FREQUENCY ANALYSIS OF THE STOCHASTIC FILTERING USING TRANSFER FUNCTIONS. PART I: SINUSOIDAL INPUT

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Volterra-series-type analyses to communication systems with stochastic resonance driven by sine waves result to. The n-hold Fourier transform of the n-th Volterra kernel plays an important role in the analysis. Methods of computing transfer functions from the system equation are described and obtaining results are considered. It is shown, if the transfer functions are known, then the output signal can be obtained by substitution the transfer functions in general formulas derived from Volterra series representation. The obtained results showed that the amplitude of the sinusoid should not be greater than one to obtain a minor third harmonic. Besides, with an increase in the frequency of the sinusoid, the power of the output signal decreases sharply. Numerical calculations of the output signal driven by sinusoidal input were made to improve the accuracy and reliability of the obtained results. Comparative analysis showed the coincidence of the results of the calculation by different methods.

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INTRODUCTION IN VOLTERRA SERIES ANALYSIS

In communication systems often it is necessary to deal with the devices executing non-linear conversions. Volterra series are usually used for calculation of such devices. Wiener introduced Volterra series into nonlinear circuit analysis [1].

The object of this paper is to present results of applying Volterra-series-type analyses to systems driven by sine waves.

Volterra series describe the output of a nonlinear system in degrees of input $x(t)$. A substantial number of the communication system can be represented as Volterra series. The series for typical system can be writing as [2]

$$y(t)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}du_1...\int_{-\infty}^{\infty}du_n g_n\left(u_1,...,u_n\right)\prod_{r=1}^{n}x\left(t-u_r\right), \quad (1)$$

where $y(t)$ is the output, $x(t)$ – the input and the kernels $g_n\left(u_1,...,u_n\right)$ describe the system. The first-order kernel $g_1(u_1)$ is simply the familiar impulse response of linear network. The higher order kernels of higher order impulse responses and serve to characterize the various orders of nonlinearity.

The coefficient $1/n!$ inserted A. Bedrosian and D. Rice [2] it because it simplifies many of equations.

The n-fold Fourier transform have the form [2, 3]

$$G_n\left(f_1,...,f_n\right)= \\ =\int_{-\infty}^{\infty}du_1...\int_{-\infty}^{\infty}du_n g_n\left(u_1,...,u_n\right)\exp\left[-j\left(f_1u_1+...f_nu_n\right)\right]. \quad (2)$$

G_0 is identically zero because our Volterra series starts with $n=1$. $G_1\left(f_1\right)$ is the transfer function of linear network. For linear systems, the possible output frequencies are the same as the frequencies in the input. For non-linear systems, however, the relationship between the input and output frequencies is more complicated [4, 5].

Thus the transform of the nth-order Volterra kernel is seen to be analogous to an nth-order Volterra transfer function. In many cases G_n can be obtained without first computing g_n.

The complete formulas are infinite series. Fortunately, in the study of communication system it is often possible to neglect terms of the Volterra series of order higher then the second or third. They are usually used because of fast increase in complexity [2, 3]. The n-fold Fourier transform considerably simplifies the solution of a large number of problems.

To calculate the transfer functions, we use the harmonic input method [2]. This method relies on the fact that a harmonic input must result in a harmonic output when (1) holds. System specified by the nonlinear differential equation considered [2, 3]

$$F(d/dt)y+\sum_{l=2}^{\infty}a_l y^l = x(t), \quad (3)$$

with the condition that system causally ($y(t)$ vanish identically when $x(t)$ does). It is assumed that one and only one such solution exists (it is proved in [3]) and the system is stable. $F(d/dt)$ is a polynomial in d/dt, and the coefficients in $F(d/dt)$ and the coefficients a_l are independent of $t, x,$ and y.

The Volterra transfer functions for (3) can be written as [2]

$$G_n(f_1,...,f_n)=-\frac{\sum_{l=2}^{n}a_l G_n^{(l)}(f_1,...,f_n)}{F(j\omega_1+...+j\omega_n)}. \quad (4)$$

The last equation is recurrence relation because $G_n^{(l)}$ is given by

$$G_n^{(l)}(f_1,...,f_n)=l!\sum_{(\nu;l,n)}\sum_{l=2}^{n}a_l G_{\nu 1}(f_1,...,f_{\nu 1})\times \\ \times G_{\nu 2}(f_{\nu 1+1},...,f_{\nu 1+\nu 2})...G_{\nu l}(f_\mu,...,f_n),$$

for the n-fold Fourier transform of the n-th kernel in the Volterra series for $[y(t)]^l$, l being a positive integer, and $1\le l\le n$. $G_n^{(l)}(f_1,...,f_n)$ is zero for $l>n$ and $G_n^{(n)}(f_1,...,f_n)$ is equal to $n!G_1(f_1)G_1(f_2)...G_1(f_n)$.

SINUSOIDAL INPUT FOR STOCHASTIC RESONANCE

Consider a general bistable dynamic system which can be described by the following stochastic differential equation [6, 7]

$$\frac{dy}{dt} = ay - by^3 + x(t)\,, \qquad (5)$$

where a and b are positive, usually given in terms of system parameters, $x(t) = s(t) + n(t)$, $s(t) = A\sin(2\pi f_0 t + \varphi)$ is the driving signal, $n(t)$ is the input noise, $E[n(t)n(t+\tau)] = 2D\delta(\tau)$, D is the noise intensity. The input signal consists of the driving signal $s(t)$ and the additive noise $n(t)$ [6, 8]. This equation describes a stochastic resonance effect (SR) [6 - 8]. Volterra transfer function for $y(t)$ are given in table 1 for the general case (the equation 3) and for SR equation.

Table 1

Volterra transfer function

	Volterra transfer function for eq. 3 [2]	Volterra transfer function for eq. 4
G_1	$1/F(j\omega_1)$	$\frac{1}{-a + j\omega_1}$
G_2	$-2a_2 G_1(f_1)G_1(f_2)/F(j\omega_1 + j\omega_2)$	0
G_3	$-\frac{2a_2 \sum_3' G_1(f_1)G_2(f_2, f_3) + 6a_3 G_1(f_1)G_1(f_2)G_1(f_3)}{F(j\omega_1 + j\omega_2 + j\omega_3)}$	$\frac{-6b}{(-a + j\omega_1)(-a + j\omega_2)(-a + j\omega_3)(-a + j\omega_1 + j\omega_2 + j\omega_3)}$

When noise is absent, input signal can be writing as $x(t) = s(t) = A\sin(2\pi f_0 t + \varphi)$. We will solve SR equation for this case. The leading terms output signal are given in Table 2.

Table 2

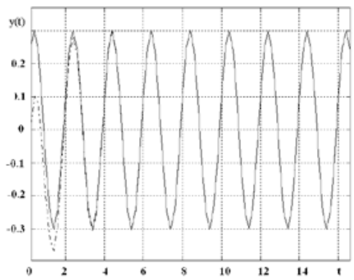
The leading terms output signal

The output signal for eq. 3 [2]	The output signal for eq. 4
$\left[\frac{P^2}{4} G_2(f_p, -f_p) + \ldots\right]$	0
$e^{jpt}\left[\frac{P}{2} G_1(f_p) + \frac{P^3}{16} G_3(f_p, f_p, -f_p) + \ldots\right] +$ $+e^{-jpt}\left[\frac{P}{2} G_1(-f_p) + \frac{P^3}{16} G_3(-f_p, -f_p, f_p) + \ldots\right]$	$\frac{A}{(a^2 + \omega_0^2)}\left[-a + \frac{3A^2 b(-a^2 + \omega_0^2)}{4(a^2 + \omega_0^2)^2}\right]\cos\omega_0 t +$ $+\frac{A}{(a^2 + \omega_0^2)}\left[\omega_0 + \frac{3A^2 b\omega_0}{2(a^2 + \omega_0^2)^2}\right]\sin\omega_0 t$
$e^{j2\pi pt}\left[\frac{P^2}{8} G_2(f_p, f_p) + \ldots\right] +$ $+e^{-j2\pi pt}\left[\frac{P^2}{8} G_2(-f_p, -f_p) + \ldots\right]$	0
$+e^{j3\pi pt}\left[\frac{P^3}{48} G_3(f_p, f_p, f_p) + \ldots\right] +$ $+e^{-j3\pi pt}\left[\frac{P^3}{48} G_3(-f_p, -f_p, -f_p) + \ldots\right] + \ldots$	$\frac{A^3 b}{4(a^2 + \omega_0^2)^3(a^2 + 9\omega_0^2)}\left[\begin{array}{l} -(a^4 - 12a^2\omega_0^2 + 3\omega_0^4)\cos 3\omega_0 t + \\ +2a\omega_0(5\omega_0^2 - 3a^2)\sin 3\omega_0 t \end{array}\right]$

We will define the output signal of non-linear system by the Runge-Kutta method and by Volterra series. Results of calculations are given in Figure for $A = 1$, $f = 0.5$ Hz.

Results from the Figure show, that in the numerical calculation by Runge-Kutta method takes place transient that lasts about two periods of a signal. Further the results of calculation of an output signal received by both methods match.

We will determine the power of an output signal. Powers of the first and third harmonica are specified in Table 3.



Analyzed results of the output signal using different methods: Volterra series (solid line); Runge-Kutta method (dotted line)

Table 3

The power of the output signal

	The power of the output signal for eq. 3 [2]	The power of the output signal for eq. 4
S_{ω_0}	$\left\lvert \frac{A}{2}G_1(f_0)+\frac{A^3}{16}G_3(f_0,f_0,-f_0)\right\rvert^2$	$\frac{a^2A^2}{4(a^2+\omega_0^2)^2}-\frac{3A^4ba}{8(a^2+\omega_0^2)^3}+\frac{9A^6b^2}{64(a^2+\omega_0^2)^4}+\frac{A^2\omega_0^2}{4(a^2+\omega_0^2)^2}$
$S_{3\omega_0}$	$\left\lvert \frac{P^3}{48}G_3(f_0,f_0,f_0)\right\rvert^2$	$\frac{A^6b^2}{64(a^2+\omega_0^2)^3(a^2+9\omega_0^2)}$

Results of Table 3 show that, with an increase in the frequency of the sinusoid, the power of the output signal decreases drastically.

We can calculate the powers relation of the first and third harmonicas as:

$$S_{\omega_0}/S_{3\omega_0}=\frac{a^2 16(a^2+\omega_0^2)(a^2+9\omega_0^2)}{A^4b^2}-\frac{24a(a^2+9\omega_0^2)}{A^2b}+\frac{9(a^2+9\omega_0^2)}{(a^2+\omega_0^2)}+\frac{16\omega_0^2(a^2+\omega_0^2)(a^2+9\omega_0^2)}{A^4b^2}.$$

The obtained results showed that the amplitude of the sinusoid A should not be greater than one to obtain a minor third harmonic.

CONCLUSIONS

The Volterra series is a powerful tool that can be used to describe a wide class of non-linear systems.

Results of applying Volterra series analysis to systems with SR effect driven by harmonic input showed what output signal contains the 1st and 3rd harmonicas, that matches the known results [6]. In addition, with an increase in the frequency of the sinusoid, the power of the output signal decreases sharply and with an increase in the frequency of the sinusoid, the power of the output signal decreases sharply.

Results of calculations showed that numerical calculation and calculation by Volterra series match. At the same time numerical calculation is followed by transient which lasts about two periods of oscillations. Transient take place in radioengineering devices [9].

In the present paper the method of transfer functions is developed. The received gear Volterra transfer function for the systems with SR effect will allow to receive further expressions for an output signal in case of input white Gaussian noise and an input additive mix of a harmonic signal and white Gaussian noise.

REFERENCES

1. N. Wiener. *Nonlinear Problems in Random Theory*. Cambridge, Mass. Technology Press; and New York Wiley, 1958.
2. E Bedrosian, S. Rice. The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs // *IEEE*. 1971, v. 59, № 12, p. 58-82.
3. K.A. Pupkov, V.I. Kapalin, A.S. Yushchenko. *Funktsional'nyye ryady v teorii nelineynykh sistem*. M.: “Nauka” 1978, 448 p. (in Russian).
4. N.V. Zernov, V.G. Karpov. *Teoriya radiotekhnicheskikh tsepey*. Moskva-Leningrad: “Energiya”, 1965, 816 p. (in Russian).
5. Yu.I. Voloshchuk. *Pidruchnik dla studentiv vich. navch. zakladiv*. Kharkiv: TOV “Kompania CMIT”, 2005, v. 3, 228 p. (in Russian).
6. V.S. Anishchenko, A.B. Neiman, F. Moss, L. Schimansky-Geier. Stochastic resonance: noise-enhanced order // *Uspekhi Fizicheskikh Nauk, Russian Academy of Sciences*. 1999, 42(1)7-36, p. 7-34 (in Russian).
7. O. Kharchenko. Simulation of the Stochastic Resonance Effect in a Nonlinear Device // *Global Journal of Researches in Engineering* -F Volume 15 Issue 7 Version 1.0 September 2015, p. 19-23.
8. Xiaofei Zhang, Niaoqing Hu, Zhe Cheng. Stochastic resonance in multi-scale bistable array // *Physics Letter*. 2013, A 377, p. 981-984.
9. B. Sklar. *Digital Communication. Fundamentals and Applications, Second Edition,* Prentice Hall PTR, 2003, 1099 p.

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ЧАСТОТНЫЙ АНАЛИЗ СТОХАСТИЧЕСКОЙ ФИЛЬТРАЦИИ С ПОМОЩЬЮ ПЕРЕДАТОЧНЫХ ФУНКЦИЙ. ЧАСТЬ I: СИНУСОИДАЛЬНЫЙ ВХОДНОЙ СИГНАЛ

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Приведен анализ на основе рядов Вольтера применительно к телекоммуникационным системам, обладающим эффектом стохастического резонанса, возбуждаемым синусоидальным сигналом. Важную роль при таком анализе играет многомерное преобразование Фурье. Описаны способы вычисления передаточных функций, исходя из уравнения системы, и рассмотрены результаты расчетов. Сравнительный анализ численных расчетов и расчетов на основе рядов Вольтера показал совпадение результатов расчета.

ЧАСТОТНИЙ АНАЛІЗ СТОХАСТИЧНОЇ ФІЛЬТРАЦІЇ ЗА ДОПОМОГОЮ ПЕРЕДАТОЧНИХ ФУНКЦІЙ. ЧАСТИНА I: СИНУСОЇДАЛЬНИЙ ВХІДНИЙ СИГНАЛ

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Наведено аналіз на основі рядів Вольтера стосовно телекомунікаційних систем, що володіють ефектом стохастичного резонансу, які збуджуються синусоїдальним сигналом. Важливу роль при такому аналізі грає багатовимірне перетворення Фур'є. Описано способи обчислення передаточних функцій, виходячи з рівняння системи, і розглянуто результати розрахунків. Порівняльний аналіз чисельних розрахунків і розрахунків на основі рядів Вольтера показав збіг результатів розрахунку.