OPTIMIZATION APPROACH TO THE SYNTHESIS OF PLASMA STABILIZATION SYSTEM IN TOKAMAK ITER

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Synthesis of the controller of ITER plasma stabilization system is considered. Stabilization system is based on tokamak diagnostic measurements, defines the voltages in tokamak coils and has the filtering properties. Known methods of synthesis of plasma regulators are advanced by the presented optimization approach. It makes possible to meet the requirements for the dynamics of the stabilization process when considering a set of some arbitrary plasma drops and disturbances. The proposed approach evaluates the ensemble of transient processes of the closed object. Based on this estimations it is possible to use wide range of optimization techniques.

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INTRODUCTION

One of the most important aspects of plasma control in tokamak is the providing the controller of plasma stabilization system [1, 2]. Such controller should be based on tokamak diagnostic measurements only, should work in real time, have reduced dimension and have the filter properties. Various plasma instabilities and drops should be fulfilled by such controller. For random perturbations, the dynamics of the transition process should meet the requirements of accuracy control. The principle of saving energy costs should also be observed. During controller synthesis, it is necessary to find the optimal balance between these conflicting criteria [3 - 5]. A presented integral criterion estimates the accuracy of stabilization and energy cost. The optimization approach involves the criterion minimization by tuning the controller values. Since the random disturbance is not known in advance, the dynamics optimization is performed for any disturbance from some disturbance ensemble. This is a set of disturbances which can happen in practice.

Linearization procedure is widely used for plasma control problem [1], [6]. Linearization gives LTI-object which is treated as a control object in deviations from the equilibrium position.

1. ITER PLASMA STABILIZATION PROBLEM

ITER tokamak has 11 control coils to provide proper magnetic configuration to plasma hold [1, 2]. These 11 voltages are treated as control object input. The ITER diagnostic system measures 6 *gaps* between hot area of plasma and tokamak chamber. These 6 gaps are treaded as control object output. The ITER chamber is divided into a large number of circuits. That defines high dimension of control object. Equations for current in circuits are linearized in the area of the equilibrium position. The initial plasma drops and external plasma conductivity drops are taken into account.

LTI-object which describes plasma dynamic in ITER in deviation from equilibrium point is represented by following equation:

$$\dot{x} = Ax + Bu + Rf(t),$$
$$x(0) = x_{0\,drop},$$
$$g_i = G_i x, \qquad i = 1, \ldots, 6,$$

$$f(t) = f_{drop}(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}, \tag{1}$$
$$f_1(t) = d_\beta e^{-(t/t_\beta)},$$
$$f_1(t) = d_j e^{-(t/t_j)},$$

where $x \in E^{112}$ is the state space vector, $u \in E^{11}$ is the vector of coils voltages, $g_1(t)$, …, $g_6(t)$ are the diagnosed gaps, matrices A, B, R are known constant matrices, G_i are one-row known constant matrices, $d_\beta, d_j, t_\beta, t_j$ are known real constants, $x_{0\,drop}$ is some arbitrary initial plasma drop, $f_{drop}(t)$ is mentioned external plasma conductivity drop. Constants $d_\beta, d_j, t_\beta, t_j$ are different for the various plasma modes.

Stabilizing controller should compute control coil voltages $u = (u_1, \ldots, u_{11})$ by measured output $g_1(t)$, …, $g_6(t)$, and provide proper dynamics quality.

2. PLASMA STABILIZATION SYSTEM FEEDBACK

Let's consider dynamic controller of decreased dimension [7 - 9]. The reduction of the dimension provides acceptable real-time computational complexity. The controller's equations have the following representation. Introduce constant matrices:

$$W_{121} = \begin{pmatrix} w_1 & \cdots & w_{11} \\ \ldots & \ldots & \ldots \\ w_{111} & \ldots & w_{121} \end{pmatrix}, \quad W_{187} = \begin{pmatrix} w_{122} & \cdots & w_{126} \\ \ldots & \ldots & \ldots \\ w_{182} & \ldots & w_{187} \end{pmatrix},$$

$$W_{308} = \begin{pmatrix} w_{188} & \cdots & w_{197} \\ \ldots & \ldots & \ldots \\ w_{298} & \ldots & w_{308} \end{pmatrix},$$

where $w_1, \ldots, w_{308}$ are the constant values of the regulator which should be found. Controller equations are

$$\dot{z} = W_{121} z + W_{187} \begin{pmatrix} g_1 \\ \ldots \\ g_6 \end{pmatrix}, \tag{2}$$
$$u = (u_1, \ldots, u_{11})^* = W_{308}\, z,$$

where $z \in E^{zn}$, $zn = 11$ is controller's state-space vector, zn is a controller dimension (it's taken 11 as an example from [1], [7]), w_1, …, w_{308} are the constant values of the regulator which should be found. Controller synthesis involves finding these constant components and minimization of quality criterion:

$$\int_0^T \sum_{i=1..6} g^2{}_i(t) + \sum_{i=1..11} u^2{}_i(t)\ dt \to \min,$$

where T is the end of the simulation interval. It's easy to notice that vector z extends the object state-space vector. This controller can be found by an optimization approach.

3. OPTIMIZATION APROACH TO THE CONTROLLER SYNTHESIS

The optimization approach involves the criterion minimization by tuning the controller values. The closed-loop ITER stabilization system has a form

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & B\,W_{308} \\ W_{187}C & W_{121} \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + Rf(t). \qquad (3)$$

To evaluate dynamics quality of closed-loop system we will compute the responses on some arbitrary disturbances x_0, f(t). Let's define the upper boundaries of the possible amplitudes of gaps and voltages for all disturbances from special set. This set is *an ensemble* of the disturbances which can be described as any x_0, f(t) which satisfy the expression

$$\begin{aligned} &(x_0, f(t)) \in \Psi_{drops} \equiv \Psi_{drops}(Y_1, Y_2(t), \mu^2), \\ &\Psi_{drops} = \{(x_0, f(t)) : \\ &\qquad x_0{}^* Y_1 x_0 + \int_{t_0}^{t} f^*(\tau)\, Y_2(\tau) f(\tau)\, d\tau \le \mu^2\}, \end{aligned} \qquad (4)$$

where Y_1, Y_2 are positive definite given matrices, μ is a positive given constant, $\mu = 1$ for this example. Let us denote the matrices of closed-loop system:

$$S_{obj} = \begin{pmatrix} A & B\,W_{308} \\ W_{187}C & W_{121} \end{pmatrix}. \qquad (5)$$

The following differential equations make it possible to obtain the desired upper boundaries [10]:

$$\begin{aligned} &\dot{D} = S_{obj}\, D + D\, S^*_{obj} + R\, Y_2^{-1} R^*, \qquad (6) \\ &D(0) = Y_1^{-1}, \\ &\dot{\Theta} = -\Theta\, S_{obj} - S^*_{obj}\, \Theta - (N^* \cdot N + K^* \cdot K), \\ &\Theta(T) = N^* N, \\ &N = (G_1, ..., G_6, \dots \ \ 0), \\ &K = \begin{pmatrix} 0 & 0 \\ 0 & W_{308} \end{pmatrix}. \end{aligned}$$

The desired upper boundaries of the possible amplitudes of gaps and voltage for $\forall (x_0, f(t)) \in \Psi_{drops}$ can be described as [10]

$$\begin{aligned} &0 \le g_i(t)^2 \le L_i(t), \quad i = 1, ..., 6, \\ &0 \le u_i(t)^2 \le V_i(t), \quad i = 1, ..., 11, \end{aligned} \qquad (7)$$

where

$$\begin{aligned} &L_i(t) = G_i\, D(t)\, G_i^*, \quad i = 1, ..., 6, \\ &V_i(t) = K_i\, D(t)\, K_i^*, \quad i = 1, ..., 11, \end{aligned}$$

where K_i is the i-th row of matrix K. By using these upper estimations it's possible to represent the integral quality criterion in form:

$$I(w_1, ..., w_{308}) = \int_0^T \sum_{i=1..6} L_i(t) + \sum_{i=1..11} V_i(t)\ dt \to \min. \qquad (8)$$

Such representation makes it possible to minimize (8) and to tune the controller values $w_1, \dots, w_{308}$ for all arbitrary initial plasma drop and external disturbances from the Ψ_{drops} set. Numerical gradient optimization is developed based on the gradient representation of functional (8):

$$\frac{\partial I}{\partial w_i} = -\mu^2 \int_0^T 2\,\mathrm{tr}\left(\Theta D \frac{\partial S^*_{obj}}{\partial w_i} + K D \frac{\partial K^*}{\partial w_i} \right) dt, \qquad (9)$$

$$i = 1, ..., 308.$$

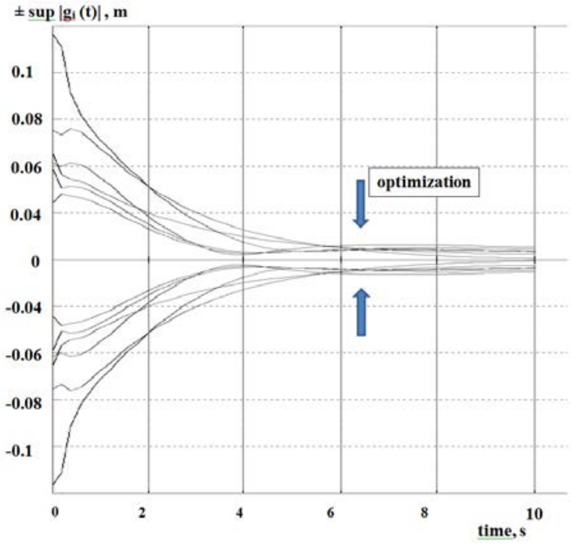
Above the trace of matrix is defined as tr. Numerical modeling of the ensemble of transient processes of the closed object is based on (7) and the ensemble quality criterion (8). For disturbances ensemble Ψ_{drops} (4) it is fair equality [10]:

$$-\sqrt{L_i(t)} = -\sup_{\Psi} |g_i(t)| \le g_i(t) \le \sup_{\Psi} |g_i(t)| = \sqrt{L_i(t)},$$

$$i = 1, ..., 6,$$

$$-\sqrt{V_i(t)} = -\sup_{\Psi} |u_i(t)| \le u_i(t) \le \sup_{\Psi} |u_i(t)| = \sqrt{V_i(t)},$$

$$i = 1, ..., 11.$$



Numerical modeling of estimations for gaps $g_1(t), \dots, g_6(t)$

It is presented (Figure) the numerical modeling of the estimations of six gaps for the transient processes ensemble in case of arbitrary disturbances from (4).

Such optimization process tunes feed-back controller and provides the proper quality for closed-loop system with any arbitrary disturbances from set of the cases encountered in practice. Many methods can be applied to minimize the functional discussed below. For example, gradient method of optimization can be implemented based on (9).

CONCLUSIONS

Optimization approach to the plasma stabilization synthesis was suggested. The equations of the plasma dynamics (1) are closed by the feedback controller (2). It's considered a set or an ensemble of some arbitrary plasma drops and disturbances (4). This set disturbs the closed system (3). In this case, the estimates of the upper boundaries of the possible gaps and voltages amplitudes are defined by the expression (7). Using expression (7), the integral criterion (8) evaluates the quality of stabilization process. Optimization approach provides the criterion minimization by tuning the controller values (2). Presented estimations, integral criterion and gradient of the functional don't depend on the system's and controller dimensions. This also allows us to close the object by regulators of various dimensions and adjust the computational complexity of the controller.

Numerical modeling of the transient processes ensemble of the closed object is based on (7) and the ensemble quality criterion (8).

Using proposed approach it is possible to apply the wide range of optimization techniques such as stochastic Monte Carlo methods, gradient optimization, particle swarm optimization, using neural network and deep learning. New optimization procedures can be developed for various applied problems. Suggested combination of methods can extend the scope of existing solutions in other practical areas [11 - 25].

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ОПТИМИЗАЦИОННЫЙ ПОДХОД К СИНТЕЗУ СИСТЕМЫ СТАБИЛИЗАЦИИ ПЛАЗМЫ В ТОКАМАКЕ ИТЭР

Д.А. Овсянников, С.В. Завадский

Рассматривается синтез регулятора для системы стабилизации плазмы в токамаке ИТЭР. Система стабилизации основана на измерениях диагностической системы токамака, она задаёт напряжения в управляющих катушках и обладает свойствами фильтрации. Известные методы синтеза регуляторов плазмы усовершенствованы предлагаемым оптимизационным подходом. Это дает возможность удовлетворить требования к качеству динамики процесса стабилизации с учётом различных неопределённостей в динамике плазмы, в том числе возмущений плазмы. Предложенный подход оценивает ансамбль переходных процессов замкнутого объекта. На основе предложенных оценок можно использовать широкий спектр методов оптимизации.

ОПТИМІЗАЦІЙНИЙ ПІДХІД ДО СИНТЕЗУ СИСТЕМИ СТАБІЛІЗАЦІЇ ПЛАЗМИ В ТОКАМАЦІ ІТЕР

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Розглядається синтез регулятора для системи стабілізації плазми в токамаці ІТЕР. Система стабілізації заснована на вимірах діагностичної системи токамака, вона задає напруги в керуючих котушках і має властивості фільтрації. Відомі методи синтезу регуляторів плазми вдосконалені запропонованим оптимізаційним підходом. Це дає можливість задовольнити вимоги до якості динаміки процесу стабілізації з урахуванням різних невизначеностей в динаміці плазми, в тому числі збурень плазми. Запропонований підхід оцінює ансамбль перехідних процесів замкнутого об'єкта. На основі запропонованих оцінок можна використовувати широкий спектр методів оптимізації.