# ON ANTI-CHUDAKOV EFFECT IN ULTRARELATIVISTIC ELECTRON-POSITRON PAIR IONIZATION LOSS IN THIN TARGET

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The ionization loss of an ultrarelativistic e<sup>+</sup>e<sup>-</sup> pair in a system of two separated targets – thick upstream, in which the pair is created, and thin downstream – is considered. The main attention is drawn to investigation of the effect of enhancement of the pair loss in thin target comparing to the sum of independent electron and positron losses (the one opposite to the Chudakov effect of pair ionization loss suppression). The problem is studied without the use of approximation of parallel electron and positron velocities which is usually applied for theoretical consideration of pair ionization loss. The dependence of the magnitude of the effect on the pair energy and its divergence angle is studied. The possibility of existence of such effect in the asymptotic case of large separations between the targets is shown. A simplified analytical expression describing the pair ionization loss dependence on distance between the targets under the conditions of manifestation of the discussed effect is obtained.

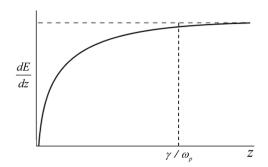
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#### 1. INTRODUCTION

When a charged particle moves through substance it experiences inelastic collisions with atomic electrons, which result in the loss of its kinetic energy. The average amount of the particle energy decrease per unit path due to such collisions is known as ionization loss of the particle. Let us note that it does not include the radiative energy loss due to particle multiple scattering on atomic nuclei but just the loss on ionization of the atoms and excitation of their electron subsystem. Ionization loss of a single particle moving in homogeneous and boundless substance is defined by Bethe-Bloch formula, taking into account its various forms for different types of particles and their energies (see [1] and references threin).

In [2] it was shown for the first time that ionization loss of a system of particles moving close to each other differs from the plain sum of independent particles losses. Here the ionization loss of a high-energy electron-positron pair was calculated in the vicinity of its creation point in substance. It was shown that in this region the pair loss is significantly suppressed comparing to the sum  $\Sigma^{\pm}$  of independent electron and positron losses, which became known as Chudakov effect. With the increase of the distance from the creation point the ionization loss monotonically grows reaching finally the limit corresponding to this sum. The reason of the Chudakov effect is the mutual screening (destructive interference) of the electron's and the positron's Coulomb fields which dis-

appears when the particles fly apart from each other on large distance. The typical dependence of the pair loss on the distance z from the creation point is presented on Fig.1. The dashed line here corresponds to the value of  $\sum^{\pm}$ ,  $\omega_p$  is the plasma frequency of the substance,  $\gamma$  is a characteristic value of electron or positron Lorenz-factor. Let us note that in [2], as well as in further papers reporting theoretical and experimental investigations of  $e^-e^+$  pairs ionization loss (for detailed list of references see, e.g. [3]), this quantity was studied in the same substance in which the pairs were created. The substance was considered as homogeneous and boundless.



**Fig.1.** Typical dependence of  $e^-e^+$  pair ionization loss in the same substance in which the pair is created on distance from the creation point

In [4] and [5] the influence of boundary effects upon  $e^+e^-$  pair ionization loss was investigated. It

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was shown that such effects are significant if an ultrarelativistic pair loses energy moving through a sufficiently thin layer of substance (thin plate). It was assumed that the pair is created in a target situated in front of the plate on some distance from it. The boundary effects in this case are caused by the transformation of the electromagnetic field around the pair after its emission from the target in which it is created. Such transformation leads to generation of the so-called transition radiation which affects upon the pair loss in thin plate. As shown, the pair ionization loss in this case increases within much larger distance from its creation point than the one  $\sim \gamma/\omega_p^{-1}$ , on which the Chudakov effect takes place in a boundless medium. Moreover the possibility of existence of the effect opposite to the Cudakov one (anti-Chudakov effect) was indicated in this case. It is the exceeding by the value of the pair ionization loss of the sum of independent electron and positron losses on some distance from the pair creation point.

Let us note that a single experiment devoted to the study of e<sup>+</sup>e<sup>-</sup> pairs ionization loss, in which the pair creation and ionization loss processes occurred in different targets, was reported in [6]. However, the investigation of boundary effects was not the aim here and relatively thick targets were used.

In the papers [4] and [5] a special simplifying approximation was used for calculation of the pair ionization loss. Its essence is the following. Let the transverse (with respect to the average direction of the pair motion) distance between the electron and the positron equal s on some distance  $z \sim \gamma s$  from the pair creation point. In the considered approximation the pair ionization loss for such z is assumed to equal the ionization loss of the electron and the positron which constantly move with parallel velocities separated by the distance s in the transverse direction. At high energies of the particles  $(\gamma \gg 1)$ such approximation is validated by the small value of the pair divergence angle and has always been used for theoretical considerations of pairs ionization loss (see Ref. in [3]). We will further call it the parallel approximation.

In [7] and [8] it was shown that under some conditions the parallel approximation is not strictly valid for consideration of pairs ionization loss in thin targets. More precise study without the use of such approximation, presented in these papers, revealed some new features of such loss concerning, particularly, its asymptotic (for  $z \to \infty$ ) behaviour at ultrahigh energies of the pair.

The present paper is devoted to theoretical study of the anti-Chudakov effect in ionization loss of an ultrarelativistic e<sup>+</sup>e<sup>-</sup> pair in thin target without the use of parallel approximation. Such generalized approach (comparing to the one applied in [5], where the discussed effect was predicted with the use of such approximation) allows to investigate the dependence

of the value of the effect on various parameters associated with the pair, such as its divergence angle, the position of its creation point inside the target etc. Such results may be valuable in connection with the problem of experimental study of such effect.

### 2. ASYMPTOTIC VALUE OF THE EFFECT

Let us consider ionization loss of an ultrarelativistic e<sup>+</sup>e<sup>-</sup> pair in the plate (thin target) situated in vacuum on distance  $z_1$  from the layer of substance (thick target) in which the pair is created, for instance, by a high-energy photon (Fig.2). In the paper [8] the expression defining the value of such loss per unit path was obtained. Let  $\eta_p$  and  $\omega_p$  be the plasma frequencies of the thin and the thick targets respectively. Let us require the thickness a of the thin target to satisfy the condition  $a \leq \eta_p^2/I$ , where I is the mean ionization potential of its atoms. The thickness of the other target is supposed to significantly exceed the value  $\omega_n^2/I$ . By  $z_0$  we denote the distance between the downstream surface of the thick target and the pair creation point inside it. We will study the simplest case corresponding to equal energy distribution between the particles of the pair. By  $\gamma$  we will denote the Lorenz-factor of a single particle. In this case the discussed expression can be presented in the next form:

$$\frac{dE}{dz} = 2\eta_p^2 e^2 \{ E_c + E_{tr} + E_{int} \}.$$
 (1)

Here the term proportional to

$$E_c = \ln \kappa_0 - \frac{1}{2} \tag{2}$$

describes the ionization produced in the plate by the proper Coulomb fields of the electron and the positron. The appearance of the value  $\kappa_0$  in (2) is associated with the fact that we consider the so-called

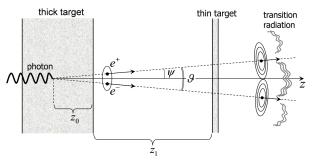


Fig.2. Traversal of the thin target by the pair created in the thick target

restricted ionization loss. It is caused by interactions of charged particles ( $e^+$  and  $e^-$  in our case) with the atomic electrons which are accompanied by the momentum transfer less than  $q_0 = \kappa_0 I/\gamma$ . The restricted loss is often more preferable for experimental measurement than the value of the total ionization loss.

<sup>&</sup>lt;sup>1</sup>It comes from the characteristic value  $1/\gamma$  of the pair divergence angle and transversal (with respect to the particle velocity direction) size  $1/\omega_p$  of a charged particle field in a polarizable medium. Here and further we take to speed of light to equal unit.

The term in (1) containing

$$E_{tr} = \ln(\omega_p \gamma / I) - 1 \tag{3}$$

describes the contribution to the ionization of the plate from the transition radiation generated during the pair emission from the target in which it is created. Here we assume that the condition

$$q_0 \gg \omega_p \gg I/\gamma$$
 (4)

is fulfilled.

The term associated with  $E_{int}$  represents the influence of interference of all the fields taking place in the considered process (proper fields of the particles and transition radiation fields generated by them) with each other upon the pair ionization loss in the plate. It has the following form:

$$E_{int} = G(\psi, \gamma) + F(z_1, \psi, \gamma), \tag{5}$$

where  $\psi$  is half of the pair divergence angle (see Fig.2) and

$$G = -W_p^4 \int_0^{\kappa_0} \frac{d\kappa \kappa}{2\pi} (\kappa^2 - \alpha^2) \int_0^{2\pi} d\varphi \frac{\cos[4\kappa \alpha Z_0 \cos \varphi]}{K_+ K_- \Omega_- \Omega_+}$$
(6)

and

$$F = -\frac{1}{2\pi} \int_{0}^{\kappa_0} d\kappa \int_{0}^{2\pi} d\varphi (f_1 + f_2 + f_3)$$
 (7)

with

$$\begin{split} f_1 &= \kappa(\kappa^2 - \alpha^2) \cos[4\kappa\alpha(Z_1 + Z_0)\cos\varphi] / (K_+ K_-), \\ f_2 &= 2W_p^2 \kappa(K_+ - 1)\cos[Z_1(1 + \kappa^2 + \\ &+ 2\kappa\alpha\cos\varphi)] / (K_+^2 \Omega_+), \\ f_3 &= -2W_p^2 \kappa(\kappa^2 - \alpha^2)\cos[2\kappa\alpha(Z_1 + 2Z_0)\cos\varphi + \\ &+ Z_1(1 + \kappa^2)] / (K_+ K_- \Omega_-) \end{split}$$

and 
$$K_{\pm} = \kappa^2 + \alpha^2 \pm 2\kappa\alpha\cos\varphi + 1$$
,  $\Omega_{\pm} = K_{\pm} + W_p^2$ ,  $W_p = \omega_p \gamma/I$ ,  $\alpha = \psi \gamma$ ,  $Z_{1,0} = z_{1,0}I/(2\gamma^2)$ .

The first two terms in (1) equal the asymptotic  $(z_1 \to \infty)$  value of the sum of independent losses of the electron and the positron in the thin target in the considered process, which we will denote as  $S^{\pm}$ . Each particle in this case is assumed to move from  $z = -\infty$  to  $z = +\infty$  normally traversing the targets. The quantity  $S^{\pm}$  differs from the corresponding quantity  $\Sigma^{\pm}$  in a boundless medium. On the one hand, such difference is associated with the contribution of transition radiation generated during the particle traversal of the thick target. On the other hand – with the fact that in the considered thin target the particle ionization loss occurs without the so-called density effect<sup>2</sup>, which is significant in boundless medium (for details see [11]).

The quantity  $E_{int}$ , which describes the difference of the pair ionization loss value (1) from  $S^{\pm}$  consists of two terms. The term F depends on the distance  $z_1$  between the plates and tends to zero at large values of this quantity. It happens due to quick oscillations of the cosines in the integrand in (7) at large  $z_1$ . In [4] and [8] it was shown that the pair ionization loss should cease to depend on  $z_1$  and become constant at  $z_1 \gg L_I \sim 2\gamma^2/I$ . The quantity  $L_I$  is of the order of so-called formation length of transition radiation waves with frequencies close to the value of I. It is these frequencies which make the main contribution to ionization loss.

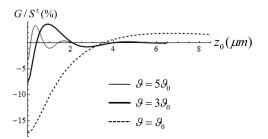
The term G in (5) does not depend on  $z_1$  and remains even in the case of rather large separation between the targets. It defines the difference between the asymptotic values of the pair ionization loss and the sum  $S^{\pm}$  of independent  $e^{+}$  and  $e^{-}$  losses in the considered process. The existence of such difference is one of the new characteristic features of ionization loss in the discussed situation which differs it from the case of "classical" Chudakov effect in a boundless medium where such difference is absent. The appearance of the term G in the expression for pair ionization loss is associated with going beyond the parallel approximation in our consideration. In [8] such term was investigated in a simplified case corresponding to pair creation in the immediate vicinity of the downstream surface of the thick target  $(z_0 \to 0)$ . It was shown that in this case the value of G is negative and slowly grows in absolute value with the increase of the energy of the pair. At the energies of about 100 GeV the relative value of such asymptotic suppression of ionization loss may exceed 50%. The approximation  $z_0 \approx 0$  works well at rather high energies of the pair at which the condition  $z_0/L_I \ll 1$  is fulfilled by wide margin. Let us note that the asymptotic difference between dE/dz of the pair and  $S^{\pm}$  in the considerd case is caused by the mutual interference of the transition radiation fields generated by the electron and the positron upon their emission from the thick target. Generally speaking, such interference does not vanish even on rather large distance from this target.

Let us consider more general case of  $z_0 \neq 0$  and investigate the dependence of the value of G on  $z_0$ . It is natural to expect such dependence to be most pronounced at rather low (however, ultrarelativistic) energies of the pair when the condition  $z_0/L_I \ll 1$  has the chance to be violated.

Fig.3 shows the ratio  $G/S^{\pm}$  as a function of  $z_0$  for the energy of the pair equal 100 MeV. It is obtained on the basis of numerical calculation of the integrals in the expression (6). The calculation is presented for three values of the pair divergence angle  $\vartheta=2\psi$ . Here and further we use the values  $\omega_p\approx 30$  eV and I=175 eV (as for silicon) of the corresponding quantities. The figure demonstrates that in the case  $z_0\to 0$  the value of G is negative and the asymptotic suppression of dE/dz comparing

<sup>&</sup>lt;sup>2</sup>It was predicted by Fermi [9] and thoroughly studied by Stermheimer [10].

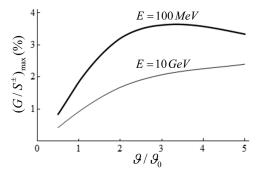
to  $S^{\pm}$  takes place. It grows with the decrease of  $\vartheta$  and for the most probable value  $\vartheta_0=2/\gamma$  of this angle becomes rather significant.



**Fig.3.** Dependence of the relative value of asymptotic difference between dE/dz and  $S^{\pm}$  on  $z_0$  for different pair divergence angles  $\vartheta$ 

However, with the slight increase of  $z_0$  (up to several microns, depending on  $\vartheta$ ) the sign of G changes and it becomes positive. This means asymptotic exceeding by the magnitude of the pair loss of the value  $S^{\pm}$ . The maximum positive value of  $G/S^{\pm}$ , on the contrary (comparing to the maximum negative one), is larger for the values of the pair divergence angle exceeding the most probable one. For the considered value of the pair energy it can reach the value of about 4%.

With the increase of the energy of the pair for the fixed value of  $\alpha = \psi \gamma$  the scale of the distance  $z_0$  on which the quantity G noticeably changes grows proportional to the square of the energy. Indeed, the analysis of the integrand in (6) shows that for the fixed values of  $\alpha$  and  $Z_0$  the single quantity here depending on the energy is  $W_p = \omega_p \gamma / I$ , which is a part of expressions for  $\Omega^{\pm}$  as well. However, taking into account the condition (4), we can approximately set  $\Omega^{\pm} \approx W_n^2$ , which leads to disappearance of  $W_p$  from (6) at all. Let us note that such simplification is valid only for  $z_0 \neq 0$  when it does not affect upon the convergence of the integrals in (6). In this case the values of G for different  $\gamma$  become equal for a fixed value of  $Z_0 = z_0 I/(2\gamma^2)$ . Therefore, with the increase of the pair energy the asymptotic anti-Chudakov effect takes place for much larger values of  $z_0$  comparing to the considered case of E = 100 MeV.



**Fig.4.** Dependence of the maximum positive relative magnitude of asymptotic difference between dE/dz and  $S^{\pm}$  on the pair divergence angle for two values of its energy

Fig.4 shows the dependence of the maximum positive relative magnitude  $(G/S^{\pm})_{max}$  of the asymptotic anti-Chudakov effect on the angle of the pair divergence for two values of its energy E. Here we see that the considered magnitude increases with the decrease the energy. It happens due to logarithmic decrease of the value  $S^{\pm}$  with the decrease of E, which follows from (2) and (3). Therefore low (however, still ultrarelativistic) energies of e<sup>+</sup>e<sup>-</sup> pairs seem more preferable for the experimental search of manifestation of the considered effect. Further for certainty we we will concentrate on the energy E = 100 MeV, for which the formation length is  $L_I \sim 20 \ \mu \text{m}$ . The point is that the measurements at lower energies might be technically very difficult due to small size of  $L_I$  in this case.

To conclude this section it is necessary to note that the consideration presented here is restricted by the values of distances between the targets for which the thin target can be considered as having unlimited transversal size. It is the case if the conditions  $z_1 \ll d\gamma$  and  $d \gg \gamma/I$  (where d is the transversal linear size of the target) are fulfilled. For instance, if E=100 MeV and d=1 cm, the presented consideration works for  $z_1 \ll 100$  cm (the second condition is fulfilled by wide margin).

### 3. ESTIMATION OF THE MAXIMUM VALUE OF THE ANTI-CHUDAKOV EFFECT

In the previous section the pair ionization loss was considered for asymptotically large separations between the targets  $(z_1 \gg 2\gamma^2/I)$ . It was shown that under some conditions the value of such loss may slightly exceed the sum  $S^{\pm}$  of independent losses of  $e^+$  and  $e^-$  traversing the same system of two targets and the value of such effect was estimated.

The maximum value of the considered effect, however, can be expected to take place for smaller values of  $z_1$ . Therefore, in order to define the optimal parameters for its observation it is necessary to consider the pair ionization loss for arbitrary separations between the targets. For very small values of such separation the pair ionization loss is significantly suppressed and grows with the increase of  $z_1$ . It is the Chudakov effect which takes place in this case. It is natural to expect the maximum value of the discussed anti-effect for  $z_1 \sim L_I$  at which dE/dz approaches its asymptotic value. Let us derive a simplified analytical expression for the term F defining the dependence of the pair ionization loss on  $z_1$  in this case.

Let us begin with the term proportional to  $f_2$  in the expression (7) for F denoting it as  $F_2$ . For the considered values of  $z_1$  it is possible to use the stationary phase method [12] for calculation of one of the integrals (let it be the one with respect to  $\varphi$ ) here. The stationary phase points  $\varphi_0$  here are defined from the equation

$$\frac{\partial}{\partial \varphi}(1+\kappa^2+2\kappa\alpha\cos\varphi)=0,$$

which has three solutions within the integration interval:  $\varphi_0 = 0, \pi, 2\pi$ . Since the points  $\varphi_0 = 0, 2\pi$  lie on the edges of this interval their contributions are half as large as the one from a point situated inside it. However, the values of the integrand  $f_2$  in these points are equal and we can substitute their contribution by a complete (not half of it) contribution, e.g. from the point  $\varphi_0 = 0$ . After some transformations the contributions from the points  $\varphi_0 = 0, \pi$  can be united into one integral which has the following form:

$$F_2 \approx 2W_p^2 \sqrt{\frac{\pi}{\alpha Z}} Re \int_{-\infty}^{+\infty} d\kappa Q(\kappa) e^{iZq(\kappa) - i\pi/4},$$
 (8)

where

$$Q(\kappa) = \frac{\sqrt{\kappa}(\kappa - \alpha)^2}{[(\kappa - \alpha)^2 + 1]^2[(\kappa - \alpha)^2 + 1 + W_p^2]}$$

and  $q(\kappa) = 2\alpha\kappa - \kappa^2 - 1$ . Here  $\sqrt{\kappa}$  is a single-valued branch of the analytic function  $w = \sqrt{\kappa}$  with the argument value  $\arg(w) = \arg(\kappa)/2$ . It equals  $|\sqrt{\kappa}|$  for  $\kappa > 0$  and  $i|\sqrt{\kappa}|$  for  $\kappa < 0$ .

In the considered case  $Z \sim 1$  and it is  $\kappa \sim 1$  that make the main contribution to the integral in (8). Thus, using the condition  $W_p \gg 1$ , which follows from (4), and taking into account that  $\alpha \sim 1$ , we can make the following simplification:  $(\kappa - \alpha)^2 + 1 + W_p^2 \approx W_p^2$ . Let us also make the substitution  $\kappa - \alpha = x$  in (8). In the result we obtain the expression for  $F_2$  in the following form:

$$F_2 \approx 2\sqrt{\frac{\pi}{\alpha Z}} Re \left\{ e^{i\chi(Z,\alpha)} \int_{-\infty}^{+\infty} dx \frac{x^2 \sqrt{x+\alpha}}{(x^2+1)^2} e^{-iZx^2} \right\},$$
(9)

where  $\chi = Z(\alpha^2 - 1) - \pi/4$ .

As shown in the previous section for the case  $Z\gg 1$ , the highest magnitudes of the considered antieffect should be expected for the values of  $\vartheta$  exceeding the most probable one (which corresponds to  $\alpha>1$ ). Therefore, taking into account that it is  $x\leq 1$  that make the main contribution to the integral in (9), we will neglect here the value of x comparing to  $\alpha$  and write:  $\sqrt{x+\alpha}\approx\sqrt{\alpha}$ . After such simplification the obtained integral can be analytically calculated (see [13]) and we finally obtain:

$$F_2 \approx -\sqrt{\frac{\pi}{Z}} Re \left\{ e^{i\chi(Z,\alpha)} \left[ \sqrt{2\pi Z} (1+i) - -\pi e^{iZ} (1+2iZ) [1 - \Phi(\sqrt{iZ})] \right] \right\}.$$
 (10)

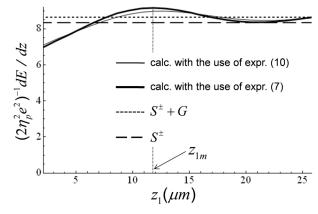
Here

$$\Phi(\sqrt{iZ}) = e^{i\pi/4} \frac{2}{\sqrt{\pi}} \int_{0}^{\sqrt{Z}} e^{-it^2} dt$$

is the error function. The analogous procedure performed for the terms in (7) associated with  $f_1$  and  $f_3$  shows that for the considered values of Z they are small compared to the term  $F_2$ . Therefore we will neglect them and write  $F \approx F_2$ .

It is natural to expect that the manifestation of the anti-Chudakov effect will be mostly pronounced at  $z_1 \sim 2\gamma^2/I$  for the values of  $z_0$  for which it has the largest value at  $z_1 \to \infty$  (see previous section). Therefore it is such values of  $z_0$  (which depend on E and  $\vartheta$ ) which we will consider further. For instance, in the case 100 MeV pair with the divergence angle  $\vartheta = 3\vartheta_0$  (see Fig.3) it is  $z_0 \approx 1~\mu \mathrm{m}$ .

Fig.5 shows the example of the dependence of pair ionization loss in thin target on distance  $z_1$  between the targets in the region  $z_1 \leq 2\gamma^2/I$ . Here we see that there exists a maximum (at the point  $z_1 = z_{1m}$ ) in  $z_1$ -dependence of dE/dz in the considered case. It is another new feature which differs the pair loss in a thin plate from its loss in a boundless medium, where such dependence is monotone (see Fig.1)<sup>3</sup>. In the considered case the magnitude of anti-Chudakov effect at the position of dE/dz maximum is more than two times larger than its asymptotic magnitude at  $z_1 \gg 2\gamma^2/I$ .



**Fig.5.** Dependence of pair ionization loss in thin target on separation between the targets for  $E = 100 \, MeV$ ,  $\vartheta = 3\vartheta_0$  and  $z_1 = 1 \, \mu m$ 

The figure shows that the approximation (10) for the term F in the interference factor (5) works rather well in the considered range of distances  $z_1$ . However, it gives slightly smaller value of ionization loss in the maximum position. Therefore, in the present work we will also use the exact expression (7) in our investigation of the maximum possible values of the anti-Chudakov effect for a single pair.

The approximation (10) may be of high importance for application of the present theory for real experimental situation. In this case in order to estimate the possible value of the considered effect it is necessary to make the procedure of averaging of the expression (1) with respect to various parameters, such as  $\vartheta$ ,  $z_0$  etc.<sup>4</sup> The use of expression (10)

<sup>&</sup>lt;sup>3</sup>In fact, nearly monotone - for details see [5].

<sup>&</sup>lt;sup>4</sup>It is necessary to note that in this case it is also desirable to take into account the unevenness of the initial photon energy distribution between the particles in the pair.

instead of (7) for such estimation might significantly reduce the calculation time.

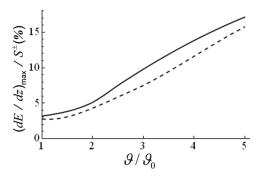


Fig. 6. Estimation of the maximum value of the anti-Chudakov effect at  $E = 100 \, \text{MeV}$  for the given parameters of the targets ( $\omega_p = 30 \, \text{eV}$ ,  $I = 175 \, \text{eV}$ ) as a function of the pair divergence angle. Solid line – calculation with the use of (7), dashed line – calculation based on (10)

The calculation results for the relative value of the considered effect for distances  $z_1 = z_{1m}$  corresponding to the maximum of dE/dz dependence are presented on Fig.6 for differnt values of  $\vartheta$ . It shows the results obtained with the use of both the expressions (7) and (10). Here we see that the magnitude of the anti-Chadakov effect at  $z_1 = z_{1m}$  may be several times larger than its asymptotic one (see Fig.3).

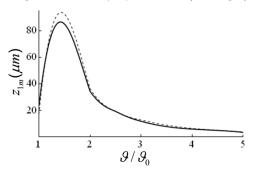


Fig.7. Dependence of the separation between the targets, corresponding to the maximum value of the effect, on the pair divergence angle. Solid line - calculation with the use of (7), dashed line - calculation based on (10)

In conclusion let us also present a figure (Fig.7) which demonstrates the value  $z_{1m}$  of separation between the targets, corresponding to the maximum magnitude of the considerd effect, as a function of the angle  $\vartheta$  of the pair divergence. The figure indicates significant nonmonotonic dependence of the separation on this angle.

#### 4. CONCLUSIONS

In the present paper the ionization loss of an ultrarelativistic e<sup>+</sup>e<sup>-</sup> pair in thin target was considered. The pair was supposed to be created in the second target situated in front of the first one on some distance from it. The main attention was drawn to the study of the so-called anti-Chudakov effect in the pair ionization loss. The effect was theoretically considered without the use of the approximation of parallel electron and positron velocities, which had been applied in the earlier paper [5] where the possibility of such effect was predicted. With use of such more general approach it was shown that under some conditions the discussed effect should be manifested even in the asymptotic case of large separations between the targets. The maximum value of the effect is achieved for the value of such separation which is proportional to the square of the energy of the pair and depends on its divergence angle as well. The effect is more pronounced at small (however, ultrarelativistic) energies of the pair for the values of such angle which exceed the most probable one.

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#### ОБ АНТИЭФФЕКТЕ ЧУДАКОВА В ИОНИЗАЦИОННЫХ ПОТЕРЯХ УЛЬТРАРЕЛЯТИВИСТСКОЙ ЭЛЕКТРОН-ПОЗИТРОННОЙ ПАРЫ В ТОНКОЙ МИШЕНИ

#### С. В. Трофименко

Рассмотрены ионизационные потери ультрарелятивистской е<sup>+</sup>е<sup>-</sup>-пары в системе из двух мишеней – толстой, в которой пара рождается, и тонкой, расположенных одна за другой на некотором промежутке. Основное внимание уделено исследованию эффекта увеличения потерь пары в тонкой мишени по сравнению с суммой независимых потерь электрона и позитрона (который является обратным эффекту Чудакова подавления ионизационных потерь пары). Задача решена без использования приближения параллельных скоростей электрона и позитрона, которое обычно применяется при теоретическом рассмотрении ионизационных потерь пар. Изучена зависимость величины данного эффекта от энергии и угла разлета пары. Показана возможность существования такого эффекта в асимптотическом случае при больших расстояниях между мишенями. Получено упрощенное аналитическое выражение, описывающее зависимость ионизационных потерь от расстояния между мишенями в условиях проявления рассматриваемого эффекта.

## ПРО АНТИЕФЕКТ ЧУДАКОВА В ІОНІЗАЦІЙНИХ ВТРАТАХ УЛЬТРАРЕЛЯТИВІСТСЬКОЇ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ В ТОНКІЙ МІШЕНІ

#### С. В. Трофименко

Розглянуто іонізаційні втрати ультрарелятивістської е<sup>+</sup>е<sup>-</sup>-пари в системі з двох мішеней – товстої, в якій пара народжується, і тонкої, що розташовані одна за одною на певному проміжку. Основну увагу приділено дослідженню ефекта збільшення втрат пари в тонкій мішені порівняно з сумою незалежних втрат електрона і позитрона (який є протилежним до ефекту Чудакова зменшення іонізаційних втрат пари). Задачу розв'язано без використання наближення паралельних швидкостей електрона і позитрона, яке зазвичай застосовується при теоретичному розгляді іонізаційних втрат пар. Вивчено залежність величини даного ефекту від енергії і кута розльоту пари. Показана можливість існування такого ефекту в асимптотичному випадку при великих відстанях між мішенями. Отримано спрощений аналітичний вираз, що описує залежність іонізаційних втрат пари від відстані між мішенями в умовах прояву ефекту, що розглядається.