GROWTH RATE OF QED CASCADES IN A ROTATING ELECTRIC FIELD

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QED cascading in a rotating electric field is studied. The cascade growth rate is derived from kinetic equations for electron-positron pairs and photons as a function of the field strength. The rate is in an agreement with the results of numerical simulations of QED cascade.

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INTRODUCTION

Strong field quantum electrodynamics (QED) attracts much attention nowadays due to development of laser technologies giving an opportunity to reach electromagnetic fields with intensities exceeding I=10^{23} W/cm^2 in the laboratory conditions [1, 2]. One of the intriguing phenomenon combining QED and classical plasma physics effects is a QED cascade in strong laser field [3 - 6]. QED cascades are based on two quantum processes – high-energy photon emission by electron and photon decay with electron-positron pair (e+e-) creation. Both these processes are possible in the presence of external electromagnetic field only and become essential with the intensities over 10^{23}W/cm^2. The sequence of such processes may lead to an avalanche-like production of an electron-positron plasma. Numerical simulations predict that the QED cascade can develop in the field of counter-propagating 10 PW laser pulses that can be generated in upcoming laser facilities like ELI, Apollon etc [7, 8].

The rotating electric field is one of the simplest field configurations which may provide QED cascading. Such field configuration corresponds to the magnetic node of the circularly polarized standing wave formed by two counterpropagating circularly polarized laser pulses. The phenomenological formula for the cascade growth rate in the rotating electric field has been obtained from numerical simulations [9, 10]. The growth rate has been also estimated in the limits of weak [11, 12] and strong laser field [5]. In this paper we present the calculation of the cascade growth rate starting from the first principles.

1. KINETIC MODEL

Dynamics of QED cascade can be described by the kinetic equations for electrons, positrons and gamma-quanta with QED effects [9, 13]

$$\begin{aligned}&\partial_t f_{e,p} + \nabla_{\mathbf{r}}\left(\frac{\mathbf{p}}{\gamma} f_{e,p}\right) + \nabla_{\mathbf{r}}\left(\mathbf{F}_L f_{e,p}\right)\\ &= \int d\overline{\mathbf{p}} f_{ph}(\mathbf{r},\overline{\mathbf{p}},t) w_p(\overline{\mathbf{p}} \to \mathbf{p})\\ &+ \int d\overline{\mathbf{p}} f_{e,p}(\mathbf{r},\overline{\mathbf{p}},t)\left[w_r(\overline{\mathbf{p}} \to \overline{\mathbf{p}} - \mathbf{p}) - w_r(\mathbf{p} \to \mathbf{p} - \overline{\mathbf{p}})\right]\end{aligned} \tag{1}$$

$$\begin{aligned}&\partial_t f_{ph} + \nabla_{\mathbf{r}}\left(\frac{\mathbf{p}}{\gamma} f_{ph}\right) = -f_{ph}(\mathbf{r},\mathbf{p},t)\int d\overline{\mathbf{p}} w_p(\mathbf{p} \to \overline{\mathbf{p}})\\ &+ \int d\overline{\mathbf{p}}\left[f_e(\mathbf{r},\overline{\mathbf{p}},t) + f_p(\mathbf{r},\overline{\mathbf{p}},t)\right] w_r(\overline{\mathbf{p}} \to \mathbf{p})\end{aligned} \tag{2}$$

where f_e, f_p, f_{ph} are the distribution functions of the electrons, the positrons and the photons, respectively, which are normalized such that

$$\int f_{e,p}(\mathbf{r},\mathbf{p},t) d\mathbf{p} d\mathbf{r} = N_{e,p,ph}(t), \tag{3}$$

where N_e, N_p, N_{ph} are the numbers of the electrons, the positrons and the photons, respectively,

$$\mathbf{F}_L = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \tag{4}$$

is the Lorentz force of the positron in the electric and magnetic field, $\mathbf{v}$, is the lepton velocity. In this paper the energy, ε, is normalized to mc^2, the momentum, p, is normalized to mc, the velocity v, is normalized to c, the field intensity is normalized to $mc\omega/e$, time is normalized to $1/\omega$, and the space coordinates are normalized to c/ω, m and e are the positron charge and mass, ω is the field frequency, c is the speed of light. $w_r(\mathbf{p} \to \overline{\mathbf{p}})$ is the probability in time unit for the electron with the momentum $\mathbf{p}$ to emit a photon and to transit to the state with the momentum $\mathbf{p} - \overline{\mathbf{p}}$, $w_p(\mathbf{p} \to \overline{\mathbf{p}})$ is the probability in time unit for the photon with the momentum $\mathbf{p}$ to decay with creation of the electron with the momentum $\overline{\mathbf{p}}$ and the positron with the momentum $\mathbf{p} - \overline{\mathbf{p}}$. Here we assume that the photon emission and the pair production occur in synchrotron regime, and we neglect the angles about or less than $1/\varepsilon_l$ where ε_l is the lepton energy. Eqs. (1), (2) are the complete set of equations which describes evolution of the distribution functions for QED cascade. The RHS of Eqs. (1), (2) describes photon emission and pair production. Namely, the first term in the RHS of Eq. (1) characterizes creation of the lepton with momentum $\mathbf{p}$ by decay of the photons with momenta $p' > p$, the second and the third terms respectively describe the increase and decrease of the number of the electrons with momentum $\mathbf{p}$ due to photon emission. The first term in the RHS of Eq. (2) characterizes photon decay, and the last two terms describe emission of new photons by the electrons and the positrons. The differential probability rate w_r can be expressed through the energy distribution of the probability rate for photon emission by ultrarelativistic lepton ($\varepsilon_l \approx p_l$) in an electromagnetic field, $dW_{rad}/d\varepsilon_{ph}$, and w_r can be expressed through energy distribution of the probability rate for direct pair creation by photon $dW_{pair}/d\varepsilon_l$ [13]:

$$w_r(\mathbf{p}\to\overline{\mathbf{p}})=\int_0^\infty d\varepsilon\frac{dW_{rad}(\mathbf{p},\varepsilon)}{d\varepsilon}\delta\left(\overline{\mathbf{p}}-\varepsilon\frac{\mathbf{p}}{p}\right), \quad (5)$$

$$w_p(\mathbf{p}\to\overline{\mathbf{p}})=\int_0^\infty d\varepsilon\frac{dW_{pair}(\mathbf{p},\varepsilon)}{d\varepsilon}\delta\left(\overline{\mathbf{p}}-\varepsilon\frac{\mathbf{p}}{p}\right), \quad (6)$$

$$\frac{dW_{rad}(\mathbf{p},\varepsilon_{ph})}{d\varepsilon}=-\frac{\alpha}{a_S p^2}\left\{\int_{\kappa(\varepsilon_{ph},\mathbf{p})}^\infty \mathrm{Ai}(\xi)d\xi \right. \quad (7)$$
$$\left.[\chi_l(\mathbf{p})]^{2/3}\,\mathrm{Ai}'(\kappa(\varepsilon_{ph},\mathbf{p}))\rho_r(\varepsilon_{ph},p)\right\},$$

$$\frac{dW_{pair}(\mathbf{p},\varepsilon_l)}{d\varepsilon}=-\frac{\alpha}{a_S p^2}\left\{\int_{\tilde\kappa(\varepsilon_l,\mathbf{p})}^\infty \mathrm{Ai}(\xi)d\xi \right. \quad (8)$$
$$\left.[\chi_{ph}(\mathbf{p})]^{2/3}\,\mathrm{Ai}'(\tilde\kappa(\varepsilon_l,\mathbf{p}))\rho_r(\varepsilon_l,p)\right\},$$

$$\kappa(\varepsilon_{ph},\mathbf{p})=\left[\frac{\varepsilon_{ph}}{(p-\varepsilon_{ph})\chi_l(\mathbf{p})}\right]^{2/3}, \quad (9)$$

$$\tilde\kappa(\varepsilon_l,\mathbf{p})=\left[\frac{p^2}{(p-\varepsilon_l)\varepsilon_l\chi_{ph}(\mathbf{p})}\right]^{2/3}, \quad (10)$$

$$\rho_r(\varepsilon_{ph},p)=2\left(\frac{p-\varepsilon_{ph}}{\varepsilon_{ph}}\right)^{2/3}+\left(\frac{\varepsilon_{ph}}{p-\varepsilon_{ph}}\right)^{1/3}\frac{\varepsilon_{ph}}{p}, \quad (11)$$

$$\rho_p(\varepsilon_l,p)=2\left(\frac{\varepsilon_l(p-\varepsilon_l)}{p^2}\right)^{2/3}-\left(\frac{p^2}{\varepsilon_l(p-\varepsilon_l)}\right)^{1/3}, \quad (12)$$

$$\chi_l=\frac{\varepsilon_l^2}{a_S^2}\sqrt{(\mathbf{E}+\mathbf{v}\times\mathbf{B})^2-(\mathbf{v}\cdot\mathbf{E})^2}, \quad (13)$$

$$\chi_{ph}=\frac{\varepsilon_{ph}^2}{a_S^2}\sqrt{(\mathbf{E}+\mathbf{n}\times\mathbf{B})^2-(\mathbf{n}\cdot\mathbf{E})^2}, \quad (14)$$

where $\mathrm{Ai}(x)$ and $\mathrm{Ai}'(x)$ are the Airy function and its derivative, respectively, v is the lepton velocity, $\mathbf{n}$ is the unity vector in the direction of the photon propagation, $a_S=eE_S/mc\omega$ and $E_S=m^2c^3/e\hbar$ is the QED critical field. The delta-function in Eqs. (4) and (5) demonstrates that the photon is emitted along the lepton momentum and the pair is created with momentum directed along the photon momentum. There are relations between w_r and W_{rad} as well as between w_p and W_{pair}:

$$W_{rad}=\int_0^\infty d\varepsilon\frac{dW_{rad}(\mathbf{p},\varepsilon)}{d\varepsilon}=\int d\overline{\mathbf{p}}w_r(\mathbf{p}\to\overline{\mathbf{p}}), \quad (15)$$

$$W_{pair}=\int_0^\infty d\varepsilon\frac{dW_{pair}(\mathbf{p},\varepsilon)}{d\varepsilon}=\int d\overline{\mathbf{p}}w_p(\mathbf{p}\to\overline{\mathbf{p}}), \quad (16)$$

in accordance with the energy conservation low: $w_{r,p}(\mathbf{p}\to\overline{\mathbf{p}})$ for $\overline{p}>p$ and $dW_{rad}(\mathbf{p},\varepsilon)/d\varepsilon=dW_{pair}(\mathbf{p},\varepsilon)/d\varepsilon=0$ for $\varepsilon>p$.

2. ELECTRON AND POSITRON DYNAMICS

It is difficult to solve these equations analytically. To simplify Eq. (1) we suppose that the lepton dynamics can be described by the radiation reaction force in the Landau-Lifshits form with QED corrections [14-16]. In this case the last two terms in Eq. (1), which are responsible for the energy losses because of the photon emission, can be described via the radiation reaction force [13]

$$\partial_t f_{e,p}+\nabla_{\mathbf{r}}\left(\frac{\mathbf{p}}{\gamma}f_{e,p}\right)+\nabla_{\mathbf{r}}\left((\mathbf{F}_L-\mathbf{v}F_R)f_{e,p}\right)$$
$$\approx\int d\overline{\mathbf{p}}f_{ph}(\mathbf{r},\overline{\mathbf{p}},t)w_p(\overline{\mathbf{p}}\to\mathbf{p}). \quad (17)$$

If the pair production is neglected ($f_{ph}=0$) and neglecting QED corrections ($\chi_l=0$) then Eq. (14) is reduced to the form

$$\partial_t f_{e,p}+\mathbf{v}\cdot(\partial_{\mathbf{r}}f_{e,p})+(\mathbf{F}_L-\mathbf{v}F_R)\cdot(\partial_{\mathbf{p}}f_{e,p})$$
$$\approx f_{e,p}\partial_{\mathbf{p}}\cdot(\mathbf{v}F_R)\approx f_{e,p}\frac{4F_R}{\varepsilon_l}, \quad (18)$$

where $F_R\approx\mu\varepsilon_l^2\left[(\mathbf{E}+\mathbf{v}\times\mathbf{B})^2-(\mathbf{v}\cdot\mathbf{E})^2\right]$ is the main term of the radiation reaction force in the Landau-Lifshits form and $\mu=(2/3)(e^2/mc^2)(\omega/c)$. The term in RHS is responsible for the phase-space contraction [17, 18]. It can be shown that $\partial_{\mathbf{p}}\cdot(\mathbf{v}F_R)\approx 4F_R/\varepsilon_l$ in the limit $\varepsilon_l\gg1$, where the remaining terms are in $\varepsilon_l^{\,2}$ less than the leading ones. Indeed, $F_R\mu^{-1}\varepsilon_l^{-2}$ can be rewritten as follows

$$F_R\mu^{-1}\varepsilon_l^{-2}=(\mathbf{E}+\mathbf{v}\times\mathbf{B})^2-(\mathbf{v}\cdot\mathbf{E})^2$$
$$=E^2+2\mathbf{E}\cdot(\mathbf{v}\times\mathbf{B})+(\mathbf{v}\times\mathbf{B})^2-(\mathbf{v}\cdot\mathbf{E})^2 \quad (19)$$
$$\approx(\mathbf{v}\times\mathbf{E})^2+(\mathbf{v}\times\mathbf{B})^2-2\mathbf{v}\cdot(\mathbf{E}\times\mathbf{B}).$$

The derived expression coincides with the expression for $4^{-1}\mu^{-1}\varepsilon_l^{-1}\partial_{\mathbf{p}}\cdot(\mathbf{v}F_R)$ calculated in Ref. [17, 18].

Kinetic equation Eq. (18) can be solved by method of characteristics. The solution of the kinetic equation for the rotating electric field can be presented in the form

$$f_{e,p}(\mathbf{r},\mathbf{p})=\sum_{i=1}^N\delta[\mathbf{r}-\mathbf{r}_i(t)]\delta[\mathbf{p}-\mathbf{p}_i(t)], \quad (20)$$

where $\mathbf{r}_i(t)$ and $\mathbf{p}_i(t)$ are the lepton trajectories in the rotating field:

$$\mathbf{E}(t)=a_0(\mathbf{e}_x\sin\omega t+\mathbf{e}_y\cos\omega t). \quad (21)$$

This can be verified by substituting Eq. (18) into Eq. (16) and taking into account the identity $x\delta'(x)=-\delta(x)$. Summation over the index i means summation over the initial position of the leptons, $\mathbf{r}_0$.

It is shown [14, 16-18] that because of the radiation friction and pahse-space contraction the lepton trajectrories in the rotating field are attracted to the asymptotic or stationary trajectory which is given by following equations

$$\mathbf{r}_i^Z(t)=\mathbf{e}_x\sin(\omega t+\varphi)+\mathbf{e}_y\cos(\omega t+\varphi). \quad (22)$$

$$p_i^Z(t)=\varepsilon_l[\mathbf{e}_x\cos(\omega t+\varphi)-\mathbf{e}_y\sin(\omega t+\varphi)]. \quad (23)$$

The lepton energy, ε_l, is the solution of the equation [14,16,19]

$$\varepsilon_l^{\,2}+\mu^2\varepsilon_l^{\,8}-a_0^{\,2}=0. \quad (24)$$

The equation for electron energy with QED corrections takes a form [16]

$$\varepsilon_l^{\,2}+\mu^2\varepsilon_l^{\,8}G(\chi_l)-a_0^{\,2}=0, \quad (25)$$

$$\chi_l=\frac{\varepsilon_l^{\,2}}{a_S}, \quad (26)$$

where

$$G(\chi_l) = -\int_0^\infty dx \frac{3 + (5/4)\chi_l x^{3/2} + 3x^3}{\left(1 + \chi_l x^{3/2}\right)^4} \mathrm{Ai}'(x) x \quad (27)$$

is the ratio of the total intensity of photon emission in QED to that in the classical regime:

$$G(\chi_l) = I(\chi_l) / I(\chi_l = 0).$$

3. CASCADE GROWTH RATE

We assume that:

(i) $f_{e,p,ph}(\mathbf{r},\mathbf{p},t) \approx \exp(\Gamma t) f_{e,p,ph}(\mathbf{r},\mathbf{p})$;

(ii) the momentum distribution of the leptons is narrow

$$f_{e,p}(\mathbf{r},\mathbf{p}) \approx n_{e,p}(\mathbf{r}) \delta(\mathbf{p} - \mathbf{p}^Z);$$

(iii) the fields and the distribution functions do not depend on y and z;

(iv) $n_e \approx n_p$;

(v) the lepton motion is suppresed in the direction, which is perpendicular to the plane of the rotation.

According to Eq. (23), $\mathbf{p}^Z$ does not depend on the initial conditions and depends only on the local strength of the electromagnetic field. Integrating Eq. (17) over $\mathbf{p}$ a continuity equation for electrons and positrons can be derived

$$\begin{aligned} \Gamma n_{e,p} &= \int d\mathbf{p} d\overline{\mathbf{p}} f_{ph}(x, \overline{\mathbf{p}}) w_p(\overline{\mathbf{p}} \to \mathbf{p}) \\ &= \int d\overline{\mathbf{p}} f_{ph}(x, \overline{\mathbf{p}}) W_{pair}(\chi_{ph}(\overline{\mathbf{p}})), \end{aligned} \quad (28)$$

where $\int f_{e,p}(\mathbf{r},\mathbf{p},t) d\mathbf{p} = n_{e,p}(\mathbf{r},t)$ is the electron and positron density, respectively, and Eq. (16) is used to derive RHS of Eq. (28). The equation for the photon (Eq. (2)) can be rewritten as follows

$$\begin{aligned} \Gamma f_{ph} + W_{pair} f_{ph} &= \int d\overline{\mathbf{p}} \left[f_e(x, \overline{\mathbf{p}}) + f_p(x, \overline{\mathbf{p}}) \right] w_r(\overline{\mathbf{p}} \to \mathbf{p}) \\ &\approx 2 n_p \int_0^\infty \frac{dW_{rad}(\mathbf{p}^Z, \varepsilon)}{d\varepsilon} \delta\left(\mathbf{p} - \varepsilon \frac{\mathbf{p}^Z}{p^Z} \right) d\varepsilon, \end{aligned} \quad (29)$$

where Eqs. (5)-(14) are used.

Solving Eq. (29) for f_{ph} and inserting the expression for f_{ph} into Eq. (28) the equation for the cascade growth rate can be derived after integration over $\overline{\mathbf{p}}$

$$\Gamma \approx 2 \int_0^\infty d\varepsilon \frac{\dfrac{dW_{rad}(\mathbf{p}^Z, \varepsilon)}{d\varepsilon} W_{pair}\left(\chi_{ph}\left(\varepsilon \dfrac{\mathbf{p}^Z}{p^Z} \right) \right)}{\Gamma + W_{pair}\left(\chi_{ph}\left(\varepsilon \dfrac{\mathbf{p}^Z}{p^Z} \right) \right)}. \quad (30)$$

The obtained expression is similar to Eq. (10) in Ref. [12]. However, in the model proposed in Ref. [12], the radiation reaction is neglected in calculation of $\mathbf{p}_l$, χ_l and $dW_{rad}(\mathbf{p},\varepsilon)/d\varepsilon$. As a result, Γ is overestimated for strong laser field (see Fig. 3a in Ref. [12]) when the radiation reaction is essential ($a_0 > 400$). In our model to calculate Γ the asymptotic trajectory, which exists due to the radiation reaction effect, is used. If W_{pair} and W_{rad} are approximated by some mean values $\overline{W}_{pair} = \mathrm{const}$ and $\overline{W}_{rad} = \mathrm{const}$, respectively, then Eq. (30) can be reduced to Eq. (20) obtained in Ref. [20].

As the mean probability for the photon emission is higher than that for the photon decay during cascading so that $\Gamma >> W_{pair}$ (see also Ref. [20]) and Eq. (30) can be reduced to the form

$$\Gamma^2 \approx 2 \int_0^\infty d\varepsilon \frac{dW_{rad}(\mathbf{p}^Z, \varepsilon)}{d\varepsilon} W_{pair}\left(\chi_{ph}\left(\varepsilon \frac{\mathbf{p}^Z}{p^Z} \right) \right), \quad (31)$$

which is similar to Eq. (11) in Ref. [12]] if the radiation reaction is neglected and agrees with Eq. (20) in Ref. [20] for Γ in the limit $\overline{W}_{rad} >> \overline{W}_{pair}$ when W_{pair} and W_{rad} are approximated by some mean values $\overline{W}_{pair} = \mathrm{const}$ and $\overline{W}_{rad} = \mathrm{const}$, respectively. In the model proposed in Refs. [5, 20] it is assumed that the leptons are almost stop after photon emission and χ_l of all leptons has to be more than 1 for cascading. This scenario is mostly relevant for extremely strong laser field $a_0 > 10^4$ and the model overestimates the intensity threshold of cascade developing for $a_0 < 10^4$ (see Fig. 3,a in Ref. [12]). Our model is relevant for moderate intensities $400 < a_0 < 10^4$ when the leptons lose their energy mainly by small portions $\varepsilon_{ph} \ll \varepsilon_l$ and the classical description of the lepton dynamics with radiation reaction is good. Moreover, Eq. (31) can be considered as a convolution of the photon spectrum, $dW_{rad}/d\varepsilon_{ph}$, and the pair production probability, W_{pair}. This implies that the small portion of the photons located within the high-energy part of the photon spectrum can be sufficient to drive QED cascade as W_{pair} exponentially decreases with the photon energy.

Further simplification of Eq. (31) can be achieved in the limit $\chi_{ph} << 1$ and $s/(1-s) >> \chi_l$ where $s = \varepsilon_{ph}/\varepsilon_l$. In this limit W_{pair} is reduced to the form [21]

$$W_{pair} \approx 0.23 \frac{\alpha a_S \chi_{ph}}{\varepsilon_{ph}} \exp\left(-\frac{8}{3\chi_{ph}} \right), \quad (32)$$

and the high-energy part of the photon spectrum can be only taken into account [21]

$$\frac{dW_{rad}}{d\varepsilon_{ph}} \approx \frac{\alpha a_S \chi_l^{1/2}}{6\sqrt{\pi}\varepsilon_l^2} \frac{1 - s + s^2}{\sqrt{s(1-s)}} \exp\left(-\frac{2s}{3(1-s)\chi_l} \right). \quad (33)$$

χ_{ph} can be estimated from Eq. (14): $\chi_{ph} \approx a_0 \varepsilon_{ph} / a_S = a_0 s \varepsilon_l / a_S$. Hence, Eq. (31) can be reduced to the form:

$$\Gamma^2 \approx C \frac{\chi_l^{1/2}}{\varepsilon_l} \int_0^1 A(s) \exp\left[B(s, \chi_l, \varepsilon_l) \right] ds, \quad (34)$$

$$A(s) = \frac{1 - s + s^2}{\sqrt{s(1-s)}}, \quad (35)$$

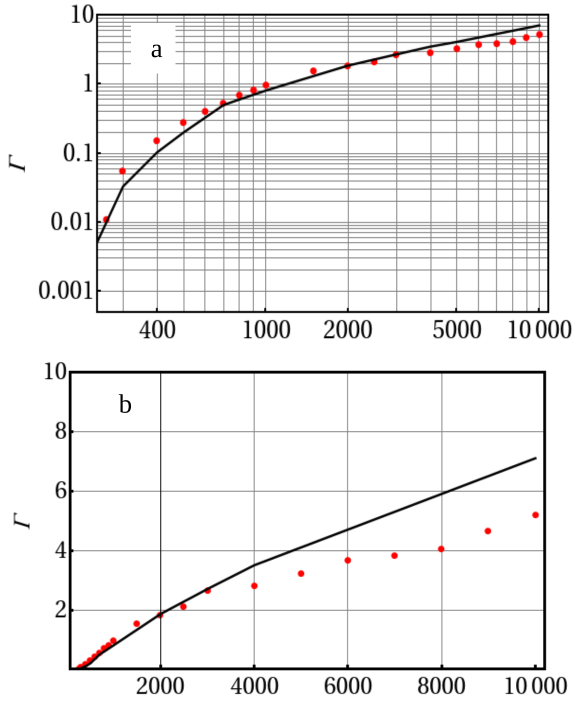
$$B(s, \chi_l, \varepsilon_l) = -\frac{2s}{3(1-s)\chi_l} - \frac{8 a_S}{3 a_0 s \varepsilon_l}, \quad (36)$$

$$C = \frac{3^{1/2}\alpha^2 a_0}{2^{9/2}\pi^{1/2} a_S}. \qquad (37)$$

It follows from Eq. (36) that $B(s) = -\infty$ and $d\Gamma^2/ds = 0$ for $s = 0$ ($\varepsilon_{ph} = 0$) and $s = 1$ ($\varepsilon_{ph} = \varepsilon_l$). In order to estimate the integral in Eq. (34) we can use the saddle point method. The main contribution to the integral comes from the neighborhood of the saddle point, $s = s_0$, specified by $dB/ds = 0$. The solution of the equation $dB/ds = 0$ is $s_0 = \tau/(1+\tau)$, $0 \le s_0 \le 1$, where $\tau^2 = 4\chi_l a_S/(a_0\varepsilon_l)$. Eq. (22) can be rewritten as follows:

$$\Gamma^2 \approx CA(s_0)\exp[B(s_0)]\int_{-\infty}^{+\infty}\exp\left[\frac{B''(s_0)}{2}(s-s_0)^2\right]ds$$
$$= CA(s_0)\sqrt{\frac{2\pi}{|B''(s_0)|}}\exp[B(s_0)] \qquad (38)$$
$$= C\sqrt{\frac{3\pi}{2}}\frac{\chi_l}{\varepsilon_l}\frac{1+\tau+\tau^2}{(1+\tau)\tau^{1/2}}\exp\left[-\frac{2\tau(2+\tau)}{2\chi_l}\right].$$

The parameters, τ and χ_l, can be calculated from Eqs. (25), (26): $\tau \approx 2(a_0^3\mu)^{-1/8}$. Further simplification of Eq. (38) is difficult as $\tau \sim 1$.



The cascade growth rate in the rotating electric field vs the field strength in the logarithmic scale (a) and in the linear scale (b). The rate is calculated numerically (red points) and analytically by Eq. (38) (black curve)

4. DISCUSSION AND CONCLUSIONS

In order to verify the obtained expressions for the cascade growth rate in the rotating electric field we perform numerical simulations based on Monte-Carlo method for modelling pair production and photon emission. The rotating frequency is equal to $2\pi c/\lambda$, where λ = 1μm. The results of the simulation are presented in Figure. It is seen from Figure that the analytical results are in an qualitative agreement with the results of the numerical simulation. The difference between the analytical results and the numerical ones at large value of a_0 can be caused by the simplification of the lepton distribution function. It is assumed in the analytical model that the lepton distribution is monoenergetic. However it follows from the numerical simulation [7] that there is a large energy spread in the lepton energy spectrum. This should be taken into account for accurate description of cascading.

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ИНКРЕМЕНТ КЭД-КАСКАДА ВО ВРАЩАЮЩЕМСЯ ЭЛЕКТРИЧЕСКОМ ПОЛЕ

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Исследовано развитие квантово-электродинамического (КЭД) каскада во вращающемся электрическом поле. Из кинетических уравнений для электрон-позитронных пар и фотонов выведен инкремент роста каскада как функция напряженности поля. Полученная формула согласуется с результатами численного моделирования.

ІНКРЕМЕНТ КЕД-КАСКАДУ В ОБЕРТАЮЧОМУ ЕЛЕКТРИЧНОМУ ПОЛІ

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Досліджено розвиток квантово-електродинамічного (КЕД) каскаду в обертаючому електричному полі. З кінетичних рівнянь для електрон-позитронних пар та фотонів виведено інкремент зростання каскаду як функція напруженості поля. Отримана формула збігається з результатами числового моделювання.