SKIN EFFECT INFLUENCE ON TIME EVOLUTION OF TRANSITION RADIATION PULSE

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The time variation of the field strength of the radiation caused by the charged particle transition through the plasma-vacuum boundary is obtained for the wave zone. It is shown that in the nearly collisionless plasma, together with the dumped oscillations at the beginning stage of pulse decay and the monotonous strength decrease by the power law with the exponent $-3/2$ at the final stage, there exists the time range with decrease as power with the exponent $-5/3$, corresponding to anomalous skin effect. The conditions are considered, for which the experimental measurements of the obtained pulse characteristics is possible.

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1. INTRODUCTION

The transition radiation arises when a particle crosses the boundary between two media [1]. It is used in the different devices. For the case of the particle transition from plasma to vacuum or vice versa, the paper [2] contains the detailed study of the influence of plasma parameters on the radiation characteristics. In the papers devoted to investigation of transition radiation the attention was mainly given to its spectral characteristics. Recently, the attention to the space-time evolution of the radiation field has grown [3]. With the aim of more precise interpretation of the nuclear physics experiments, in the paper [4], the characteristics of space-time evolution are studied not only in the wave zone, but in the pre-wave zone, where the radiation is formed, too. With the aim to create the generators of transition radiation, the characteristics of the radiation generated by the modulated beams are obtained [5], and the experimental investigations of the wide-range transition radiation generated with use of the pulsed accelerators of direct action are carried out [6]. The transition radiation as an elementary mechanism is the base of operation of some other devices, in particular, of monotron [7].

In the present work, it is considered the time dependence of the electric field strength in the wave zone. A pulse may be presented in the form of integral Fourier with real frequencies. Skin effect takes place at sufficiently low frequencies, where the absolute values of the radiated pulse amplitudes are almost the same for the different frequencies, whereas skin effect type depends on the ratio of the collision and plasma frequencies and on the characteristic plasma electron velocity value. So, in the frequency domain, the characteristics of transition radiation are almost the same for any skin effect type (if skin effect takes place). But in the time domain, there are the stages of the pulse decay characterized by the different rates, which correspond to anomalous and normal skin effect. In the following sections, the problem for the radiation caused by a particle transition through the plasma-vacuum boundary is formulated and reduced to the form convenient for solution construction, the approximate equalities for the different stages of the strength time evolution are obtained, and the possibility of experimental measurement of the considered pulse characteristics is discussed.

2. EQUATIONS FOR FIELD COMPONENTS IN PLASMA

Let a particle with charge $Z_0 e_0$ moves along OZ axis with velocity $\beta_0 c\vec{e}_z$, where $\vec{e}_z$ is unit vector of OZ axis, c is the speed of light, e_0 is elementary charge, $\beta_0 \in (-1,1)$, $\beta_0 \neq 0$, the particle is relativistic, but not ultrarelativistic, $\beta_0 \sim 1-\beta_0$, the half-space $z<0$ is empty, and the half-space $z>0$ is filled with plasma. In plasma, Maxwell equations may be written in the form

$$\begin{gathered}\mathrm{rot}\vec{E} + c^{-1}(\partial/\partial t)\vec{H} = 0,\\ \mathrm{rot}\vec{H} - c^{-1}(\partial/\partial t)\vec{E} - 4\pi c^{-1}(\vec{j}+\vec{j}_0) = 0,\end{gathered} \tag{1}$$

where $\vec{j}_0 = Z_0 e_0 \delta(x)\delta(y)\delta(z-\beta_0 ct)\beta_0 c\vec{e}_z$, $\vec{j} = e_0\int d^3\vec{v}\vec{v}f_1$, the perturbation, $f_1 = f_1(\vec{v},\vec{r},t)$, of electron distribution function obeys the equality

$$\begin{gathered}(\partial/\partial t)f_1 + \vec{v}(\partial/\partial\vec{r})f_1 +\\ +(e_0/m)\vec{E}(\partial/\partial\vec{v})f_0 + \nu f_1 = 0,\end{gathered}$$

m is electron mass, ν is collision frequency, the plasma electron motion is assumed nonrelativistic, the unperturbed electron distribution function f_0 is assumed isotropic, $f_0 = n_\mathrm{e}\bar{f}_0(|\vec{v}|/v_\mathrm{e})/v_\mathrm{e}^3$, n_e is the unperturbed electron density, v_e is characteristic

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velocity of plasma electrons, $v_{\rm e}\ll c$, v is the velocity normalized with respect to $v_{\rm e}$, and the function $\bar{f}_0(v)$ obeys the condition $4\pi\int_0^\infty dvv^2\bar{f}_0(v)=1$. In the case of Maxwell distribution (MD), there is $\bar{f}_0(v)=\exp(-v^2)/\pi^{3/2}$ for all positive v. In the case of Fermi distribution with zero temperature (FD), there is $\bar{f}_0(v)=3/(4\pi)$ at $v\in(0,1)$ and $\bar{f}_0(v)=0$ at $v>1$. Electron flow from the boundary into the plasma is characterized by the part, $p\in(0,1)$, of electrons scattered from the boundary specularly (the rest is scattered diffusely), and the boundary condition $f(v_z)=pf(-v_z)$ is held, for $v_z>0$ at $z=0$. To the time-dependent components of the field strength the Fourier transformation is applied with the factor $\exp[i\omega c^{-1}(ct-k_xx-k_yy-k_zz)]$ and with integration over the intervals $t\in(-\infty,+\infty)$, $x,y\in(-\infty,+\infty)$, $z\in(0,+\infty)$. The values of ω, k_x, and k_y are real. It may be put $k_y=0$ (due to possibility of relevant revolution of XOY-plane).

Using the letter u instead of k_z in the arguments of some functions, let us introduce the functions

$$\begin{gathered}\bar{S}_0(\vartheta,\eta)=-2\pi\int_0^\infty dv\times\\ \times v^3(1-\eta v\cos\vartheta)^{-1}(d/dv)\bar{f}_0(v),\end{gathered}$$

$$\begin{gathered}q_\lambda(\eta)=\int_0^\pi d\vartheta\sin\vartheta\cos^2\vartheta\bar{S}_0(\vartheta,\eta),\\ q_\tau(\eta)=\int_0^\pi d\vartheta\sin^3\vartheta\bar{S}_0(\vartheta,\eta)/2,\end{gathered}$$

$$\tilde{\eta}(u)=\beta(u^2+k_x^2)^{1/2},$$

$$\begin{gathered}Q_\lambda(u)=1-\Omega^2q_\lambda(\tilde{\eta}(u)),\\ Q_\tau(u)=1-k_x^2-u^2-\Omega^2q_\tau(\tilde{\eta}(u)),\end{gathered}$$

$$\begin{gathered}\Psi_\lambda(u)=\omega c^{-1}[k_xE_x(u)+uE_z(u)],\\ \Psi_\tau(u)=\omega c^{-1}[k_xE_z(u)-uE_x(u)],\end{gathered}$$

$$\begin{gathered}\Phi_\lambda(u)=uI_z(u)+\\ +Q_\lambda(u)[\Psi_\lambda(u)+p\Psi_\lambda(-u)],\end{gathered}\tag{2}$$

$$\begin{gathered}\Phi_\tau(u)=k_xI_z(u)+\\ +Q_\tau(u)[\Psi_\tau(u)-p\Psi_\tau(-u)].\end{gathered}\tag{3}$$

Here $I_z(u)=I_{z0}/(u-k_{z0})$, $I_{z0}=I_{z1}/\omega$, $k_{z0}=\beta_0^{-1}$, $I_{z1}=4\pi\mathrm{sign}(\beta_0)Z_0e_0$, $\Omega=\omega_{\rm e}[\omega(\omega+i\nu)]^{-1/2}$, $\beta=\beta_{\rm e}\omega(\omega+i\nu)^{-1}$, $\omega_{\rm e}=(4\pi e_0^2n_{\rm e}/m)^{1/2}$, $\beta_{\rm e}=v_{\rm e}/c$. The functions $\Psi_\lambda(u)$ and $\Psi_\tau(u)$ (and the functions $E_x(u)$ and $E_z(u)$) should be analytical in the half-plane $\mathrm{Im}(u)<0$ and in the point $u=-k_{z0}$. The functions $\Phi_\lambda(u)$ and $\Phi_\tau(u)$ correspond to some linear combinations of the left hand sides of (1), and they should be analytical in the half-plane $\mathrm{Im}(u)>0$ and in the point $u=k_{z0}$, in connection with validity of the equations in the half-space $z>0$. The functions $\Psi_\lambda(u)$ and $\Psi_\tau(u)$ should be bounded in the half-plane $\mathrm{Im}(u)<0$, and also, the equalities

$$\Psi(\pm ik_x)=0\tag{4}$$

for the function $\Psi(u)=k_x\Psi_\lambda(u)-u\Psi_\tau(u)$ should be held. The introduced functions $q_{\lambda,\tau}(\eta)$ are even, $q_{\lambda,\tau}(-\eta)=q_{\lambda,\tau}(\eta)$, they are analytical in the half-planes $\mathrm{Im}(\eta)>0$ and $\mathrm{Im}(\eta)<0$ of the complex plane η, they are bounded and continuous on the whole imaginary axis $\mathrm{Re}(\eta)=0$, and the equalities $q_{\lambda,\tau}(0)=1$ take place. In the case of FD, the functions $q_{\lambda,\tau}(\eta)$ also are analytical on the interval $(-1,1)$ of real axis. Let γ_0 is minimum of such values that $\bar{f}_0(v)=0$ for all $v>1/\gamma_0$ (so that $\gamma_0=1$ for FD, $\gamma_0=0$ for MD), and let Γ_0 is the interval $(\gamma_0,+\infty)$. For $\eta\in\Gamma_0$, denoting with $\Delta_{\lambda,\tau}(\eta)$ the difference of relevant function values at the different sides of the cut carried out along Γ_0, $\Delta_{\lambda,\tau}(\eta)=q_{\lambda,\tau}(\eta-i0)-q_{\lambda,\tau}(\eta+i0)$, one gets

$$\begin{gathered}\Delta_\lambda(\eta)=-4\pi^2i\eta^{-3}\bar{f}_0(1/\eta),\\ \Delta_\tau(\eta)=-4\pi^2i\eta^{-1}\int_{1/\eta}^\infty dvv\bar{f}_0(v).\end{gathered}$$

In particular,

$$\begin{gathered}\Delta_\lambda(\eta)=-3\pi i\eta^{-3},\\ \Delta_\tau(\eta)=3\pi i(\eta^{-3}-\eta^{-1})/2,\end{gathered}$$

for $\eta>1$, in the case of FD, and

$$\begin{gathered}\Delta_\lambda(\eta)=-4i\pi^{1/2}\eta^{-3}\exp(-\eta^{-2}),\\ \Delta_\tau(\eta)=-2i\pi^{1/2}\eta^{-1}\exp(-\eta^{-2}),\end{gathered}$$

in the case of MD. The functions $Q_{\lambda,\tau}(u)$ have the branching points $\pm b$, where $b=[(\gamma_0/\beta)^2-k_x^2]^{1/2}$, $\mathrm{Im}(b)>0$. If the cut G in the half-plane $\mathrm{Im}(u)>0$ from the point b to infinity is made then analytical extension of the functions $Q_{\lambda,\tau}(u)$ with the different path-tracing around the point b gives the different values of the functions, so that

$$\begin{gathered}Q_{\lambda,\tau}(u(1-i0))-Q_{\lambda,\tau}(u(1+i0))=\\ =-\Omega^2\Delta_{\lambda,\tau}(\tilde{\eta}(u)).\end{gathered}$$

At $z\to0+$, for the boundary values of relevant field components, $\widetilde{E}_z(z)$, $\widetilde{E}_x(z)$, and $\widetilde{H}_y(z)$, which are the functions of z obtained with integration of the time dependent components only with respect to t, x and y, with the factor $\exp[i\omega c^{-1}(ct-k_xx-k_yy)]$, one has the equalities

$$\widetilde{E}_z(0+)=i\Psi_\lambda(\infty),$$

$$\widetilde{E}_x(0)=-i\Psi_\tau(\infty),\widetilde{H}_y(0)=-i\Psi'_\tau,\tag{5}$$

where $\Psi'_\tau=\lim_{u\to\infty}\{u[\Psi_\tau(u)-\Psi_\tau(\infty)]\}$. The values of $\widetilde{E}_z(0+)$ and $\widetilde{E}_z(0-)$ for $p\neq1$ may be different, in connection with existence of infinitely thin charge layer varying with time at the sharp plasma boundary [8]. The waves with $k_x>1$ are not emitted into the free half-space $z<0$, as they decreases there exponentially with $z\to-\infty$, so, only the case $k_x\in(0,1)$ is considered.

3. REDUCING OF THE PROBLEM TO INTEGRAL EQUATIONS

With the method similar to one used in [9], [10], and [11] for the problem of wave incidence on the plasma medium, each of the equalities, (2) or (3), together with the requirements of analyticity, is reduced to Riemann-Hilbert boundary problem for the pair of functions, $\{\Phi_{\lambda,\tau}(u),\ \Psi_{\lambda,\tau}(-u)\}$ and $\{\Phi_{\lambda,\tau}(-u),\ \Psi_{\lambda,\tau}(u)\}$, analytical in the different half-planes. And this problem is solved with reducing to integral equations.

Let us introduce the functions

$$S^{\pm}_{\lambda,\tau}(\pm u) = \pm(2\pi i)^{-1}\times \\ \times \int_L dw(w\mp u)^{-1} q_{\lambda,\tau}(\tilde{\eta}(w))\Psi_{\lambda,\tau}(-w),$$

where L is a straight line on the complex plane, passing through the point 0 from left to right, lower the points u, b, and k_{z0}. Moving the path-tracing to the cut G, one can obtain the equality

$$S^{-}_{\lambda,\tau}(u) = (2\pi)^{-1} i\times \\ \times \int_G dw(w-u)^{-1}\Delta_{\lambda,\tau}(\tilde{\eta}(w))\Psi_{\lambda,\tau}(-w), \tag{6}$$

for any $u \notin G$. For the analytical extensions of the functions of u on the line L one gets the equalities

$$S^{+}_{\lambda,\tau}(u) + S^{-}_{\lambda,\tau}(u) = q_{\lambda,\tau}(\tilde{\eta}(u))\Psi_{\lambda,\tau}(-u),$$

$$\begin{gathered}\Phi_\lambda(u) + (1-p)\Psi_\lambda(-u) - uI_z(-u) + \\ +(1-p)\Omega^2[S^-_\lambda(-u) - S^+_\lambda(u)] = \\ = u[I_z(u) - I_z(-u)] + \\ +Q_\lambda(u)[\Psi_\lambda(u) + \Psi_\lambda(-u)] + \\ +(1-p)\Omega^2[S^-_\lambda(u) + S^-_\lambda(-u)],\end{gathered} \tag{7}$$

$$\begin{gathered}\Phi_\tau(u) - (1-p)(1-k_x^2-u^2)\Psi_\tau(-u) + \\ +(1-p)\Omega^2[S^+_\tau(u) + S^-_\tau(-u)] - k_x I_z(-u) = \\ = k_x[I_z(u) - I_z(-u)] + \\ +Q_\tau(u)[\Psi_\tau(u) - \Psi_\tau(-u)] + \\ +(1-p)\Omega^2[S^-_\tau(-u) - S^-_\tau(u)].\end{gathered} \tag{8}$$

The left hand sides of (7) and (8) are analytical upper of L, the right hand side of (7) is even by u, and one of (8) is odd by u. So, both sides of both equations should be analytical on the whole complex plane of the variable u, and they should be equal to the analytical functions determined with asymptotic at $u\to\infty$. As a result, one comes to the equalities

$$\begin{gathered}Q_\lambda(u)[\Psi_\lambda(u) + \Psi_\lambda(-u)] + \\ +(1-p)\Omega^2[S^-_\lambda(u) + S^-_\lambda(-u)] + \\ +u[I_z(u) - I_z(-u)] = 2I_{z0} + 2\Psi_\lambda(\infty).\end{gathered} \tag{9}$$

$$\begin{gathered}Q_\tau(u)[\Psi_\tau(u) - \Psi_\tau(-u)] + \\ +(1-p)\Omega^2[S^-_\tau(-u) - S^-_\tau(u)] + \\ +k_x[I_z(u) - I_z(-u)] = -2u\Psi'_\tau.\end{gathered} \tag{10}$$

The next steps consist of dividing of both sides of (9) on $Q_\lambda(u)$ and ones of (10) on $Q_\tau(u)$, and splitting of each term on the sum of the functions analytical in one of half-planes, upper or lower of L, according to the following example: $F(u) = F^+(u) + F^-(u)$, where

$$F^{\pm}(\pm u) = \pm(2\pi i)^{-1}\int_L dv(v\mp u)^{-1}F(v), \tag{11}$$

for u upper the line L. The terms analytical in the different half-planes, upper and lower of L, should be transposed to the different sides of the equalities, so that both sides of the obtained equalities should be equal to zero, as they are analytical on the whole complex plane and their terms come to zero at $u\to\infty$. For u upper the line L, introducing the functions

$$V^+_\lambda(u) = \int_L dv[\pi i(v-u)]^{-1}\{[Q_\lambda(v)]^{-1} - 1\},$$

$$V^+_\tau(u) = \int_L dv[\pi i(v-u)Q_\tau(v)]^{-1},$$

$$V_\lambda(u,w) = (w^2-u^2)^{-1}\times \\ \times[wV^+_\lambda(u) - uV^+_\lambda(w) + w - u],$$

$$V_\tau(u,w) = (w^2-u^2)^{-1}\times \\ \times[uV^+_\tau(u) - wV^+_\tau(w)],$$

$$K'_{\lambda,\tau}(u,w) = (2\pi)^{-1} i\Omega^2\Delta_{\lambda,\tau}(\tilde{\eta}(w))V_{\lambda,\tau}(u,w),$$

one comes to the equations

$$\begin{gathered}\Psi_\lambda(-u) + (1-p)\int_G dwK'_\lambda(u,w)\Psi_\lambda(-w) = \\ = [1+V^+_\lambda(u)]\Psi_\lambda(\infty) + I_{z0}k_{z0}V_\lambda(u,k_{z0}),\end{gathered} \tag{12}$$

$$\begin{gathered}\Psi_\tau(-u) + (1-p)\int_G dwK'_\tau(u,w)\Psi_\tau(-w) = \\ = -I_{z0}k_xV_\tau(u,k_{z0}) + \\ +\Psi_\tau(\infty) + [uV^+_\tau(u) - V'_\tau]\Psi'_\tau,\end{gathered} \tag{13}$$

where $V'_\tau = \lim_{u\to\infty}[uV^+_\tau(u)]$. In deducing of the equations, it is applied the change of the integration order, with respect to w in (6) and with respect to v in the integral of the type (11) used in the splitting of each equation term on the sum of the functions analytical in the different half-planes. The solving of integral equations gives the functions $\Psi_{\lambda,\tau}(-u)$ for the given $\Psi_\lambda(\infty)$, $\Psi_\tau(\infty)$, and Ψ'_τ. To find the last three quantities, there are three equations: two equations correspond to the conditions (4), and the third one may be obtained from the field consideration in the half-space $z<0$, free of plasma, with account of the continuity of the functions $\widetilde{E}_x(z)$ and $\widetilde{H}_y(z)$ at $z=0$. With account of (5), the equation may be written in the form

$$\Psi_\tau(\infty) + w_z\Psi'_\tau = k_x(k_{z0}-w_z)^{-1}I_{z0}, \tag{14}$$

where $w_z = (1-k_x^2)^{1/2}$. Also, the field consideration in the half-space $z<0$ gives the equality $\widetilde{E}^{\mathrm{r}}_z(0-) = ik_x\Psi'_\tau - ik_x^2(k_{z0}^2-w_z^2)^{-1}I_{z0}$, for the boundary value of relevant field component of the wave radiated there from the boundary.

4. TIME EVOLUTION OF RADIATION FIELD STRENGTH

For the time dependence of the electric field strength in the wave zone, taking into account an axial symmetry of the problem and carrying out the integration with respect to transverse wave number with use of the saddle-point method, one can get the expression

$$E_\theta \approx (4\pi^2 c)^{-1} r^{-1} \int_{-\infty}^{+\infty} d\omega E'(\omega) \exp(-i\omega t_r), \quad (15)$$

where r is the distance from the origin, θ is the angle between the vector $\vec{r}$ and the negative part of OZ axis, $t_r = t - r/c$, $E'(\omega) = i\omega \cot\theta \widetilde{E}_z^{\mathrm{r}}(0-)$ (with $\widetilde{E}_z^{\mathrm{r}}(0-)$ dependent on ω), and in all used relationships it should be taken $\sin\theta$ for k_x. The quantity t_r is the time counted from the instant of the transition radiation signal arriving to the given point at the distance r from the origin. The trends of the field time variation are connected with the singularities of the function $E'(\omega)$ on the complex plane of ω.

It is worthy to point out that an attempt to use the frequency with the positive imaginary part at the stage of the problem formulation (as it may be done in the problem for the field given at initial time instant) leads to divergence of some integrals at $t \to -\infty$, in the problem considered here. But at the stage of solving of the problems, having been formulated for the frequencies, one can move ω from the semi-axes $\omega > 0$ and $\omega < 0$ to the half-plane $\mathrm{Im}(\omega) > 0$ or $\mathrm{Im}(\omega) < 0$. For any time-dependent real quantity $\widetilde{f}(t)$, which may be subjected to Fourier transformation, the transforms, $f(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \widetilde{f}(t)$, obeys the equality $f(\omega) = f^*(-\omega^*)$ (where * indicates complex conjugate). If the transformation may be carried out with real frequencies then the inverse transformation, $\widetilde{f}(t) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} f(\omega)$, gives the equality

$$(d/dt)^n \widetilde{f}(t) = \pi^{-1} \mathrm{Re} \int_0^{\infty} d\omega e^{-i\omega t} (\omega/i)^n f(\omega), \quad (16)$$

for $n \geq 0$, if the integral converges. If the analytical extension from the real axis of ω to any (positive or negative) part of imaginary axis gives purely real values of $f(\omega)$ then in some part of relevant half-plane ($\mathrm{Im}(\omega) > 0$ or $\mathrm{Im}(\omega) < 0$) symmetric with respect to the imaginary axis the analytical extensions from the semi-axes $\omega > 0$ and $\omega < 0$ give the same analytical function, and then, in connection with integrability of the possible singularities on real axis, the path-tracing may be changed from $(-\infty, +\infty)$ to one, partially disposed in the relevant half-plane.

For $t_r < 0$, the time dependencies are connected with the singularities of integrands in the half-plane $\mathrm{Im}(\omega) > 0$. If such singularities were present there, the growth of perturbations would take place, which is impossible in the equilibrium plasma state. And the solution of the equations (4), (12), (13), and (14) in the case $\{\mathrm{Re}(\omega) = 0, \mathrm{Im}(\omega) > 0\}$ gives purely real value of integrand in (15). So, at $t_r < 0$ there is no radiation field in the given point in the wave zone.

For $t_r > 0$, the path-tracing in (15) may be moved to the half-plane $\mathrm{Im}(\omega) < 0$, where the singularities are present. The jump of the quantity (16) at $t_r = 0$ is given with relevant integral over infinitely remote semicircle (instead of the semi-axis $(0, \infty)$). In the case $\{\omega \gg \omega_{\mathrm{e}},\ \omega \gg \nu,\ \Omega^2 \ll \beta_{\mathrm{e}} \ll 1\}$ the kernels in integral equations (12) and (13) are small, and the approximate solutions of the equations may be obtained with the substitution of free summands into integrals, instead of unknown functions. In such a way, one can obtain $E'(\omega) \approx E_2' \omega^{-2}$, where

$$\begin{aligned} E_2' = {} & I_{z1} \sin\theta \omega_{\mathrm{e}}^2 [2(1 - \beta_0^2 \cos^2\theta)]^{-1} \times \\ & \times \{\beta_0 - \beta_0^3 (1 + \beta_0 \cos\theta)^{-1} - \beta_{\mathrm{e}} \bar{v} \times \\ & \times [(1-p)\beta_0 \cos\theta + 2(1+p)(1-\beta_0^2)]/4\}, \end{aligned}$$

$\bar{v} = 4\pi \int_0^{\infty} dv v^3 \bar{f}_0(v)$ ($\bar{v} = 3/4$ for FD, $\bar{v} = 2/\pi^{1/2}$ for MD). As a result, the strength E_θ at $t_r = 0$ varies continuously, and its time derivative has the jump, which is approximately equal to $(-2\pi c)^{-1} r^{-1} E_2'$.

As plasma is nonrelativistic, $\beta_{\mathrm{e}} \ll 1$, then, neglecting the terms, which are small together with β_{e}, one can obtain the approximate equality

$$\begin{aligned} E'(\omega) \approx {} & I_{z1} \sin\theta \cos\theta \beta_0 \Omega^2 (1 - \beta_0^2 + \beta_0 k_{z\mathrm{e}}) \times \\ & \times [(k_{z\mathrm{e}} \cos\theta + \sin^2\theta)(k_{z\mathrm{e}} + \cos\theta)]^{-1} \times \\ & \times [(1 - \beta_0^2 \cos^2\theta)(1 + \beta_0 k_{z\mathrm{e}})]^{-1}, \end{aligned} \quad (17)$$

where $k_{z\mathrm{e}} = (\cos^2\theta - \Omega^2)^{1/2}$. Using (17), one can estimate the time variation of the field strength at the beginning stage of the pulse dumping. If the collision frequency is low, $\nu \ll \omega_{\mathrm{e}}$, and if θ is not very close to 0 or $\pi/2$, then, after relevant moving of path-tracing, the integral in (15) may be estimated by the contribution of the vicinity of the branching point at $\omega \approx \omega_{\mathrm{e}}/\cos\theta$, near which the value of $k_{z\mathrm{e}}$ is small. For $\omega_{\mathrm{e}}^{-1} \ll t_r \ll \nu^{-1}$, the estimation gives

$$\begin{aligned} E_\theta \approx {} & (2\pi c)^{-1} r^{-1} I_{z1} \omega_{\mathrm{e}} \beta_0 \cos^2\theta \times \\ & \times (1 - \beta_0 \cos\theta - \beta_0^2 \sin^2\theta) \times \\ & \times [\sin^3\theta (1 - \beta_0 \cos\theta)]^{-1} \times \\ & \times \{2\pi^{-1} \cos\theta (\omega_{\mathrm{e}} t_r)^{-3}\}^{1/2} \times \\ & \times \cos[(\omega_{\mathrm{e}}/\cos\theta) t_r - (\pi/4)]. \end{aligned} \quad (18)$$

At $t_r \gg \nu^{-1}$ the contribution of the mentioned branching point to the field strength value should be modified with the factor $\exp(-\nu t_r/2)$, which makes this contribution relatively small. For the strength value $\langle E_\theta \rangle$ obtained from (18) with averaging by the cycle of oscillation one can get the estimation

$$|\langle E_\theta \rangle| \sim (2\pi c)^{-1} r^{-1} I_{z1} \omega_{\mathrm{e}} (\omega_{\mathrm{e}} t_r)^{-5/2}. \quad (19)$$

At $t_r \to \infty$ the strength is determined by the contribution of the branching point $\omega = 0$. For $\{\omega \ll \nu,\ \omega \ll \omega_{\mathrm{e}}\}$, and any ν/ω_{e}, from (17) one can get

$$\begin{aligned} E'(\omega) \approx {} & -I_{z1} \sin\theta \beta_0 \{(1 - \beta_0^2 \cos^2\theta)^{-1} + \\ & + k_{z\mathrm{e}} [\Omega^2 \cos\theta (1 - \beta_0 \cos\theta)]^{-1}\}, \end{aligned}$$

and estimation of the integral over the path-tracing near $\omega = 0$ for $\{t_r \gg \omega_e^{-1},\ t_r \gg \nu^{-1}\}$ gives

$$\begin{aligned} E_\theta \approx &- (2\pi c)^{-1} r^{-1} I_{z1} \beta_0 \sin\theta \times \\ &\times [2\cos\theta(1-\beta_0\cos\theta)]^{-1} \times \\ &\times \{\pi^{-1}\nu\omega_e(\omega_e t_r)^{-3}\}^{1/2}. \end{aligned} \tag{20}$$

But if there exists the frequency range, corresponding to the anomalous skin effect conditions, in which $\nu \ll \omega \ll \beta_e\omega_e$ (with $\beta_e \ll 1$), then in this range the relationship

$$\begin{aligned} E'(\omega) \approx &- I_{z1}\sin\theta\beta_0(1-\beta_0\cos\theta)^{-1} \times \\ &\times [(1+\beta_0\cos\theta)^{-1} + \\ &+ \exp(2i\pi/3)\bar{Z}\beta_e(A\cos\theta)^{-1}] \end{aligned} \tag{21}$$

takes place, in which $A = (\beta\Omega)^{2/3}$,

$$\bar{Z} = (48/\pi^2)^{1/6}[\sin(\alpha_p/3)/\sin(\alpha_p/2)]^2, \tag{22}$$

$\alpha_p = \arccos(p)$. And the estimation of relevant integral for $(\beta_e\omega_e)^{-1} \ll t_r \ll \nu^{-1}$ gives

$$\begin{aligned} E_\theta \approx &- (2\pi c)^{-1} r^{-1} I_{z1}\beta_0 \sin\theta \times \\ &\times 3^{1/2}\Gamma(5/3)\bar{Z}[2\pi\cos\theta(1-\beta_0\cos\theta)]^{-1} \times \\ &\times \omega_e\beta_e^{1/3}(\omega_e t_r)^{-5/3}, \end{aligned} \tag{23}$$

where Γ is Euler function. The relationship (21) may be obtained in impedance approximation [2] with use of normalized impedance $\bar{Z}$ value (22) found in [12] for any p value. Somewhat more detailed consideration of the problem for the anomalous skin effect conditions is presented in [13]. At $(\beta_e\omega_e)^{-1} \ll t_r \ll \nu^{-1}$ the value of E_θ given by (23) is much greater than the value of E_θ given by (20) and the value of $\langle E_\theta \rangle$ given by (19).

A possibility of experimental measurement of time evolution of the transition radiation pulse is restricted by quantum effects. The expression for the radiated energy [14] contains the dimensional factor, $e_0^2\omega_e c^{-1}$, which may be written in the form $\alpha\hbar\omega_e$, where $\alpha = e_0^2(\hbar c)^{-1} \approx 1/137$. And the probability of presence of photon with the frequency of order of plasma one in the transition radiation formed by a single elementary charge crossing a plasma boundary is small. But if the boundary is crossed with compact bunch of particles, which number is much greater than $1/\alpha$, then the electric field strength evolution may be described in the frames of classic electrodynamics.

Considering a generation of transition radiation by the bunches obtained on the pulsed accelerator, it is worthy to estimate the parameters of the bunch and plasma medium, for which the generation may be energetically efficient. An increase of particle number in the bunch leads to increase of the ratio of the radiated energy to the bunch kinetic energy. When the particle number is small the ratio is approximately proportional to it. The ratio is connected with the efficiency of generation of transition radiation, and it is saturated when the removing of the bunch from plasma is accompanied with considerable decrease of the bunch kinetic energy, and the part of the energy may be spent on radiation. To obtain the radiated energy value of the order of the kinetic energy value of the relativistic electron bunch, transiting from plasma to vacuum, for the given bunch dimension D and the particle number N, it is necessary to increase N up to the value, at which $N^2e_0^2/D \sim Nmc^2$, so, $D \sim Nr_e$, where r_e is classic electron radius, $r_e = e_0^2/(mc^2) \approx 2.8 \cdot 10^{-13}$cm. The main part of the energy is radiated at the frequency of the order of plasma one (assuming that bunch is not ultrarelativistic), and for its effective generation the relationship $D \sim c/\omega_e$ should be held, and so, it should be $N \sim \omega_{e0}/\omega_e$, where $\omega_{e0} = c/r_e \approx 1.06 \cdot 10^{23}\mathrm{s}^{-1}$. The charge of bunch is equal to Ne_0. When the bunch goes out of plasma the peak current is of order of Alfven current, $I_A = e_0\omega_{e0} \approx 1.7 \cdot 10^4$A. For example, the effective generation of wide-band signal with characteristic frequency 1 GHz, may be achieved for the following parameters: plasma frequency $\omega_e \sim 6 \cdot 10^9\mathrm{s}^{-1}$, plasma density $n_e \sim 10^{10}\mathrm{cm}^{-3}$, bunch dimension $D \sim 5$ cm, particle number in the bunch $N \sim 2 \cdot 10^{13}$, charge of the bunch $Ne_0 \sim 3 \cdot 10^{-6}$C, particle density in the bunch $ND^{-3} \sim 2 \cdot 10^{11}\mathrm{cm}^{-3}$.

5. CONCLUSIONS

It is considered the time variation of the field strength of the radiation caused by the charged particle transition through the plasma-vacuum boundary, with plasma consideration in the kinetic approximation. Before the signal about crossing the boundary by the particle comes to the given point in the wave zone, the radiation field is absent. The signal coming is accompanied with the jump of the strength time derivative. Then there take place the oscillations of the strength with amplitude decreasing with time by the power law with the exponent $-3/2$. At the final stage there takes place the monotonous strength decrease as the power with exponent $-3/2$. If the collision frequency in plasma is much less than plasma one then there exists the time range, in which, together with the dumped oscillations, the monotonous decay by the power law with the exponent $-5/3$ is present. It is the realization of anomalous skin effect in the corresponding frequency range. The possibility of experimental measurement of the obtained pulse characteristics is restricted with quantum effects, and it can be achieved with the sufficiently large particle number in the compact bunch going out of plasma medium. The parameters of the electron bunch and plasma medium required for effective generation of transition radiation in the given frequency range are determined by the conditions that the bunch dimension should be of order of the reciprocal wave number and the potential energy of electrostatic interaction of the bunch should be of order of the bunch kinetic energy.

References

1. V. Ginzburg, I. Frank. Radiation of the uniformly moving electron arising with its transition from one medium to other // *JETP.* 1946, v.16, N.1, p.15-28.
2. E.A. Kaner, V.M. Jakovenko. To the theory of transition radiation // *JETP.* 1962, v.42, N.2, p.471-478.
3. B.M. Bolotovskii, A.V. Serov. Peculiarities of transition radiation field // *UFN.* 2009, v.179, N.5, p.517-524.
4. N.F. Shul'ga, S.V. Trofymenko. High-energy wave packets. 'Half-bare' electron // *Journ. of Kh. Nat. Univ., Phys. Series "Nuclei, Particles, Fields".* 2013, iss. 1(57), p.59-69.
5. V.A. Balakirev, G.L. Sidel'nikov. *Transition radiation of the modulated electron beams in nonhomogeneous plasma. Review.* Kharkov, KIPT, 1994, 104p.
6. V.A. Balakirev, N.I. Gaponenko, A.M. Gorban', et al. Excitement TEM-horn antenna by impulsive relativistic electron beam // *PAST. Series "Plasma Physics" (5).* 2000, N.3, p.118-119.
7. V.A. Buts, I.K. Koval'chuk. Elementary mechanism of oscillations excitation by electron beam in a cavity // *Ukr. Fiz. Zh.* 1999, v.44, N.11, p.1356-1363.
8. A.N. Kondratenko. *Surface and volume waves in the bounded plasma.* Moscow: "Energoatomizdat", 1985, 208 p. (in Russian).
9. G.E.H. Reuter, E.H. Sondheimer. The theory of the anomalous skin effect in metals // *Proc. Roy. Soc.* 1949, v.195, p.336-364.
10. V.P. Silin, A.A. Rukhadze. *Electromagnetic properties of plasma and plasma-like media.* Moscow: "Gosatomizdat", 1961, 244 p. (in Russian).
11. V.I. Miroshnichenko. Electromagnetic properties of half-infinite plasma for diffusive electron scattering from the boundary // *ZhTF.* 1966, v.36, N.6, p.1008-1016 (in Russian).
12. L.E. Hartmann, J.M. Lattinger. Exact solution of the integral equation for the anomalous skin effect and cyclotron resonance in metals // *Phys. Rev.* 1966, v.151, N.2, p.430-433.
13. V.I. Miroshnichenko, V.M. Ostroushko. Skin-effect influence on transition radiation // *PAST. Series "Nuclear Physics Investigations" (64).* 2015, N.3(97), p.103-108.
14. V.L. Ginzburg, V.N. Tsytovich. Some questions of transition radiation and transition scattering theory // *UFN.* 1978, v.126, N.4, p.553-608 (in Russian).

ВЛИЯНИЕ СКИН-ЭФФЕКТА НА ЭВОЛЮЦИЮ ИМПУЛЬСА ПЕРЕХОДНОГО ИЗЛУЧЕНИЯ СО ВРЕМЕНЕМ

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Изменение со временем напряжённости поля излучения, вызванного переходом заряженной частицы через границу плазма-вакуум, получено для волновой зоны. Показано, что в почти бесстолкновительной плазме, наряду с затухающими колебаниями на начальной стадии затухания импульса и монотонным уменьшением напряжённости по степенному закону с показателем $-3/2$ на конечной стадии, существует диапазон времени со степенным уменьшением с показателем $-5/3$, соответствующим аномальному скин-эффекту. Рассматриваются условия, при которых возможно экспериментальное измерение полученных характеристик импульса.

ВПЛИВ СКІН-ЕФЕКТУ НА ЕВОЛЮЦІЮ ІМПУЛЬСУ ПЕРЕХІДНОГО ВИПРОМІНЮВАННЯ З ЧАСОМ

В. Остроушко

Зміну з часом напруженості поля випромінювання, викликаного переходом зарядженої частинки через межу плазма-вакуум, отримано для хвилевої зони. Показано, що у майже беззіткненній плазмі, поряд зі спадними коливаннями на початковій стадії загасання імпульсу та монотонним зменшенням напруженості за степеневим законом з показником $-3/2$ на кінцевій стадії, існує діапазон часу зі степеневим зменшенням з показником $-5/3$, відповідним аномальному скін-ефекту. Розглядаються умови, за яких можливе експериментальне вимірювання отриманих характеристик імпульсу.