TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN DISSIPATIVE MEDIA BY SEQUENCE OF ELECTRON BUNCHES

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To increase the transformation ratio, determining maximum energy of accelerated electrons, when wakefield is excited by sequence of electron bunches in plasma, it is profitable to use a certain difference of bunch repetition frequency and plasma frequency. First, field is excited resonantly. Then bunches shift into a small decelerating field. Bunches lose as much energy as accelerated bunches takes up.

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INTRODUCTION

The efficiency of electron bunch acceleration by wakefield, excited in two-beam accelerator by sequence of electron bunches is determined by transformation ratio (see [1 - 12]). The larger energy of bunches of accelerated electrons can be at larger transformation ratio. The transformation ratio, defined as ratio $TR=E_{ac}/E_{dec}$ of the wakefield E_{ac}, which is excited in accelerator by sequence of the identical electron bunches, to the field E_{dec}, in which an electron bunch is decelerated, is considered in dissipative media: collisional plasma or in accelerating dielectric structure with finite Q factor. The lengths of the bunches are selected to be equal less than half of wavelength $\xi_b<\lambda/2$. We consider the non-resonant case, when the frequency of the excited wave ω is not equal to the repetition frequency of bunches $\omega\neq\omega_b$. It is shown that at a certain rate of dissipation after reaching saturation of wakefield amplitude the asymptotic bunches get in small decelerating wakefield, providing a large transformation ratio. Asymptotic transformation ratio is determined in dependence on the rate of dissipation and current ratio of witness-bunches and driver-bunches.

First wakefield is excited by sequence of bunches approximately according to resonant scenario, namely the first bunches, which excite wakefield up to amplitude, close to saturation one, are approximately in the maximum decelerating field. Then in collisional plasma within the time, approximately equal to the inverse frequency of collisions of plasma electrons $\Delta t\approx 1/\nu_e$, during which a few beats of wakefield occur, in the non-resonant case bunches are shifted on phase relative to wave in phase, where a decelerating field is small. Steady state is established such that the bunches are in this phase of the field in which driver-bunches lose as much energy as total energy is dissipated in the system and as much energy as accelerated bunches take away energy.

It is shown that in order to increase the transformation ratio, it is advisable to use the corresponding difference of the injection frequency of bunches and electron plasma frequency, i.e. it is advisable to use the corresponding nonresonance regime.

1. PHASE SHIFT OF BUNCHES RELATIVE TO THE WAKEFIELD AT ITS EXCITATION IN THE PLASMA BY LONG NON-RESONANT SEQUENCE OF IDENTICAL RELATIVISTIC ELECTRON BUNCHES IN COLLISIONAL PLASMA

Nonresonant case $\omega_m\neq\omega_{pe}$ is considered with a large transformation ratio (TR) at excitation of wakefield E_z in collisional plasma $\nu_e\neq 0$ by long sequence of identical relativistic electron bunches. ω_m is the frequency of injection of bunches, ω_{pe} is the electron plasma frequency, ν_e is the electron collision frequency. Transformation ratio can be estimated as the ratio of the maximum accelerating field to the maximum decelerating field in the middle of the bunch.

As a result of decomposition of the charge (current) density of the bunch sequence of relativistic electrons in the Fourier series and separation of the main first periodic term, one can obtain the equation for the amplitude of the excited longitudinal electric wakefield

$$\frac{d^2E_{z0}}{d\tau^2}+\nu_e\frac{dE_{z0}}{d\tau}+\omega_{pe}^2E_{z0}=\omega_{pe}^2E_b\cos(\omega_m\tau),\ \tau=t-z/V_b\,. \quad (1)$$

Here E_b is the constant which is proportional to the maximum current of sequence of bunches and it depends on the spatial structure of the bunches and on the transverse structure of the longitudinal field.

Partial solution for excited wakefield looks like

$$E_z(t)=E_{z0}(t)\cos(\omega_m\tau+\varphi(\tau)). \quad (2)$$

Here E_{z0} is the amplitude of the wakefield, $\varphi(\tau)$ is the relative phase of excited wakefield and bunches, which excite wakefield.

At wakefield excitation in plasma by a nonresonant sequence of bunches of relativistic electrons $\omega_m\neq\omega_{pe}$, the relative phase φ of the first bunch, which excite the field E_z and the wakefield is equal to (Fig. 1) $\varphi=0$.

At further wakefield excitation $E_z(\tau)=E_{z0}(\tau)\cos(\omega_m\tau+\varphi(\tau))$ by a nonresonant sequence of electron bunches the relative phase φ due to $\omega_m\neq\omega_{pe}$ and amplitude of wakefield E_{z0} grow.

Due to the finite frequency of collisions $\nu_e\neq 0$ and the resonance detuning $\omega_m\neq\omega_{pe}$, the wakefield amplitude E_{z0} reaches saturation

$$E_{z0}=\frac{E_b\omega_{pe}^2}{\sqrt{\left(\omega_{pe}^2-\omega_m^2\right)^2+\nu_e^2\omega_{pe}^2}}. \quad (3)$$

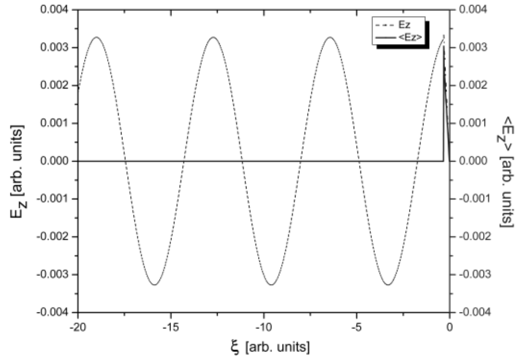

Fig. 1. The relative position of the first exciting bunch and the excited wakefield E_z (red)

Maximum amplitude of wakefield $E_{z0}\approx E_b\,\omega_{pe}/\nu_e$ is achieved at frequency

$$\omega_m^{(res)}=\omega_{pe}\sqrt{1-\nu_e^2/2\omega_{pe}^2}. \quad (4)$$

At reaching the maximum amplitude, a relative phase shift φ is established between the bunches and the excited field, which is determined by the ratio

$$\mathrm{tg}\varphi=\left(\omega_{pe}^2-\omega^2\right)/\nu_e\omega_{pe}. \quad (5)$$

From the latter one can see that in the resonant case $\omega_m=\omega_{pe}$, the relative phase remains $\varphi=0$ the same as for the first bunch. One can see also that in the nonresonance case $\Delta\omega=\omega_{pe}-\omega_m\neq 0$ at

$$2\Delta\omega>>\nu_e \quad (6)$$

phase can reach $\varphi\approx\pi/2$. That is, in the extreme case $2\Delta\omega>>\nu_e$, the regime is achieved when all bunches, beginning with a certain bunch, are in almost zero decelerating field, created by the previous bunches. Then a large transformation factor is achieved $TR>>1$.

2. DEPENDENCE OF THE TRANSFORMATION RATIO ON THE COLLISION FREQUENCY OF PLASMA ELECTRONS

For a case, corresponding to the maximum amplitude of wakefield

$$\Delta\omega/\omega_m=\left(\omega_0-\omega_m^{(res)}\right)/\omega_m\approx\nu_e^2/4\omega_{pe}^2<<\nu_e/2\omega_{pe},$$

the condition $2\Delta\omega>>\nu_e$ for achieving a large transformation ratio is not fulfilled. Indeed, although the excited accelerating field is a large $E_{z0}\approx E_b\,\omega_{pe}/\nu_e$, decelerating field

$$E_z^{(dec)}=E_{z0}\cos(\varphi)\approx E_{z0} \quad (7)$$

for driver-bunches is approximately equal to the accelerating field, so that

$$TR=\omega_{pe}E_b/\nu_e E_z^{(dec)}\approx 1. \quad (8)$$

For different values ν_e and $\Delta\omega$ the relative phase in the regime of the amplitude saturation can be equal to $0\le\varphi\le\pi/2$. In this case, the saturation amplitude can be equal to

$$E_{z0}=\frac{E_b\omega_{pe}^2}{\sqrt{\left(\omega_{pe}^2-\omega_m^2\right)^2+\omega_{pe}^2\nu_e^2}},\ \frac{\omega_{pe}}{2\Delta\omega}E_b\le E_{z0}\le\frac{\omega_{pe}}{\nu_e}E_b. \quad (9)$$

For a nonresonant case, when $2\Delta\omega>>\nu_e$ is true, the decelerating field for driver-bunches is small

$$E_z^{(dec)}\Big|_{t>\pi/2\Delta\omega}=E_{z0}\cos(\varphi)\approx E_{z0}\,\nu_e/2\Delta\omega<<E_{z0}. \quad (10)$$

Then the asymptotic at times $t>\pi/2\Delta\omega$ transformation ratio $TR=E_{z0}/E_z^{(dec)}\big|_{t>\pi/2\Delta\omega}$ is large

$$TR=E_{z0}/E_z^{(dec)}\Big|_{t>\pi/2\Delta\omega}=2\,\Delta\omega/\nu_e>>1. \quad (11)$$

In this case, the accelerating field equals

$$E_{z0}\approx E_b\,\omega_{pe}/2\Delta\omega>>E_b. \quad (12)$$

Now let's consider the collisionless plasma, but with certain witnesses. Namely, accelerated electron bunches with small charges are in relation to bunches with large charges, which excite a wakefield, slightly less than through $\pi/2$. Then the first bunches excite wakefield according to resonance mechanism, and bunches with small charges get into the phases of the wakefield, where the longitudinal wakefield is small $E_z\approx 0$. Then, due to a small initial nonresonance $\omega_m\neq\omega_{pe}$ or due to nonlinear frequency shift, the resonance is violated. The quasi-stationary asymptotic behavior, when the amplitude of the wakefield reaches saturation is as follows. As a result of the resonance disturbance the bunches, which excite wakefield, appear in the small longitudinal wakefield $E_{z1}\ge 0$, and accelerated bunches appear in the maximum wakefield $E_z^{(max)}$. Then the asymptotic transformation ratio is large

$$TR=E_z^{(max)}/E_{z1}>>1.$$

Thus, we consider the case of a collisionless plasma, when, after reaching a large amplitude, accelerated bunches get into maximum accelerating wakefield so that as the decelerated bunch loses energy, the subsequent accelerated bunch absorbs the same energy. As a result, if the quasi-stationary state is supported, then decelerated bunches lose energy in the decelerating field, which is equal to

$$E_{dec}^{(w)}=E_{z0}\,I_w/I_{dr}. \quad (13)$$

I_w, I_{dr} are the currents of bunches, which is accelerated and decelerated. If inequality $I_w/I_{dr}>>\nu_e/2\Delta\omega$ is true, then the asymptotic transformation ratio is equal to

$$TR=I_{dr}/I_w \quad (14)$$

and large in the case $I_w<<I_{dr}$.

3. EXCITATION OF WAKEFIELD FROM NULL TO LARGE AMPLITUDE

Nonresonant sequence of bunches $\omega_m<\omega_{pe}$ is considered. Wakefield after one Gaussian bunch, whose current density equals

$$j_b=\left(\frac{I_0}{\pi\sigma_r^2}\right)\exp\left(-\frac{r^2}{\sigma_r^2}\right)\exp\left[-\frac{(z-V_b t)^2}{\sigma_z^2}\right], \quad (15)$$

equals

$$E_z^{(1)}=\left(\frac{4I_0\sqrt{\pi}\sigma_z}{V_b\sigma_r^2}\right)\exp\left(-\frac{\omega^2\sigma_z^2}{4V_b^2}\right)R_z(r)\cos(\omega\tau),$$

$$\tau = t - \frac{z}{V_b}, \quad (16)$$

V_b, I_0 are the velocity and current of the beam, $R_z(r)$ is the transversal structure of the longitudinal field, $\sigma_z = V_b t_b$ is the length of the bunch, σ_r is the radius of the bunch, j_b is the current density of the beam.

The contribution of each bunch to the wakefield excitation is different, since this contribution is proportional to $\cos(\varphi_N)$, $\varphi_N = \varphi_M (N-1)/(N_c-1)$ is the phase of N-th bunch in the wakefield. $N_c = 1/2(1-\omega_m/\omega_{pe}) = \omega_{pe}/2\Delta\omega$ corresponds to a bunch, on which the wakefield amplitude reaches a maximum E_M, at which the relative phase becomes equal φ_M, which satisfies

$$tg(\varphi_M) = Q(\omega_{pe}^2 - \omega_m^2)/\nu_e \omega_{pe}. \quad (17)$$

That is, one can see that almost to the saturation amplitude the wakefield is excited rapidly, almost according to the resonance mechanism. Summing in sequence of N bunches, we derive that the Gaussian non-resonant $(N+1)$-th bunch gets into the wakefield equal to

$$E_z = \left(\frac{4I_0\sqrt{\pi}\sigma_z}{V_b\sigma_r^2}\right) R_z(r)\cos(\omega_m\tau + \varphi_{N+1}(\tau)) \times$$
$$\times \sum_{q=1}^{N} \cos\left[\frac{\varphi_M(q-1)}{(N_c-1)}\right] \exp\left[-\left(\frac{\omega_m\sigma_z}{2V_b}\right)^2\right] = \quad (18)$$
$$= \left(\frac{2I_0\sqrt{\pi}\omega_m\sigma_z}{\varphi_M V_b \sigma_r^2 \Delta\omega}\right) R_z(r)\cos(\omega_m\tau + \varphi_{N+1}(\tau)) \times$$
$$\times \sin\left(\frac{\varphi_M N}{N_c}\right) \exp\left[-\left(\frac{\omega_m\sigma_z}{2V_b}\right)^2\right],$$
$$\Delta\omega = \omega_{pe} - \omega_m, \ \tau = t - \frac{z}{V_b},$$
$$N_c = \frac{\omega_{pe}}{2\Delta\omega}, \ \varphi_{N+1} = \varphi_M \frac{N}{(N_c-1)}.$$

Then we derive

$$E_M = \left(\frac{2I_0\sqrt{\pi}\omega_m\sigma_z}{\varphi_M V_b \sigma_r^2 \Delta\omega}\right) \sin(\varphi_M) \exp\left[-\left(\frac{\omega_m\sigma_z}{2V_b}\right)^2\right]. \quad (19)$$

Thus, for N-th bunch TR_N, estimated as

$$TR_N = \cos^{-1}\left[\varphi_M (N-1)/(N_c-1)\right], \quad (20)$$

is equal at $2\Delta\omega >> \nu_e$ to

$$\cos(\varphi_M) \approx \frac{\nu_e}{2\Delta\omega} << 1,$$
$$TR_N = \cos^{-1}\left[\pi(N-1)/2(N_c-1)\right]. \quad (21)$$

One can see that $TR_1 = 1$, TR_N grows with growth N and it becomes large at $N = N_c = \omega_{pe}/2\Delta\omega$, and at $N \to N_c$

$$TR_N\big|_{N\to N_c} = \cos^{-1}\left[\frac{\pi(N-1)}{2(N_c-1)}\right] \approx \frac{2(N_c-1)}{\pi(N_c-N)} >> 1. \quad (22)$$

Now let's consider the average transformation ratio, taking into account the stage of the wakefield excitation. We take into account that at the excitation stage of the wakefield, the decelerating field and the accelerating field are approximately the same NE_b. N is the number of injected bunches. We also take into account that at the asymptotic stage the accelerating field is equal to E_{z0}, and the decelerating field is equal to $E_z^{(dec)} \approx E_b \omega/2\Delta\omega >> E_b$. Then TR approximately equals

$$TR_N \approx \frac{\left[N_c E_{z0} + (N-N_c)E_{z0}\right]/N}{\left[N_c E_{z0} + (N-N_c)E_z^{(dec)}\right]/N}\Big|_{N\to\infty} =$$
$$= \frac{E_{z0}}{E_{z0} N_c/N + E_z^{(dec)}(N-N_c)/N}\Big|_{N\to\infty} >> 1, \quad (23)$$

N is the total number of bunches, $N_c = \omega/2\Delta\omega$ is the number of bunches, during which wakefield increases. $N >> N_c$, because $N \approx 6000$, $N_c \approx 300$, $E_{z0} \approx N_c E_b$.

Thus, it has been shown that a significant transformation ratio can be achieved at the excitation of a wakefield in a collisionless plasma or in a plasma with a certain sequence of wintesses by a long sequence of identical bunches of relativistic electrons.

4. RESULTS OF NUMERICAL SIMULATION

Numerical simulation has been performed using 2d3v-code lcode [13].

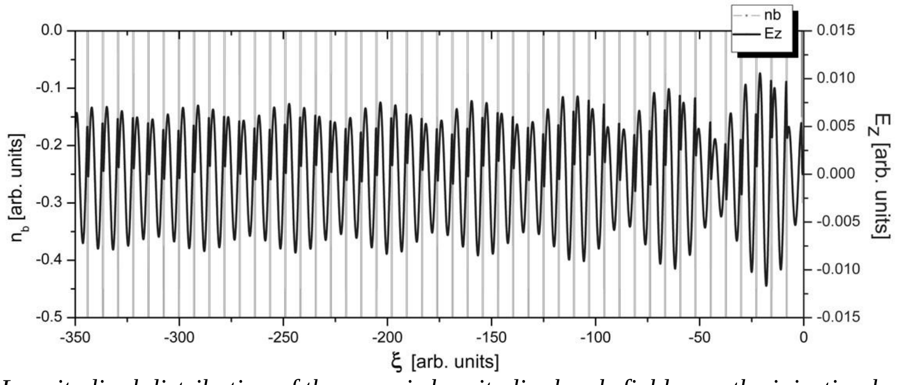

Fig. 2. Longitudinal distribution of the on-axis longitudinal wakefield near the injection boundary of bunches of the non-resonance sequence

The main purpose of the numerical simulation of the wakefield excitation by the sequence of electron bunches is demonstration that, in the asymptotic stationary state, the driver-bunches get into the phases of the wakefield, where the decelerating field is small (Fig. 2). At the same time, if at the front of the sequence of bunches, where the wakefield is beatings, some bunches lose energy, while others bunches take it, then in the asymptotic stationary state all bunches lose energy. It was assumed that the bunches have a rectangular shape with a uniform distribution of density.

CONCLUSIONS

Thus it has been shown that in collisional plasma or in dielectric accelerating structure with finite Q factor the state occurs naturally with large transformation ratio of energy of driver-bunches into energy of witness-bunches at wakefield excitation by a sequence of identical electron bunches. The expression for transformation ratio has been derived in dependence on the rate of dissipation and current ratio of witness-bunches and driver-bunches.

It is shown that in order to increase the transformation ratio at the wakefield excitation by a long non-resonant sequence of identical relativistic electron bunches in a collisional plasma it is expedient to use a certain difference of the injection frequency of bunches and the electron plasma frequency, that is, it is expedient to use a certain nonresonance regime.

The dependence of the transformation ratio is established, which determines the maximum energy of the accelerated particles, from the collision frequency of plasma electrons at the wakefield excitation in it by a long nonresonant sequence of identical relativistic electron bunches.

It is also shown that a large transformation ratio is achieved in a quasi-steady state in the non-resonant case of the wakefield excitation in a collisionless plasma, if after reaching a large amplitude, accelerating bunches are injected in the maximum accelerating field, so that energy, lost by decelerated bunches, equals to energy, absorbed by accelerated bunches of low density.

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КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ В ДИССИПАТИВНОЙ СРЕДЕ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ЭЛЕКТРОННЫХ СГУСТКОВ

И.П. Левчук, В.И. Маслов, И.Н. Онищенко

Для увеличения коэффициента трансформации, определяющего максимальную энергию ускоренных электронов, при возбуждении кильватерного поля последовательностью электронных сгустков в плазме выгодно использовать определенную разницу частоты инжекции сгустков и плазменной частоты. Сначала поле возбуждается резонансно. Затем сгустки смещаются в небольшое тормозящее поле. Ускоряемые сгустки забирают столько энергии, сколько тормозящиеся сгустки теряют.

КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ В ДИСИПАТИВНОМУ СЕРЕДОВИЩІ ПОСЛІДОВНІСТЮ ЕЛЕКТРОННИХ ЗГУСТКІВ

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Для збільшення коефіцієнта трансформації, що визначає максимальну енергію прискорених електронів, при збудженні кільватерного поля послідовністю електронних згустків у плазмі вигідно використовувати певну різницю частоти інжекції згустків і плазмової частоти. Спочатку поле збуджується резонансно. Потім згустки зміщуються в невелике гальмуюче поле. Прискорювані згустки забирають стільки енергії, скільки згустки, що гальмуються, втрачають.