APPLICATIONS OF THE KHARKOV POTENTIAL IN THE THEORY OF NUCLEAR FORCES AND NUCLEAR REACTIONS

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The Kharkov potential is a recent field theoretical model of nucleon-nucleon (NN) interaction that has been built up in the framework of the instant form of relativistic dynamics starting with the total Hamiltonian of interacting meson and nucleon fields and using the method of unitary clothing transformations (UCTs). The latter connect the representation of "bare" particles (BPR) and the representation of "clothed" particles (CPR), i.e., the particles with physical properties. Unlike many available NN potentials each of which is the kernel of the corresponding nonrelativistic Lippmann-Schwinger (LS) equation this potential being dependent in momentum space on the Feynman-like propagators and covariant cutoff factors at the meson-nucleon vertices is the kernel of relativistic integral equations for the NN bound and scattering states. We show our calculations with the Bonn and Kharkov potentials for such quantities as the phase shifts in the neutron-proton scattering up to the pion production threshold, the binding energies of deuteron and triton, the nucleon momentum distributions in these nuclei and some Nd elastic scattering observables. Special attention is paid to finding from the contemporary n-p phase shift analysis some optimum values of the adjustable parameters included.

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INTRODUCTION

In many textbooks on nuclear physics we encounter the following form

$$H = K + V$$

of the nuclear Hamiltonian, where K is the one-body operator of kinetic energy and the interaction between nucleons

$$V = \sum_{i<j} V(i,j) + \sum_{i<j<k} V(i,j,k) + ...(i,j,k = 1,2,...,N) \quad (1)$$

consists of the two-body $V(i,j)$, three-body $V(i,j,k)$ and more complex forces. The UCT method [1, 2] allows us to construct such interactions on one and the same physical footing.

Our departure point is the Hamiltonian of interacting meson and nucleon fields in case of mesodynamics with the Yukawa-type couplings between mesons (π, η, ρ, ω) and nucleons (antinucleons). As an illustration, in case of the vector mesons we separate out the scalar H_{sc}(x) and nonscalar H_{nonsc}(x) contributions

$$H_{sc}(\vec{x}) = g_v \bar{\psi}(\vec{x})\gamma_\mu \psi(\vec{x})\varphi_v^\mu(\vec{x}) + \frac{f_v}{4m}\bar{\psi}(\vec{x})\sigma_{\mu\nu}\psi(\vec{x})\varphi_v^{\mu\nu}(\vec{x}), \quad (2)$$

$$H_{nonsc}(\vec{x}) = \frac{g_v^2}{2m_v^2}\bar{\psi}(\vec{x})\gamma_0\psi(\vec{x})\bar{\psi}(\vec{x})\gamma_0\psi(\vec{x}) + \frac{f_v^2}{4m^2}\bar{\psi}(\vec{x})\sigma_{0i}\psi(\vec{x})\bar{\psi}(\vec{x})\sigma_{0i}\psi(\vec{x}) \quad (3)$$

to the Hamiltonian density. Here

$$\varphi_v^{\mu\nu}(\vec{x}) = \partial^\mu \varphi_v^\nu(\vec{x}) - \partial^\nu \varphi_v^\mu(\vec{x})$$

the tensor of vector field $\varphi_v^\mu(\vec{x})$ $(\mu = 0,1,2,3)$ in the Schrodinger (S) picture. In its turn, the fields involved can be expressed through the creation and destruction operators that meet

$$\left[a(p',s'), a^\dagger(p,s)\right]_\pm = p_0\delta(\vec{p} - \vec{p}\,')\delta_{ss'},$$
$$\left[a(p,s), a(p',s')\right]_\pm = \left[a^\dagger(p,s), a^\dagger(p',s')\right]_\pm = 0, \quad (4)$$

where $p_0 = \sqrt{p^2 + m^2}$, m the mass of particle and s its spin index, if any.

The method in question is aimed at expressing a field Hamiltonian through the so-called clothed-particle creation (annihilation) operators $\{\alpha_c\}$, e.g., $a_c^\dagger(a_c)$ for bosons, $b_c^\dagger(b_c)$ for fermions and $d_c^\dagger(d_c)$ for antifermions via UCTs $W(\alpha_c) = W(\alpha) = \exp(R)$, $R = -R^\dagger$ in the similarity transformation

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)\,, \quad (5)$$

that connects a primary set α in BPR with the new operators in CPR.

A key point of the clothing procedure of interested is to remove the so-called bad terms from the Hamiltonian.

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c). \quad (6)$$

By definition, such terms prevent the physical vacuum $|\Omega\rangle$ (H lowest eigenstate) and one-clothed-particle states $|n_c\rangle = a_c^\dagger(n)|\Omega\rangle$ to be H eigenvectors for all n considered. In this context all primary Yukawa-type (trilinear) couplings shown above should be eliminated as a whole.

1. RELATIVISTIC INTERACTIONS IN MESON-NUCLEON SYSTEM

Omitting some details, we arrive to a new form of our field theoretical Hamiltonian

$$H = K(\alpha_c) = K_F + K_I\,, \tag{7}$$

with a new free part

$$K_F = H_F(\alpha_c) \text{ □□} \sim a_c^\dagger a_c + b_c^\dagger b_c + d_c^\dagger d_c$$

and interaction K_I between the clothed particles.

Doing so, we get the following operator structure

$$\begin{aligned}
K_I &\sim a_c^\dagger b_c^\dagger a_c b_c (\pi N \to \pi N) + \\
&+ b_c^\dagger b_c^\dagger b_c b_c (NN \to NN) + d_c^\dagger d_c^\dagger d_c d_c (\bar{N}\bar{N} \to \bar{N}\bar{N}) + \\
&+ b_c^\dagger b_c^\dagger b_c^\dagger b_c b_c b_c (NNN \to NNN) + \ldots \\
&+ [a_c^\dagger a_c^\dagger b_c d_c + H.c.](N\bar{N} \leftrightarrow 2\pi) + \ldots \\
&+ [a_c^\dagger b_c^\dagger b_c^\dagger b_c b_c + H.c.](NN \leftrightarrow \pi NN) + \ldots
\end{aligned} \tag{8}$$

separate terms of which are responsible for different processes in the given system.

In this context, we will confine ourselves to the nucleon-nucleon interaction operator, viz., after normal ordering of the fermion operators we derive $NN \to NN$ interaction operator generated by the one-pion-exchange, i.e., exchange by the intermediate pseudoscalar boson with the physical mass μ (Fig. 1).

$$K(NN \to NN) = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}_1' d\vec{p}_2' V_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) b_c^\dagger(\vec{p}_1') b_c^\dagger(\vec{p}_2') b_c(\vec{p}_1) b_c(\vec{p}_2), \tag{9}$$

$$\begin{aligned}
V_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) &= -\frac{1}{2}\frac{g^2}{(2\pi)^3}\frac{m^2}{\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}E_{\vec{p}_1'}E_{\vec{p}_2'}}}\delta(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2) \times \\
&\times \bar{u}(\vec{p}_1')\gamma_5 u(\vec{p}_1)\frac{1}{(p_1 - p_1')^2 - \mu^2}\bar{u}(\vec{p}_2')\gamma_5 u(\vec{p}_2).
\end{aligned} \tag{10}$$

The corresponding relativistic and properly symmetrized NN quasipotential is

$$\left\langle \Omega b_c^\dagger(\vec{p}_1') b_c^\dagger(\vec{p}_2') \,|\, K(NN \to NN) \,|\, b_c^\dagger(\vec{p}_1) b_c^\dagger(\vec{p}_2)\Omega \right\rangle,$$

$$\begin{aligned}
\tilde{V}_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) &= -\frac{1}{2}\frac{g^2}{(2\pi)^3}\frac{m^2}{2\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}E_{\vec{p}_1'}E_{\vec{p}_2'}}}\delta(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2) \times \\
&\times \bar{u}(\vec{p}_1')\gamma_5 u(\vec{p}_1)\frac{1}{2}\left\{\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_2 - p_2')^2 - \mu^2}\right\}\bar{u}(\vec{p}_2')\gamma_5 u(\vec{p}_2) - (1 \leftrightarrow 2).
\end{aligned} \tag{11}$$

A distinctive feature of this potential is the presence of the covariant (Feynman-like) "propagator"

$$\frac{1}{2}\left\{\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_2 - p_2')^2 - \mu^2}\right\}.$$

On the energy shell for the NN scattering, that is $E_i \equiv E_{\vec{p}_1} + E_{\vec{p}_2} = E_{\vec{p}_1'} + E_{\vec{p}_2'} \equiv E_f$, this expression is converted into the genuine Feynman propagator. It is typical of other interactions.

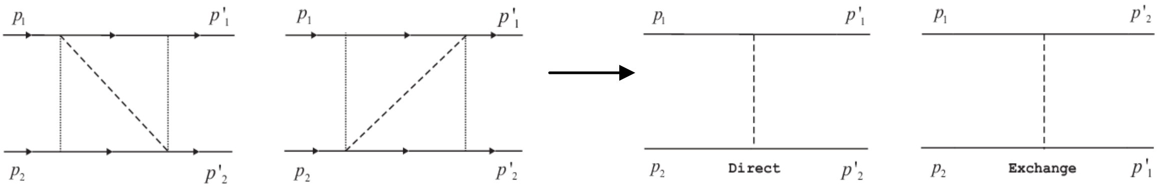

Fig. 1. The one-meson-exchange off energy-shell graphs (left) and Feynman diagrams (right) for NN scattering

2. CALCULATIONS OF THE PHASE-SHIFTS FOR n-p-SCATTERING

We will compare our calculations [3] with the Kharkov potential and those by the Bonn group. To clarify some similarities and differences between them, one needs keep in mind that the potential B by the Bonn group can be obtained from the Kharkov potential with help of the replacements for boson propagators

$$\left[\left(p' - p\right)^2 - \mathrm{m}_b^2\right]^{-1} \to -\left[\left(\vec{p}' - \vec{p}\right)^2 - \mathrm{m}_b^2\right]^{-1},$$

cut-off functions

$$\left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 - \left(p' - p\right)^2}\right]^{n_b} \to \left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 + \left(\vec{p}' - \vec{p}\right)^2}\right]^{n_b},$$

and neglecting off-energy-shell corrections and tensor-tensor term

$$\begin{aligned}
&\frac{f_\nu^2}{4m^2}(\mathrm{E}_{p'} - \mathrm{E}_p)^2 \bar{u}(\vec{p}\,')\left[\gamma_0\gamma_\nu - g_{0\nu}\right] \times \\
&\times u(\vec{p})\bar{u}(-\vec{p}\,')\left[\gamma_0\gamma_\nu - g_{0\nu}\right]u(-\vec{p}) \to 0.
\end{aligned}$$

It has turned out that the values of the adjustable parameters (coupling constants and cut-off factors) which provide a fair treatment of the available n-p scattering data can be considerably different for the both models (Table 1).

In Fig. 2 we show the energy dependence of the phase shifts and the mixing parameter ε_1 that regulates the ${}^3S_1 - {}^3D_1$ transitions, while Fig. 3 allow us to see the off-energy-shell differences for the corresponding half-off-shell $R-$ matrices. Recall that on-shell $R-$ matrix elements $R(p_{0,}p_{0,})$ are proportional to $\tan\delta(p_0)$.

Table 1

The best-fit parameters for the two models

Meson		Bonn	UCT3	UCT4
π	$g_\pi^2/4\pi$	14.4	14.67	14.31
	Λ_π	1700	2497	2364.25
	m_π	138.03	138.03	138.03
η	$g_\eta^2/4\pi$	3	6.11	4.67
	Λ_η	1500	955.0	1188.87
	m_η	548.8	548.8	548.8
ρ	$g_\rho^2/4\pi$	0.9	1.54	1.38
	Λ_ρ	1850	1483	1469.78
	f_ρ/g_ρ	6.1	5.2	5.75
	m_ρ	769	769	769
ω	$g_\omega^2/4\pi$	24.5	28.13	28.25
	Λ_ω	1850	2061	2017.27
	m_ω	782.6	782.6	782.6
δ	$g_\delta^2/4\pi$	2.488	2.04	1.85
	Λ_δ	2000	2349.97	2004.05
	m_δ	983	983	983
σ, $T = 0$, ($T = 1$)	$g_\sigma^2/4\pi$	18.3773, (8.9437)	18.576, (11.11)	19.20, (10.93)
	Λ_σ	2000, (1900)	1611.54, (1986)	1727.02, (2241.14)
	m_σ	720, (550)	713.04, (565.4)	721.58, (567.03)

Column UCT3 (UCT4) fits the Bonn potential (WCJ1 potential from [4]).

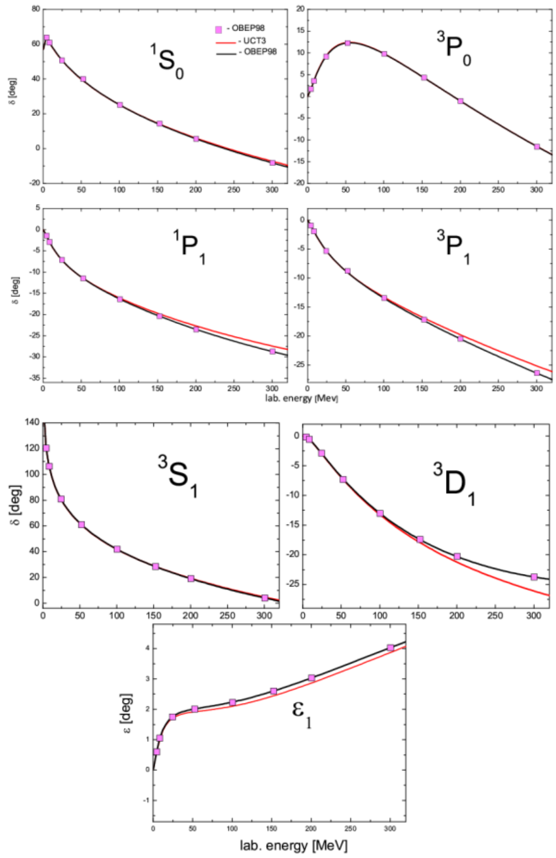

Fig. 2. Neutron-proton phase-shifts for the uncoupled (a) and coupled (b) partial waves versus the nucleon kinetic energy in the lab. frame with the UCT3 parameters

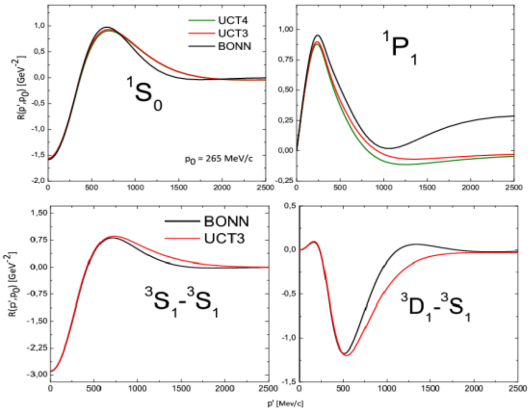

Fig. 3. Half-off-shell R-matrices for uncoupled waves at lab. energy equal to 150 MeV (p_0 =265 MeV)

3. DEUTERON AND TRITON PROPERTIES

Some results of our calculations are collected in Tables 2 and 3.

Table 2

Deuteron and low-energy parameters. The experimental values are from Table 4.2 in [5]

Parameters	Bonn B	UCT3	Experiment
a_s (fm)	-23.71	-23.724	-23.748 ± 0.010
r_s (fm)	2.71	2.725	2.75 ± 0.05
a_t (fm)	5.426	5.41	5.419 ± 0.007
r_t (fm)	1.761	1.772	1.754 ± 0.008
ε_d (MeV)	2.223	2.243	2.224575
P_D (%)	4.99	5.35	-

Table 3

Triton binding energies of Kharkov potential compare with other popular solutions (in MeV)

Solution	Relativistic (Nonrelativistic)	Difference
Kharkov (UCT3)	-7.72 (-7.838)	0.066
Bonn	-8.14	0.099
CD-Bonn	-8.150 (-8.249)	-
Experiment	8.48	

The shift observed in these p – dependences of the deuteron wave function component $u(p)$ (Fig. 4) is of interest for further explorations. These wave functions have following normalization

$$\int_0^\infty p^2 dp\left[\psi_0^2(p)+\psi_2^2(p)\right]=1\,.$$

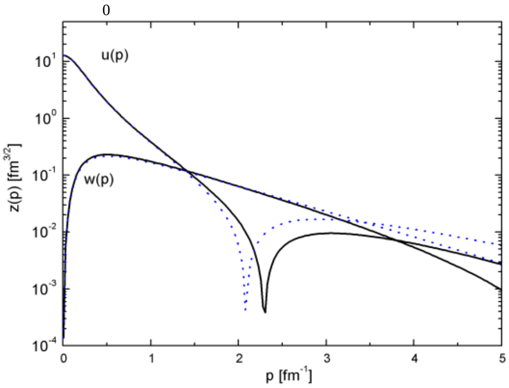

Fig. 4. Deuteron wave functions $\psi_0(p)=u(p)$ and $\psi_2(p)=w(p)$. Solid (dotted) curves for Bonn Potential B (Kharkov) potential

In addition, we compare in Fig. 5 the nucleon momentum distributions for the triton (left) and the deuteron (right).

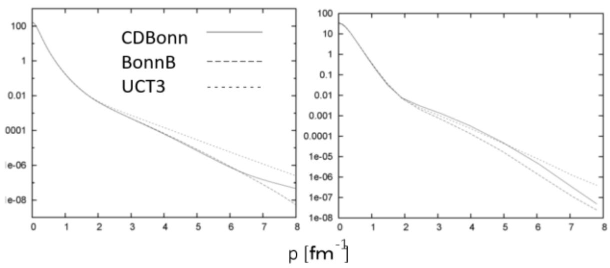

Fig. 5. Deuteron (left) and triton (right) nucleon momentum distributions

4. SUMMARY

Starting from a total Hamiltonian for interacting meson and nucleon fields, we come to Hamiltonian and boost generator in CPR whose interaction parts consist of new relativistic interactions responsible for physical (not virtual) processes, particularly, in the system of bosons (π-, η-, ρ-, ω- – mesons) and fermions (nucleons and antinucleons). The corresponding quasipotentials (these essentially nonlocal objects) for binary processes $NN \to NN$, $\bar{N}\bar{N} \to \bar{N}\bar{N}$, etc. and are Hermitian and energy independent. It makes them attractive for various applications in nuclear physics. They embody the off-shell and recoil effects (the latter in all orders of the $1/c^2$ – expansion) without addressing to any off-shell extrapolations of the S – matrix for the NN scattering.

Triton wave function is solved by the Faddeev 3-body theory with Kharkov potential. The magnitude of the binding energy of the triton (7.77 MeV) is smaller than data (8.48 MeV). This is the same situation of the case which one have calculated using the nonrelativistic potential, e.g., CDBonn. Therefore we may need the so-called 3-body force.

As a whole, persistent clouds of virtual particles are no longer explicitly contained in CPR, and their influence is included in properties of clothed particles (these quasiparticles of UCT method). In addition, we would like to stress that problem of the mass and vertex renormalizations is intimately interwoven with constructing the interactions between clothed nucleons. Renormalized quantities are calculated step by step in course of clothing procedure unlike some approaches, where they are introduced by "hands".

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ПРИМЕНЕНИЯ ХАРЬКОВСКОГО ПОТЕНЦИАЛА В ТЕОРИИ ЯДЕРНЫХ СИЛ И ЯДЕРНЫХ РЕАКЦИЙ

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Харьковский потенциал, недавно предложенная теоретико-полевая модель нуклон-нуклонного (NN) взаимодействия, был построен в рамках так называемой мгновенной формы релятивистской динамики, имея гамильтониан взаимодействующих мезонных и нуклонного полей и используя метод унитарных одевающих преобразований. Эти преобразования связывают представление “голых” частиц с представлением “одетых” частиц, т.е. частиц с наблюдаемыми (физическими) свойствами. В отличие от многих NN-потенциалов, каждый из которых является ядром соответствующего нерелятивистского уравнения Липманна-Швингера, этот потенциал, зависящий в импульсном пространстве от фейнман-подобных пропагаторов и ковариантных обрезающих факторов в мезон-нуклонных вершинах, является ядром релятивистских интегральных уравнений для NN-связанных состояний и состояний рассеяния. Мы покажем наши вычисления с Боннским и Харьковским потенциалами для таких величин, как фазовые сдвиги в нейтрон-протонном рассеянии до порога рождения пионов, энергий связи дейтрона и тритона, импульсных распределений нуклонов в этих ядрах и некоторых наблюдаемых в упругом Nd-рассеянии. Особое внимание уделяется определению из современного n-p-фазового анализа оптимальных значений для подгоночных параметров.

ЗАСТОСУВАННЯ ХАРКІВСЬКОГО ПОТЕНЦІАЛУ В ТЕОРІЇ ЯДЕРНИХ СИЛ І ЯДЕРНИХ РЕАКЦІЙ

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Харківський потенціал, у недавній час запропонована теоретико-польова модель нуклон-нуклонної (NN) взаємодії, був побудований в рамках так званої миттєвої форми релятивістської динаміки, маючи гамільтоніан взаємодіючих мезонних і нуклонного полів і використовуючи метод унітарних одягаючих перетворень. Ці перетворення пов'язують зображення “голих” частинок з зображенням “одягнених” частинок, тобто частинок з спостережуваними (фізичними) властивостями. На відміну від багатьох NN-потенціалів, кожен з яких є ядром відповідного нерелятивістського рівняння Ліпмана-Швінгера, цей потенціал, що залежить у імпульсному просторі від фейнман-подібних пропагаторів і коваріантних обрізуючих факторів у мезон-нуклонних вершинах, є ядром релятивістських інтегральних рівнянь для NN-пов'язаних станів і станів розсіювання. Ми покажемо наші обчислення з Боннським та Харківським потенціалами для таких величин, як фазові зрушення в нейтрон-протонному розсіюванні до порога народження піонів, енергій зв'язку дейтрона і тритона, імпульсних розподілів нуклонів у цих ядрах і деяких спостережуваних у пружному Nd-розсіюванні. Особлива увага приділяється визначенню з сучасного n-p-фазового аналізу оптимальних значень для підгоночних параметрів.