INTERACTION OF A RELATIVISTIC ELECTRON BEAM WITH ELECTROMAGNETIC FIELDS IN AZIMUTHALLY CORRUGATED WAVEGUIDE

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Interaction of a relativistic electron beam with electromagnetic fields in the cylindrical waveguide with the perfectly conducting and sinusoidally corrugated in azimuth wall is theoretically investigated. Influence of geometrical parameters of the waveguide on characteristics of propagating in it fields is investigated. Regarding the approximation of small corrugation, analytical dependences of growth rates and resonant frequencies from beam and waveguide parameters are defined.

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INTRODUCTION

Interaction of charged particles beams with electromagnetic waves in electrodynamic structures is widely used for creation of powerful generators of the microwave radiation applied to heating of plasma, acceleration of charged particles and etc. (e.g. see [1 - 5]). Due to the development of intensive relativistic electronic beams formation waveguide with smoothly changing periodic surface represent considerable interest. For generation of electromagnetic waves by relativistic electronic beams effective conversion of beam energy to energy of electromagnetic radiation, as it is known, is provided by slowing down structures of type of waveguides with the corrugated walls.

The paper describes the excitation of electromagnetic waves by the monoenergetic relativistic electronic beam in the cylindrical waveguide with azimuthally corrugated walls.

1. PROBLEM STATEMENT AND SOLUTION METHOD

Let's consider the cylindrical waveguide with perfectly conducting wall of radius R(φ) which changes with angular coordinate φ according to

$$R(\varphi) = R_0\left[1 + q\cos(N\varphi)\right],$$

where R_0 is the mean radius of the waveguide, and N is the positive integer, q<1, $0 \le \varphi \le 2\pi$.

Let the infinitely thin annular monoenergetic relativistic electron beam with charge density $n_b = n_S\delta(r - r_0)$ move in a guide steady uniform magnetic field $\mathbf{H}^{(ext)} = \mathbf{e}_z H_0$ directed along the waveguide axis. Here n_S is the surface charge density, r_0 is the equilibrium radius of the beam, $r_0 \le R_0(1-q)$, $\mathbf{e}_z$ is the unit vector along the waveguide axis. The beam moves along the waveguide axis with equilibrium velocity v_{z0} and rotates about this axis with equilibrium velocity $v_{\varphi 0} = r_0\omega_{H\perp}$, where $\omega_{H\perp} = |e|H_0/(mc\gamma_0)$, $\gamma_0 = \left(1 - \beta_{\varphi 0}^2 - \beta_{z0}^2\right)^{-1/2}$, $\beta_{\varphi 0} = v_{\varphi 0}/c$, $\beta_{z0} = v_{z0}/c$.

The original system of equations. Describing interaction of a beam with electromagnetic fields in a waveguide, consists of the Maxwell's equations for a field of waves and the relativistic equations of motion and the continuity equation for the electrons in cylindrical coordinates.

Let's consider excitation of a TE-field by an electron beam.

As the waveguide radius is periodical on φ the exited fields in a waveguide can be expanded in terms of azimuthal harmonics, by Floquet's theorem:

$$F(r,\varphi,z,t) = \sum_{n=-\infty}^{\infty} A_n f_n(r)\exp(i\psi_n),$$

where $\psi_n = l_n\varphi + k_z z - \omega t$. $l_n = s + nN$, s is an integer due to the periodicity of the whole system in φ with the period 2π.

The components of the TE- field can be expressed in terms of the longitudinal magnetic field as follows:

$$E_{rn} = -\frac{kl_n}{k_\perp^2 r}H_{zn},\quad E_{\varphi n} = -\frac{ik}{k_\perp^2}\frac{\partial}{\partial r}H_{zn},$$

$$H_{rn} = -\frac{k_z l_n}{k_\perp^2}\frac{\partial}{\partial r}H_{zn},\quad H_{\varphi n} = -\frac{k_z l_n}{k_\perp^2 r}H_{zn},$$

where $k = \omega/c$, $k_\perp = \sqrt{k^2 - k_z^2}$.

The fields resulting from Maxwell's equations satisfy the boundary conditions at the beam-vacuum interface and a perfectly conducting wall, respectively. On a beam surface at $r=r_0$ boundary conditions reduce to the conditions that E_φ becomes continuous. Changes in the magnetic field across the boundary are associated with surface current:

$$E_\varphi^{(1)} = E_\varphi^{(2)},\quad H_z^{(1)} - H_z^{(2)} = (4\pi/c)\,j_{\varphi S}, \qquad (1)$$

where $j_{\varphi S}$ is the surface current density of the beam.

On the wall the boundary conditions reduce to the requirement that the tangential electric field vanishes, i.e., $E_\tau[R(\varphi)] = 0$. For quiet dependence of the function describing a profile of a surface from coordinate this boundary condition on a wall of a waveguide, expressed through electric field components, becomes:

$$\frac{dR(\varphi)}{d\varphi}E_r^{(2)} + R(\varphi)E_\varphi^{(2)} = 0. \qquad (2)$$

Superscript refers to the particular region of waveguide being considered.

The components of longitudinal magnetic field take the form:

$$H_{zn}^{(1)} = A_n J_{l_n}(k_\perp r),\ \text{at}\ 0 \le r \le r_0$$

$$H_{zn}^{(2)} = B_n J_{l_n}(k_\perp r) + C_n N_{l_n}(k_\perp r),\ \text{at}\quad r_0 \le r \le R(\varphi).$$

On the basis of the equations of the electrons motion in an external magnetic field and in the field of the electromagnetic wave excited by a beam, it is possible to obtain the expressions for small perturbations of velocity and respectively the trajectory of electrons and express them through the components of the electromagnetic field in the considered structure [6, 7].

Using the continuity equations, connecting the perturbation of density with the perturbation of velocity of electrons, we can find the azimuthal component of current density of a beam. Then substituting these expressions for fields and current density in boundary conditions (1), (2) and performing the relevant calculations (see [8]) we obtain an infinite system of the algebraic equations for A_n

$$\sum_{n=-\infty}^{\infty} a_{mn} A_n = 0, \quad -\infty \le m \le \infty, \tag{3}$$

where

$$a_{mn} = \int_{-\pi/N}^{\pi/N} \{k_\perp R_0 [1 + q\cos(N\varphi)]^2 W'_{l_n}[k_\perp R(\varphi)] + $$
$$+ iql_n N \sin(N\varphi) W_{l_n}[k_\perp R(\varphi)]\} e^{iN(n-m)\varphi} d\varphi,$$

$W_l(x) = J_l(x) + \mu_l N_l(x)$, $J_l(x)$, $N_l(x)$ are the Bessel functions of the first and second kind, $W'_l(x) = (d/dx) W_l(x)$.

Considering the excitation of electromagnetic waves caused by resonance $\overline{\omega}_l = \omega - k_z \mathrm{v}_{z0} - l_n \omega_{H\perp} \approx 0$, the expression for μ_l can be written as:

$$\mu_l = -\frac{\pi}{2}\left[\frac{\omega_{b\perp}}{\overline{\omega}_l c}\omega_{H\perp} k_\perp r_0^2 J'_l(k_\perp r_0)\right]^2 \left(1 - \frac{l^2}{k_\perp^2 r_0^2}\right),$$

where $\omega_{b\perp} = \sqrt{4\pi e^2 n_S / (m\gamma_0 r_0)}$.

The nontrivial solution of Eq. (3) gives the dependence of frequency ω on a wave number k_z and azimuthal number s

$$\|a_{mn}(\omega)\| = 0, \tag{4}$$

In a limiting case of small depth of corrugation (q<<1), with accuracy of the second order in q, this equation can be written as,

$$\sum_{n=-\infty}^{\infty} \frac{a_{n,n+1}}{a_{n,n}} \cdot \frac{a_{n+1,n}}{a_{n+1,n+1}} = 1, \tag{5}$$

where the coefficients a_{mn} are defined by formulas:

$$a_{mn} = \left\{ W'_{l_n}(\alpha) - \frac{q^2\alpha}{4} \times \right.$$
$$\left. \times \left[\alpha\left(1 - \frac{l_n^2}{\alpha^2}\right) W'_{l_n}(\alpha) + \left(3 - \frac{l_n^2}{\alpha^2}\right) W_{l_n}(\alpha)\right]\right\} \delta_{n,m} +$$
$$+ \frac{q}{2}\left[W'_{l_n}(\alpha) - \alpha\left(1 - \frac{l_n l_m}{\alpha^2}\right) W_{l_n}(\alpha)\right](\delta_{m,n+1} + \delta_{m,n-1}),$$

where $\alpha = k_\perp R_0$, $\delta_{m,n}$ is the Kronecker delta.

2. ANALYSIS OF THE DISPERSION RELATION

Let us consider the solution of equation (5) as an explicit analytical formulae since the quantity q is small. To determine the dependence of frequency ω on a wave number k_z, only the interaction of the fundamental mode s (n=0) with the n=1, and n= -1 harmonics of the azimuthally corrugated waveguide needs be considered.

In the absence of a beam, in the zero-order approximation on corrugation, analysis of eq. (5) leads to the equation: $J'_s[\alpha(\omega_0)] = 0$ and to the dispersion relation for the normal-mode spectrum for cylindrical waveguide with perfectly conducting wall. The result is:

$$\omega_0^2 / c^2 - k_z^2 = \chi_{s,m}^2 / R_0^2,$$

where $\chi_{s,m}$ is the m-th root of derivative of the sth-order Bessel function of the first kind.

Accurate within the second order in q inclusively, we obtain the following correction $\Delta\omega$ to the frequency ω_0 of a smooth waveguide due to corrugation from the equation (5):

$$\Delta\omega = -\frac{q^2 c^2 \chi_{s,m}^2}{4R_0^2 \omega_0}\left(1 - \frac{s^2}{\chi_{s,m}^2}\right)^{-1} \times$$
$$\times \left\{1 + \frac{s^2}{\chi_{s,m}^2} + \chi_{s,m}\left[1 - \frac{s(s+N)}{\chi_{s,m}^2}\right]^2 \frac{J_{s+N}(\chi_{s,m})}{J'_{s+N}(\chi_{s,m})} + \right. \tag{6}$$
$$\left. + \chi_{s,m}\left[1 - \frac{s(s-N)}{\chi_{s,m}^2}\right]^2 \frac{J_{s-N}(\chi_{s,m})}{J'_{s-N}(\chi_{s,m})} + \right\}.$$

From expression (6) it follows that at a finite nonzero depth of corrugations the correction to the frequency is proportional to q^2.

If N is an even number at s=N/2, the denominator of the last term in the right-hand side of Eq. (9) is zero. At these values of s the expression (6) is incorrect. In calculating the correction of frequency at s=N/2 it is necessary to take into account the coupling of fundamental mode and n= -1 harmonic of the field. The expression for the correction $\Delta\omega$ in this case becomes:

$$\Delta\omega = \pm\frac{qc^2\chi_{s,m}^2}{2R_0^2\omega_0}\frac{\left(1 + s^2/\chi_{s,m}^2\right)}{\left|1 - s^2/\chi_{s,m}^2\right|}. \tag{7}$$

Thus, at s=N/2 there is a spectrum splitting due to finite depth of corrugations. At the fixed value of longitudinal wave number there is an opacity band which width is proportional to q.

The account in the eq. (5) terms, depending on the beam density, leads to occurrence resonant terms at $\omega = k_z \mathrm{v}_{z0} + (s + nN)\omega_{H\perp}$

Let's consider the excitation of the electromagnetic wave under the resonance of particles with the fundamental electromagnetic mode: $\omega = k_z \mathrm{v}_{z0} + s\omega_{H\perp}$. Writing down $\omega = \omega_0 + \Delta\omega + \delta\omega$ and taking into account only resonant terms in the eq. (5), we obtain the following expression for the growth rate

$$Jm(\delta\omega) = \left(\sqrt{3}/2\right)\left(\omega_{b\perp}^2 \omega_{H\perp}^2 K_0 / \omega_0\right)^{1/3}, \tag{8}$$

$$K_0 = \frac{\pi}{2}\chi_{s,m}^3\xi^4\left(1-\frac{s^2}{\chi_{s,m}^2\xi^2}\right)\frac{\left[J_s'(\chi_{s,m}\xi)\right]^2}{1-s^2/\chi_{s,m}^2}\frac{N_s'(\chi_{s,m})}{J_s(\chi_{s,m})}.$$

Here, to simplify the formulae, we omit the terms proportional to q, $\xi = r_0/R_0$.

In a case of excitation of an electromagnetic wave under the resonance with the first harmonic of the azimuthally corrugated waveguide

$$\omega = k_z v_{z0} + (s+N)\omega_{H\perp}$$

the expression for growth rate is

$$Jm(\delta\omega) = \left(\sqrt{3}/2\right)\left(q^2\omega_{b\perp}^2\omega_{H\perp}^2K_{s+N}/\omega_0\right)^{1/3}, \quad (9)$$

$$K_{s+N} = \frac{\chi_{s,m}^4\xi^4}{4}\left(1-\frac{(s+N)^2}{\chi_{s,m}^2\xi^2}\right)\frac{\left[1-s(s+N)/\chi_{s,m}^2\right]^2}{1-s^2/\chi_{s,m}^2}\times$$

$$\times\left[\frac{J_{s+N}'(\chi_{s,m}\xi)}{J_{s+N}'(\chi_{s,m})}\right]^2 .$$

CONCLUSIONS

The analytical theory of interaction of a relativistic electron beam with electromagnetic waves in a cylindrical waveguide with sinusoidally corrugated in azimuth wall is investigated.

It is shown, in particular, that corrugation of wall leads to change of a smooth waveguide dispersion and occurrence of opacity bands. In a limiting case of small depth of corrugations analytical dependences of frequencies on waveguide parameters (depth of corrugations, radius of a waveguide, etc.) are obtained. The width of the first opacity band is proportional to q.

The growth rates of instability under the cyclotron resonance of electron beam with the fundamental mode, and also with the first harmonic of the azimuthally corrugated waveguide are found. In the latter case growth rate is proportional to $q^{2/3}$. Thus smaller external magnetic fields are necessary for generation of high-frequency electromagnetic radiation, than in waveguides with smooth walls.

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ВЗАИМОДЕЙСТВИЕ РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ПУЧКА С ЭЛЕКТРОМАГНИТНЫМИ ПОЛЯМИ В АЗИМУТАЛЬНО-ГОФРИРОВАННОМ ВОЛНОВОДЕ

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Теоретически исследовано взаимодействие релятивистского электронного пучка с электромагнитными полями в цилиндрическом волноводе с синусоидально-гофрированными по азимуту идеально-проводящими стенками. Исследовано влияние геометрических параметров волновода на характеристики распространяющихся в нем волн. В приближении малой глубины гофра определены аналитические зависимости инкрементов неустойчивостей и резонансные частоты от параметров пучка и волновода.

ВЗАЄМОДІЯ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ПУЧКА З ЕЛЕКТРОМАГНІТНИМИ ПОЛЯМИ В АЗИМУТАЛЬНО-ГОФРОВАНОМУ ХВИЛЕВОДІ

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Теоретично досліджена взаємодія релятивістського електронного пучка з електромагнітними полями в циліндричному хвилеводі із синусоїдно-гофрованими по азимуту ідеально-провідними стінками. Досліджено вплив геометричних параметрів хвилеводу на характеристики хвиль. У наближенні малої глибини гофри визначені аналітичні залежності інкрементів нестійкостей і резонансні частоти від параметрів пучка та хвилеводу.