of chemical elements origin inside stars, if all connections between nucleons are remain at process of new elements formation. *But, if we admit breaks of some connections between nucleons, then the construction of new elements will be possible.*

By above-mentioned theory three atoms $^4He$ under high temperature and pressure unite at the atom of $^{12}C$.

$$3\,^4He \rightarrow \,^{12}C,$$

which nucleus by our hypothesis is represented at form of icosahedron. Consider the possibility of such transformation. The tetrahedron and icosahedron have one common property. Their sides are equilateral triangles.

Let us suppose that at every tetrahedron at the process one connection is destroying, or at first one can imagine that nucleus of $^4He$ has forme of two equilateral triangles with one common side. Then the construction of a icosahedron will be possible. We obtain three pairs of triangles, which can be applicable on icosahedron (Fig.2).

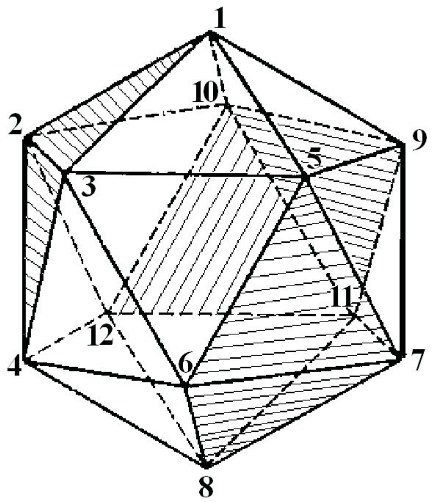


***Fig.2.*** *Touch sides represent applicable pairs of triangles*

The first pair has vertexes (1,2,3,4), second one has vertexes (5,6,7,8) and third pair has vertexes (9,10,11,12).

For destruction of connections between nucleions at $^4He$ the high energy is necessary. Therefore, the process is going only with high temperature and pressure. But, under amalgamation of nucleons at new form the energy emanates at larger quantity. And process is going later.

Later, let us go to the next reactions of the chain for creation of chemical elements, see, for example, [7], p.108.

$$^{12}C + \alpha \rightarrow \,^{16}O + \gamma,$$

$$^{16}O + \alpha \rightarrow \,^{20}Ne,$$

$$^{20}Ne + \alpha \rightarrow \,^{24}Mg.$$

The first reaction with $^{12}C$ one can represent as penetration of $\alpha$-particle inside icosahedron. As the sides of icosahedron consists from triangles, which we can separate on pair, that are $\alpha$-particles, then it is possible some $\alpha$-particle to press inside of polyhedron and change it on $\alpha$-particle from outside. Also, following G. Gamow one can speak about tunnel effect for penetration of $\alpha$-particle inside nucleus of $^{12}C$.

For the nucleus $^{20}Ne$ we propose two constructions: 1) at form: icosahedron + cube, 2) at form: dodecahedron.

The first form we obtain by transformation, when $\alpha$-particle penetrates into nucleus of $^{16}O$.

The second form is represented on Fig.3, where the dodecahedron is constructed with the help of 5 $\alpha$-particles.

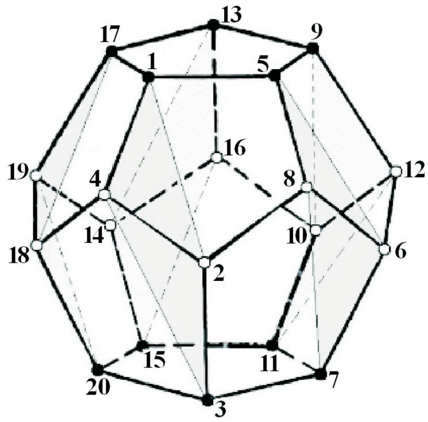


***Fig.3.*** *Dark non flat 4-angles represent $\alpha$-particles*

On this figure $\alpha$-particles are represented at the forms of 5 pairs of triangles with vertexes (1,2,3,4), (5,6,7,8), (9,10,11,12), (13,14,15,16), (17,18,19,20). At first time these $\alpha$-particles are separately, but under rapprochement each other the new connections between nucleons (1,5),(2,8) (3,7) and so on arise and by this way the whole dodecahedron can be obtained.

Under consideration of the reaction for receipt of $^{24}Mg$ one can suppose that $\alpha$-particle penetrate inside the dodecahedron or at the center of the construction: icosahedron+cube.

Under other hypothesis chemical elements can be formed at under star state of Universe existence, see [7], p.102. The forming mechanism could be quite different, maybe, similar to forming of crystals. Under this situation it was possible successive growing of layers at form of regular polyhedrons.

Remark, that at the book [8] , chapter XII episodically the constructions with regular polyhedrons one can meet. For example, on p.225 it is telling that "$\alpha$-geometry of $^{40}Ca$ has form of "octahedron inside tetrahedron with 24 connections". The nucleus of $^{24}Mg$ has form of octahedron with $\alpha$-particles at its vertexes.

J.A. Wheeler in the work [9] connects with wave function of nucleus of $^{16}O$ the disposition of $\alpha$-associations at form of regular tetrahedron and with $^{12}C$ the disposition at form of equilateral triangle. The last representation of nucleus of $^{12}C$ has good agreement with our representation at form of icosahedron on Fig.2. We can indicate the disposition of vertexes of Wheeler's triangle on Fig.2 at points , which are middles of segments (2,3), (6,7) and (10,11). Applications with icosahedron gives us whole picture of nearest connections between nucleons in form of edges of this polyhedron. Remark, that interaction between $\alpha$-particles is founded on interaction between nucleons. Our representation shows these connections completely. Besides, the values of all connections between nucleons under our consideration are equal.

Remark, that much works are devoted to investigations of light nuclei with the help of $\alpha$-particle model, see, for example, the article by Moshkowsky "Nuclear models" at the book [11]. In the book by L. Pauling and P. Pauling [12] on p.627 the nucleus of $^{20}Ne$ is represented at form of threegonal bipyramid and on the p. 203 on Fig.7.6 this representation is given in term of 5 $\alpha$-particles [13].

## 4. CONCLUSIONS

At the work for light nuclei geometrical approach one proposed for the space positions of nucleons. These positions we associated with the vertexes of regular polyhedrons. The hypothesis gives new explanation of "magic" numbers existence. Consideration shows good agreement with the theory of chemical elements origin.

## ACKNOWLEDGEMENTS

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## ОДНА ГИПОТЕЗА О СТРУКТУРЕ АТОМНОГО ЯДРА

***Ю. А. Аминов***

Работа посвящается связи между теорией многогранников и теорией строения ядер, особенно правильных многогранников и лёгких ядер. Для подтверждения высказанной гипотезы мы сравниваем экспериментальные значения ядерных радиусов и их теоретические значения, вычисленные на основе гипотезы. Далее мы исследуем согласие гипотезы с теорией происхождения химических элементов.

## ОДНА ГІПОТЕЗА ПРО СТРУКТУРУ АТОМНОГО ЯДРА

***Ю. А. Амінов***

Робота присвячена зв'язку між теорією многогранників та теорією ядерної будови, особливо правильних многогранників та легких ядер. Для підтвердження висловленої гіпотези ми порівнюємо експериментальні величини ядерних радіусів та їх теоретичні величини, обчислені на підставі гіпотези. Далі ми досліджуємо узгодження гіпотези з теорією походження хімічних елементів.