MAGNETIZED PARTICLE DIFFUSION IN A RANDOM ELECTRIC FIELD WITH JUMPING PHASE

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Diffusion of charged particles across a constant magnetic field caused by random varying electric field is considered. The electric field is assumed to be a superposition of waves with a fixed frequency, various wave vectors and randomly jumping phase. The dependence of a diffusion coefficient on a field correlation time is calculated. The effect of particle trapping which is more pronounced for a field with a low frequency of phase jumps is taken into account. It is shown that the results of the statistical approach are consistent with the direct numerical simulation.

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INTRODUCTION

In recent times, there is still considerable interest in waves with stochastic jumps of phases [1], the generation of which was implemented experimentally. It is due to their ability to penetrate into overdense plasma [2, 3], to maintain a low pressure discharge [4, 5], to heat plasmas.

Previously, we considered the heating and longitudinal diffusion of particles in the electric field of waves with jumping phase [6]. It has been shown that phase jumps not only increase the intensity of resonance particle heating but also involve nonresonance particles in this process from a wide interval of their initial velocities.

In this paper an attention is focused on the transverse diffusion of magnetized particles under the influence of random electric fields with jumping phases. We are interested in the dependence of particle diffusivity on a frequency of phase jumps. The frequency of jumps determines the decay rate of the Lagrangian correlation function of an electric field, and thus affects its integral characteristic such as a diffusion coefficient. It can be expected that the mean square displacement of particles in a field with jumping phases after initial stage becomes linear in time, i.e. diffusion resembles the classical process. By reducing the frequency of phase jumps, all the more influential is the particle trapping, and diffusion become increasingly different from the classical regime. To study the dependence of a particle transverse diffusion coefficient on a correlation time which is determined by a frequency of phase jumps, we generalize our approach formulated earlier for frozen (constant in time) random fields [7, 8] on varying random fields with stochastically jumping phase.

1. EQUATION OF MOTION AND ELECTRIC FIELD WITH RANDOMLY JUMPING PHASE

Two components of particle drift velocity $v_{x,y}$ (zero Larmor radius approximation) in a plane perpendicular to a magnetic field under action of an electric field is governed by the equations

$$v_x = -\frac{\partial}{\partial y}\varphi(\mathbf{r},t), \quad v_y = \frac{\partial}{\partial x}\varphi(\mathbf{r},t), \qquad (1)$$

where φ is proportional to the potential of the electric field with a coefficient dependent on a mass and charge of a particle and a magnitude of constant magnetic field. An electric field depends on two coordinate

$$\mathbf{r} = (x,y) = (r\cos\theta, r\sin\theta)$$

in a plane transverse to magnetic field and time *t*. If the electric field is random, the motion of particles has the character of wandering, and the characteristic of a rate of particle excursion from a starting point is the diffusion coefficient. Such diffusion resembles classical wandering only at small correlation times of a random field $\varphi(\mathbf{r},t)$, or coming over to dimensionless characteristics, at Kubo numbers much smaller than unity. For large Kubo numbers (i.e. long correlation times), particles in the course of the movement react on a local electric field profile. In the limit of a constant electric field and zero Larmor radiuses their movement becomes finite when almost all particles move along closed trajectories. Then the strongest effect is a particle trapping by an electric field. The particle trapping changes the character of diffusion from classical to anomalous at least on some temporal interval; here particle mean square displacement deviates from linear law.

Previously, we considered the problem of a random *frozen* electric field, in which the effect of particle trapping is most pronounced. The statistically isotropic stream function was taken as a superposition of $N_k \times N_\phi$ harmonics with random phases α_i, α_j

$$\varphi(\mathbf{r}) = \varphi(r,\theta) = \sum_{i=1}^{N_k}\sum_{j=1}^{N_\phi} \varphi_i \times \\ \times \cos(k_i r\cos(\phi_j - \theta) - \alpha_i - \alpha_j), \qquad (2)$$

where $k_i = (k_{\max}/N_k)\cdot i$, and $\phi_j = (2\pi / N_\phi)\cdot j$. The total intensity is distributed over a partial harmonics by the Gaussian law

$$\varphi_i^2 = \frac{2}{\sqrt{\pi}}\varphi_0^2\frac{\delta k}{\Delta k}\exp\left(-\frac{k^2{}_i}{\Delta k^2}\right)$$

with a spectrum width Δk and the interval between neighboring wave numbers δk

$$\Delta k = k_{\max} / 2.5, \qquad \delta k = k_{\max} / N_k .$$

Statistical description of particle spread in frozen field (2) proposed in [7, 8] gave us the key characteristic such as the Lagrangian correlation function of velocities which was shown to be in agreement with direct simulation.

In this paper we consider an electric field varying in time. The time dependence is twofold. First, we assume the field is time harmonic with a frequency ω. Second, a variation of field is also introduced through stochastic jumps of the random phase $\beta(t)$.

Let us consider low frequency electric field with ω much less than a cyclotron frequency when a polarization drift may be neglected. Then particle drift motion is governed the same Eq.(1), and for φ we take:

$$\varphi\left(r,\theta,t\right)=\sum_{i=1}^{N_k}\sum_{j=1}^{N_\phi}\varphi_i\times \\ \times\cos(\omega t-k_i r\cos(\phi_j-\theta)+\alpha_i+\alpha_j+\beta(t)). \tag{3}$$

The set of random phases $\alpha_i+\alpha_j$ remains constant for each field realization, while the common phase $\beta(t)$ stochastically jumps in a course of particle motion. The phase $\beta(t)$ is changed abruptly by an arbitrary value with the frequency of a jump *f* and the probability p. It can be shown that the relation between a correlation time of the field τ and parameters of the phase jumps is following

$$\tau=-\frac{1}{f\ln(1-p)}. \tag{4}$$

Depending on values of f and p, the correlation time can vary from zero to infinity.

2. NUMERICAL SIMULATION

Orbits of 10^4 particles moving in a field with stochastically jumping phases and governed by Eqs.(1), (3) were calculated. After statistical averaging of simulation data, a temporal dependence of a mean square displacement for the various combinations of parameters *p* and *f* was obtained, and a diffusion coefficients corresponding to these parameters were calculated. For asymptotic diffusion coefficient was taken its value at instant several times greater than the field correlation time.

3. STATISTICAL EQUATION

Previously, we have proposed a statistical description of particles [7] moving in accordance with the Eqs (1) in a random field (2). Then it was refined with involvement of a subensemble concept [8].

For each partial particle subensembles, marked by the value of stream function there is the relation between the mean square displacement $\langle r^2\rangle$ and the diffusion coefficient $D\left(t\right)$:

$$\langle r^2\rangle=2\int_0^t D(t)\,dt. \tag{5}$$

In turn the diffusion coefficient $D\left(t\right)$ and the Lagrangian correlation function $V_L\left(t\right)$ obey the Taylor relation:

$$D(t)=\int_0^t V_L(\tau)\,d\tau. \tag{6}$$

The Lagrangian correlation function $V_L\left(t\right)$ is unknown, and it should be found from the Eulerian correlation function $V_E\left(t\right)$ defined in the laboratory coordinate system

$$V_E(r^2)=\langle v_x(\mathbf{r})\,v_x(\mathbf{0})\rangle+\langle v_y(\mathbf{r})\,v_y(\mathbf{0})\rangle. \tag{7}$$

For given velocity field the Eulerian correlation function can be obtained by averaging over a statistical ensemble. The Eulerian correlation function of velocities (1) corresponding to the stream function (2) is found explicitly by averaging over an ensemble of random phases.

The most important step was to establish a relation between Eulerian and Lagrangian correlation functions

$$V_L(t)=V_E(\langle r^2(t)\rangle). \tag{8}$$

The system of equations (5)-(8) is closed and can be solved by numerical methods. The efficiency of such a closure is verified by agreement between solutions $\langle r(t)^2\rangle$, $D\left(t\right)$, $V_L\left(t\right)$ and results of direct simulation.

Transition from constant fields with fixed random phases (2) to time harmonic fields (3) with jumping phase $\beta(t)$ introduce additional time dependence to the Lagrangian correlation function

$$V_L(t)\to V_L(t)\cos(\omega\,t)\exp\left(-t\,/\,\tau\right). \tag{9}$$

Then the diffusion of particles in a field with jumps of phase is governed by equations of the same form as (5)-(8), but with account for the additional dependence of the Lagrangian correlation function on time (9).

4. RESULTS

In this section we compare numerical solutions of statistical equations (5) - (9) with results obtained from direct simulation of particle motion governed by Eqs. (1), (3). In Figs. 1-3 results for particular choice of jumping phase $\beta(t)$ that occur with frequency $f=1$ and probability $p=0.03$ are given. According to Eq.(4) the correlation time corresponding to this parameters of phase jumps is $\tau=32.83$. In Fig. 4 results collected from a set of calculation for various correlation times are presented.

It is shown in Fig. 1 that the decay of the correlation function $<\cos\beta(0)\cos\beta(t)>$ for stochastically jumping phase $\beta(t)$ generated in numerical simulation is exponential. This validates the form of the correlation function $V_L\left(t\right)$ (9) that is taken in statistical equations.

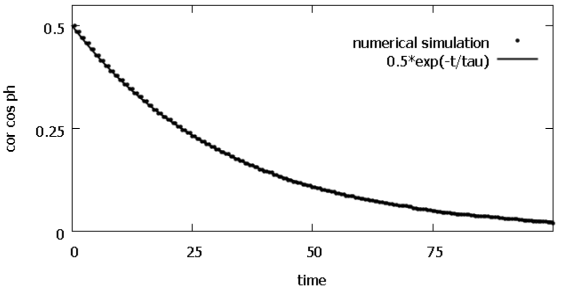

Fig. 1. Exponential decay of the correlation function $<\cos\beta(0)\cos\beta(t)>$, $\tau=32.83$

In Fig. 2 the Lagrangian correlation function of drift velocity $V_L\left(t\right)$ found as a solution of statistical equation is compared with one obtained in numerical simulation.

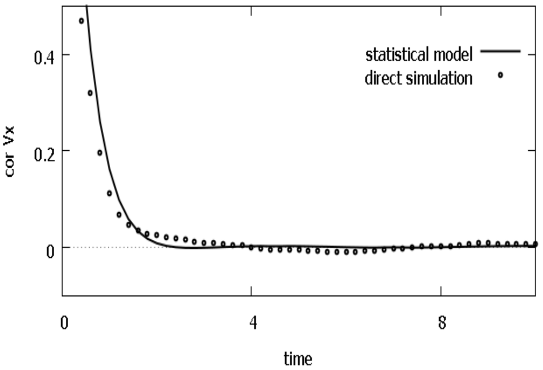

Fig. 2. Lagrangian velocity correlation function $V_L(t)$ *for* $\tau = 32.83$; *statistical model (line) vs numerical simulation (dots)*

Double integration of the Lagrangian velocity correlation function over time in statistical description gives dispersion – particle mean square excursion from an initial point. Particle dispersion obtained in statistical model is compared with direct simulation in Fig. 3.

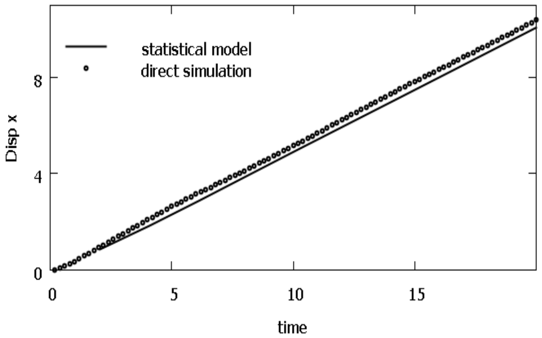

Fig. 3. Particle mean square displacement for $\tau = 32.83$; *statistical model (line) vs numerical simulation (dots)*

Beyond the initial interval, the dispersion of particles becomes linear over time, and its time derivative that is proportional to the running diffusion coefficient becomes constant. Thus a linear dispersion at times lager than field correlation time is characterized by a constant asymptotic diffusion coefficient.

The asymptotic diffusion coefficient dependence from inverse field correlation time $1/\tau$ is shown in Fig. 4. Solutions of the analytical model are compared with results of direct numerical simulation in a wide range of electric field correlation time

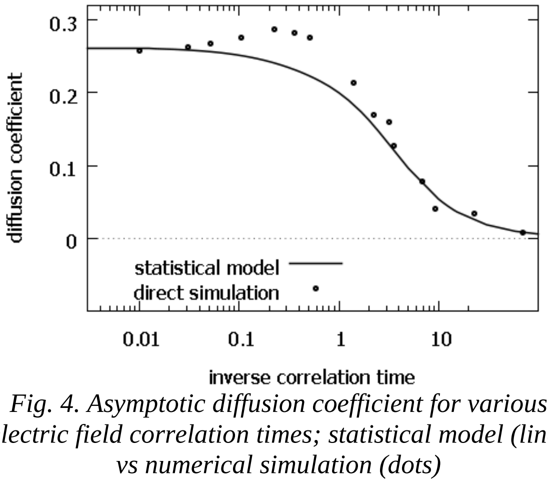

Fig. 4. Asymptotic diffusion coefficient for various electric field correlation times; statistical model (line) vs numerical simulation (dots)

CONCLUSIONS

While the diffusion of particles in fields with small correlation time was studied in details, the development of statistical descriptions for large correlation times, when the particle trapping effect is not negligible, faced difficulties. We have proposed a method of statistical equation closure to overcome them. Earlier the method was applied to study diffusion of magnetized particles in random frozen electric field.

Here it is used to examine diffusion of particles across a magnetic field undergoing time-harmonic electric field with jumping phase. Jumps of phase cause the exponential decay of field correlation function. A correlation time is determined by a frequency and probability of phase jumps.

Main statistical characteristic of particle ensembles such as the Lagrangian correlation function of particle drift velocity, running and asymptotic diffusion coefficients, and mean square particle displacement were found as solutions of statistical equations and as well in direct numerical simulations. It is shown that solutions of statistical equations agree with results obtained in direct numerical simulations in wide interval of correlation times. The statistical approach accounts for particle trapping effect pronounced for low frequency electric field and low frequency phase jumps.

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ДИФФУЗИЯ ЗАМАГНИЧЕННЫХ ЧАСТИЦ В СЛУЧАЙНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ С ПРЫЖКАМИ ФАЗЫ

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Рассмотрена диффузия заряженных частиц поперек постоянного магнитного поля, обусловленная случайным переменным электрическим полем. Предполагается, что электрическое поле представляет собой суперпозицию волн с фиксированной частотой, различными волновыми векторами и случайными прыжками фазы. Рассчитана зависимость коэффициента диффузии от времени корреляции поля. Учтен эффект захвата частиц, наиболее выраженный для полей с малой частотой скачков фазы. Показано, что результаты статистического подхода согласуются с прямым числовым моделированием.

ДИФУЗІЯ ЗАМАГНІЧЕНИХ ЧАСТИНОК У ВИПАДКОВОМУ ЕЛЕКТРИЧНОМУ ПОЛІ ЗІ СТРИБКАМИ ФАЗИ

В.І. Засенко, О.М. Черняк

Розглянуто дифузію заряджених частинок поперек постійного магнітного поля, обумовлену випадковим змінним електричним полем. Вважається, що електричне поле є суперпозицією хвиль з фіксованою частотою, різними хвильовими векторами та випадковими стрибками фази. Розрахована залежність коефіцієнта дифузії від часу кореляції поля. Враховано ефект захоплення частинок, який є більш вираженим для полів з малою частотою стрибків фази. Показано, що результати статистичного підходу узгоджуються з прямим числовим моделюванням.