EFFICIENT APPROACH TO ANALYSIS OF TM MODES IN COAXIAL GYROTRON CAVITY WITH CORRUGATED INSERT

T.I. Tkachova[1], V.I. Shcherbinin[1], V.I. Tkachenko[1,2]
[1]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
[2]V.N. Karazin Kharkov National University, Kharkov, Ukraine
E-mail: t.i.tkachova@gmail.com

Electromagnetic characteristics of TM modes of a coaxial gyrotron cavity with a corrugated inner conductor are investigated. Coupled-mode method based on re-expansion of the cavity fields in terms of the Gegenbauer polynomials is applied to reduce the computation time required for solving the eigenvalue problem with desired accuracy. Dispersion equation is obtained for calculating the eigenvalues of the TM modes. For these modes, transverse field distribution is investigated. Convergence of computations is shown for cavity eigenvalues and eigenfields.

PACS: 84.40.-x, 84.40.Ik, 52.35.Hr

INTRODUCTION

Nowadays gyrotrons are the most powerful sources of continuous-wave (CW) millimeter radiation [1]. Among their applications, one of the most important is electron-cyclotron heating of magnetically confined plasma in controlled thermonuclear fusion devices. In particular, a number of 170 GHz MW-class CW gyrotrons are now under development worldwide for use in the heating system of the International Thermonuclear Experimental Reactor (ITER).

The key gyrotron performances are the frequency and the output power. The frequency of 170 GHz is required for ITER, while the output power of an individual gyrotron should be as high as possible to reduce both the complexity and the cost of the gyrotron-based heating system. At present, the ITER-relevant 170 GHz CW gyrotrons with conventional cylindrical cavities are able to produce up to 1.2 MW of the output power [2]. However, further increase in their power is limited due to interrelated problems of mode competition and thermal cavity loading. Insertion of corrugated coaxial conductor into gyrotron cavity can alleviate these problems. Owing to this, the record-breaking power of 2.2 MW [3] has been recently achieved with the 170-GHz coaxial cavity gyrotron operating at short pulses. Its cavity is depicted schematically in Fig. 1.

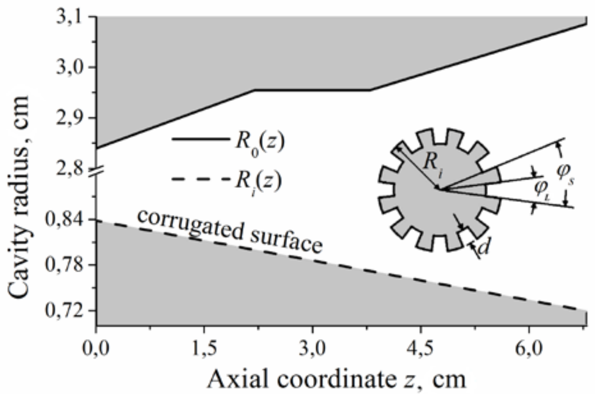


Fig. 1. The cavity of the 170-GHz 2-MW coaxial gyrotron [3] (d = 0.044 cm, φ_L/φ_S = 0.5, number of corrugations equals 75)

It is clear that the corrugated conductor adds complexity to the transverse cross-section of the gyrotron cavity and thus makes its electromagnetic analysis more difficult. The main difficulty consists in the coupling [3] arising between different azimuthal modes due to the corrugations. Because of this, the electromagnetic analysis of a gyrotron cavity must involve a large number of these modes to ensure high accuracy of the computation for the cavity eigenfrequencies and eigenfields.

There are several approaches to this analysis. The simplest and the most popular one is the surface impedance model [4]. It is a single-mode method, which treats corrugated coaxial insert as a smooth rod with uniform effective surface impedance. The validity of this method requires the number of corrugations to be large enough. Otherwise it may fail due to the strong coupling between azimuthal modes.

A rigorous approach to the electromagnetic analysis of a coaxial gyrotron cavity with corrugated insert is the method of singular integral equations (SIE) [5, 6]. This method is advantageous in that it takes into account the complete infinite set of coupled azimuthal modes. Owing to this, the resulting eigenfields converge uniformly everywhere inside the cavity, including regions near the corrugation edges. The chief shortcoming of SIE is that it is associated with time-consuming numerical computations. For this reason, it can be used as a checking numerical tool, but is hardly feasible for gyrotron design and optimization studies.

Compared to SIE, another coupled-mode approach known as space harmonics method (SHM) [4, 7] is simpler, but still rather accurate. It considers truncated number of coupled azimuthal modes. On the one hand, such simplification makes SHM suitable for electromagnetic analysis of various complex microwave structures. On the other hand, it represents an obvious drawback, which can lead [8] to inconsistency between the computations of the cavity eigenfields and the well-known Meixner's edge condition [9]. As a result, these computations may suffer from poor convergence and Gibbs phenomenon.

To correct this situation, the cavity eigenfields at the corrugation (groove) aperture can be re-expanded in terms of the orthogonal functions satisfying Meixner's edge condition. Gegenbauer polynomials (GP) can be used as such functions. This has been demonstrated in a number of investigations devoted to the guiding structures for electromagnetic [10, 11] and acoustic [12, 13] waves. For normal TE electromagnetic modes of the coaxial gyrotron cavity with corrugated insert, the re-expansion method (or GP method) was successfully used in [8]. In gyrotrons, such modes are the operating and the competing modes.

However, the gyrotron cavity (Fig. 1) is longitudinally nonuniform. Therefore, TE operating and competing cavity modes can be converted partially into TM modes [14, 15]. This explains the importance of the electromagnetic analysis for normal TM modes of the coaxial gyrotron cavity with corrugated insert. In [6] and [7] this analysis has been performed on the basis of SIE and SHM methods, respectively. In this study, the GP method is used for the same purpose.

1. MATHEMATICAL MODEL

Consider TM mode of the dielectric-filled coaxial gyrotron cavity shown in Fig. 1. The cavity has the smooth outer wall of the radius $R_0(z)$ and the corrugated insert with the radius $R_i(z)$ and the groove depth *d*. It is well known that in the cavity region filled with dielectric of permittivity ε the electromagnetic field of TM mode is expressed in terms of a single membrane function Φ as

$$E_r = ik_z \frac{\partial \Phi}{\partial r},\ \ E_\varphi = \frac{ik_z}{r}\frac{\partial \Phi}{\partial \varphi},\ \ E_z = k_\perp^2 \Phi,$$
$$B_r = -\frac{i\varepsilon k}{r}\frac{\partial \Phi}{\partial \varphi},\ \ B_\varphi = i\varepsilon k \frac{\partial \Phi}{\partial r},\ \ B_z = 0, \tag{1}$$

where k_z and $k_\perp = \sqrt{\varepsilon k^2 - k_z^2}$ are the longitudinal and the transverse wave numbers, respectively, $k = \omega/c$, ω is the wave frequency. Hereinafter the phase factor $e^{i(k_z z - \omega t)}$ is omitted.

The membrane function Φ satisfies the Helmholtz equation:

$$\left(\Delta_\perp + k_\perp^2\right)\Phi = 0, \tag{2}$$

with the Dirichlet boundary condition:

$$\Phi\big|_C = 0, \tag{3}$$

where *C* is the contour of transverse cross section of the gyrotron cavity.

Inside the cross-section area it is possible to introduce two adjacent regions, one for the waveguide channel ($R_i < r < R_0$) and another for the grooves ($R_i - d < r < R_i$). For generality, the waveguide channel and the grooves are assumed to be filled with different dielectrics of permittivity ε_1 and ε_2, respectively. In view of the Dirichlet boundary conditions (3), the membrane function in the waveguide channel (Φ_1) and inside the grooves (Φ_2) can be expanded into series of space Bloch and Fourier harmonics, respectively. Thus we have

$$\Phi_1 = \sum_{n=-\infty}^{\infty} A_n f_n(r) e^{ik_n\varphi},\ \ R_i < r < R_0$$
$$\Phi_2 = \sum_{l=1}^{\infty} X_l g_l(r) \sin\left(\xi_l\left(\varphi + \frac{\varphi_L}{2}\right)\right),\ \ R_i - d < r < R_i \tag{4}$$

where

$$f_n(r) = \frac{Z_{k_n 1}(k_{\perp 1} r)}{Z_{k_n 1}(k_{\perp 1} R_i)},\ \ k_n = m + nN,$$
$$g_l(r) = \frac{Z_{\xi_l 2}(k_{\perp 2} r)}{Z_{\xi_l 2}(k_{\perp 2} R_i)},\ \ \xi_l = \frac{\pi l}{\varphi_L},$$
$$Z_\nu^{1(2)}(x) = J_\nu(x) + \alpha_\nu^{1(2)} N_\nu(x),$$
$$\alpha_\nu^1 = -\frac{J_\nu(k_{\perp 1} R_0)}{N_\nu(k_{\perp 1} R_0)},$$
$$\alpha_\nu^2 = -\frac{J_\nu(k_{\perp 2}(R_i - d))}{N_\nu(k_{\perp 2}(R_i - d))},$$
$$k_{\perp 1,2} = \sqrt{\varepsilon_{1,2} k^2 - k_z^2}.$$

To account for the field singularity near the corrugation edges, we introduce new function $F(\varphi)$

$$E_{z1}\big|_{r=R_i} = E_{z2}\big|_{r=R_i} = F(\varphi), \tag{5}$$

or $\ \ k_{\perp 1}^2 \Phi_1\big|_{r=R_i} = k_{\perp 2}^2 \Phi_2\big|_{r=R_i} = F(\varphi).$

The function $F(\varphi)$ is subject to the Dirichlet boundary conditions (3) and the Meixner's edge condition [9]. The last-mentioned condition implies the finite magnitude of the electromagnetic field energy stored in any finite volume near the edge. For TM modes this results in the following field singularity:

$$E_\varphi \sim \rho^{\tau-1}, \quad E_z \sim \rho^\tau \tag{6}$$

where ρ is the distance to the edge. In view of (6), $F(\varphi)$ can be re-expanded as

$$F(\varphi) = \left(1 - \left(\frac{2\varphi}{\varphi_L}\right)^2\right)^\tau \sum_{k=0}^{N_1} a_k P_k^\sigma\left(\frac{2\varphi}{\varphi_L}\right), \tag{7}$$

where P_k^σ are the Gegenbauer polynomials, which are orthogonal on the interval $\left(-\varphi_L/2, \varphi_L/2\right)$ for $\sigma = \tau + 1/2$, $(N_1 + 1)$ is the number of Gegenbauer polynomials under consideration. It can be shown [16] that τ = 2/3 for the wedge-shaped corrugations under consideration.

From (3), (5), (6), and the orthogonality conditions for the Bloch (Fourier) harmonics follows the relationship between A_n (X_l) and a_k. Using these relationships, the continuity condition for B_φ

$$\varepsilon_1 \frac{\partial \Phi_1}{\partial r}\bigg|_{r=R_i} = \varepsilon_2 \frac{\partial \Phi_2}{\partial r}\bigg|_{r=R_i}, \tag{8}$$

and the orthogonal properties of GP, we obtain the system of equations for the unknown a_k:

$$\sum_{k=0}^{N_1}\sum_{j=0}^{N_1} a_k d_{jk} = 0, \tag{9}$$

$$d_{jk} = \frac{\varphi_L}{2\varphi_S}\frac{\varepsilon_1 k^2}{k_{\perp 1}^2}\sum_{n=-\infty}^{\infty} f_n t_{nk} t_{nj}^* - \frac{\varepsilon_2 k^2}{k_{\perp 2}^2}\sum_{l=1}^{\infty} g_l s_{lk} s_{lj},$$

where $f_n = f_n'(R_i)$, $g_l = g_l'(R_i)$,

$$t_{nk} = \int_{-1}^{1}\left(1 - t^2\right)^{2/3} e^{-il_n t} P_k^{7/6}(t)\,dt,$$

$$s_{lk} = \int_{-1}^{1}\left(1 - t^2\right)^{2/3} \sin\left(q_l(t+1)\right) P_k^{7/6}(t)\,dt.$$

The nontrivial solution of the system of equations (9) exists, if the determinant of the matrix d_{jk} equals zero:

$$\det\left\|d_{jk}\right\| = 0, \tag{10}$$

Truncating the infinite sums, we obtain:

$$d_{jk} = \frac{\varphi_L}{2\varphi_S} \frac{\varepsilon_1 k^2}{k_{\perp 1}^2} \sum_{n=-N_2}^{N_2} f_n t_{nk} t_{nj}^* - \frac{\varepsilon_2 k^2}{k_{\perp 2}^2} \sum_{l=1}^{N_3} g_l s_{lk} s_{lj}, \quad (11)$$

where the number of Bloch and Fourier harmonics under consideration equals ($2N_2$+1) and N_3, respectively.

The dispersion equation (10) gives the eigenfrequencies $\omega(k_z)$ of TM modes for dielectric-filled coaxial waveguide with corrugated insert.

2. NUMERICAL RESULTS

The calculations are performed for the $TM_{34,19}$ mode of the coaxial cavity of the 2.2-MW 170-GHz gyrotron [3]. For this gyrotron, ultra-high vacuum is maintained under actual experimental conditions. Thus we can assume $\varepsilon_{1,2} = \varepsilon = 1$ and $k_{\perp 1,2} = k_\perp$. Fig. 2 shows the variation of the eigenvalue $\chi = k_\perp R_0$ of the $TM_{34,19}$ mode along the cavity axis. For comparison purpose, the alternative results of SHM are also presented. As can be seen from Fig. 2,a, the computations of the GP method (N_1, N_2, N_3) and SHM (N_2, N_3) are in close agreement. This agreement is improved with increase in the number of initial (Bloch and Fourier) and new (Gegenbauer polynomials) basic functions (Fig. 2,b).

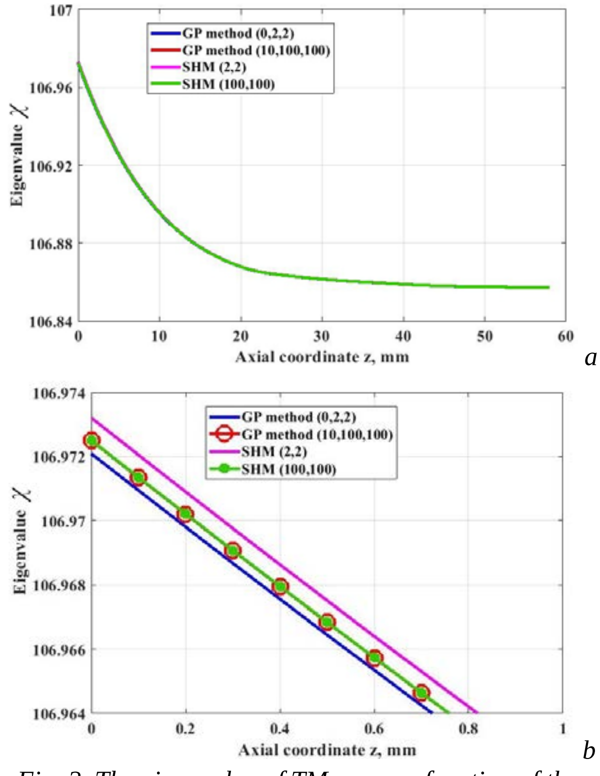


Fig. 2. The eigenvalue of $TM_{34,19}$ as a function of the axial coordinate z: a – the full length of the cavity; b – the region near the cavity entrance

Despite this, there is a notable distinction between the GP method and SHM. It is in the resulting size of the determinant involved in the dispersion equation. For SHM (see (11)) this size equals the total number of the Bloch and the Fourier harmonics $(2N_2+1)+N_3$, while it equals (N_1+1) for the GP method. Our computations show that N_1 can be selected ten times smaller than N_2 and N_3 to guarantee an excellent agreement of the GP method and SHM. For this reason, among these two methods, the GP method more rarely suffers from the problem of large-size matrices and features much smaller computation time.

The convergence of the GP method can be seen from Fig. 3, which shows the dependences of the $TM_{34,19}$ mode eigenvalue (Fig. 3,a) and the relative error of calculations (Fig. 3,b) on number of Gegenbauer polynomials. The relative error is evaluated as:

$$\delta(N_1) = \left| \frac{\chi(N_1+1) - \chi(N_1)}{\chi(N_1)} \right|, \quad (12)$$

where N_1 is related to N_2 and N_3 by the equation $N_2 = N_3 = 10(N_1+1)$.

It is easy to see that the relative error does not exceed $7 \cdot 10^{-6}$, even though the dispersion equation (10) includes a single-element matrix ($N_1+1=1$). Thus the GP method provides very good convergence for the eigenvalues of cavity modes.

Despite this, some numerical errors can be presented in the cavity eigenfields. This can be seen from Fig. 4, which shows the distribution of the membrane functions Φ_1 and Φ_2 along the groove aperture. Ideally, these functions must be identical. However, in actual practice there is a certain mismatch between them. This mismatch is most evident near the corrugation edges $\pm \varphi_L/2$ and can be reduced with increase in N_1, N_2, and N_3 (Fig. 4,b,c).

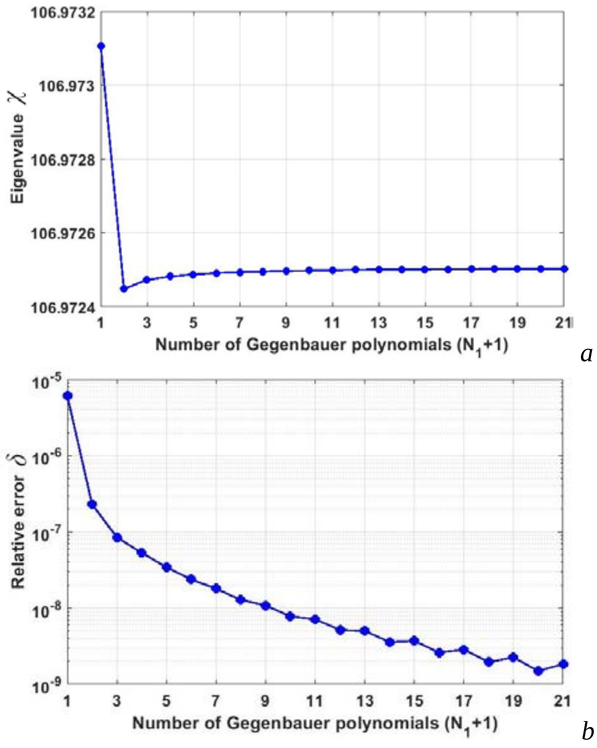
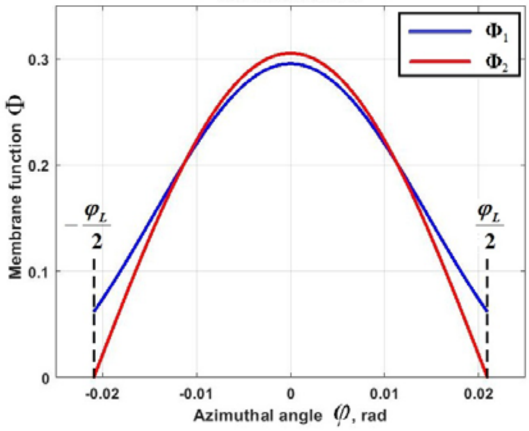
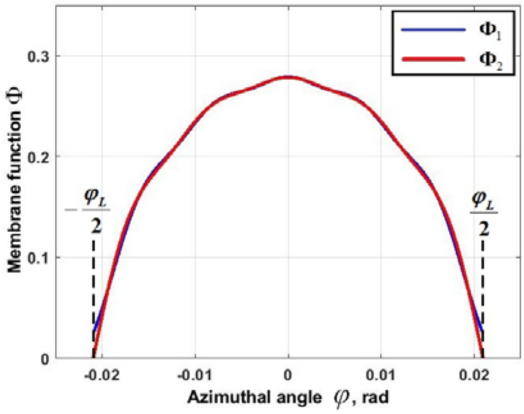


Fig. 3. The eigenvalue (a) and relative error of calculations (b) as a function of the GP number

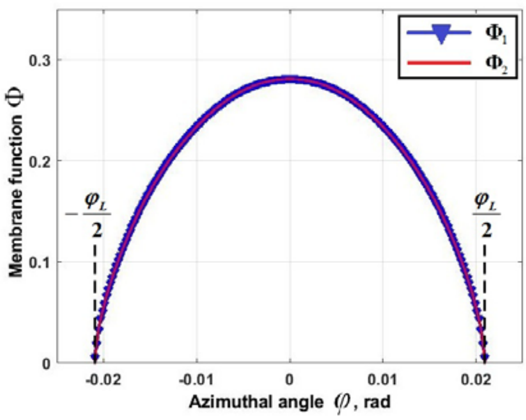
The form of the membrane function of the $TM_{34,19}$ mode shown in Fig. 4,c is similar to that obtained by the rigorous SIE technique in [6] (see Fig. 4 in [6]). It is consistent with the required boundary condition (3) and the Meixner's edge condition [9].



a



b



c

Fig. 4. The dependence of membrane function on azimuthal angle at the aperture of the groove: a – GP method (0, 2, 2); b – GP method (3, 10, 10); c – GP method (10, 100, 100)

The absolute value of the longitudinal electric field $|E_z|$ of the $TM_{34,19}$ mode is shown in Fig. 5 for the input transverse cross-section of the gyrotron cavity. From this figure, the number 19 of the radial field variations can be clearly seen. Such field distribution agrees closely with the calculations [6] based on SIE technique.

It can be seen from Fig. 5 that the corrugations have a slight effect on the mode field. To demonstrate this effect we consider the cavity region in the vicinity of isolated groove. Fig. 6 shows the distribution of $|E_z|$ over such region.

Fig. 6,a corresponds to the case when the resultant matrix of (10) consists of a single element ($N_1+1=1$). In this case the field discontinuity is evident at the boundary between the waveguide channel and the groove region. The field mismatch is of the largest value near the groove edge. It can be reduced with increase in value of the numbers N_1, N_2, and N_3 (Fig. 6,b). Thus from Fig. 6 the good convergence of the cavity field can be clearly seen.

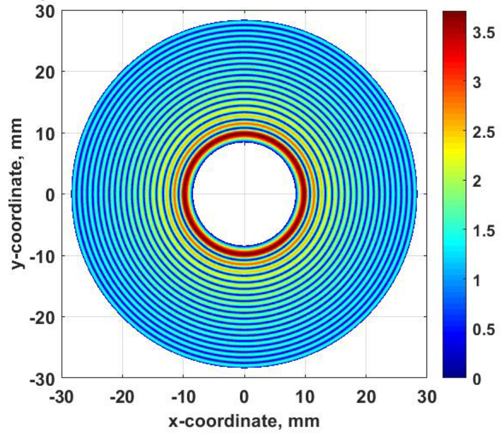
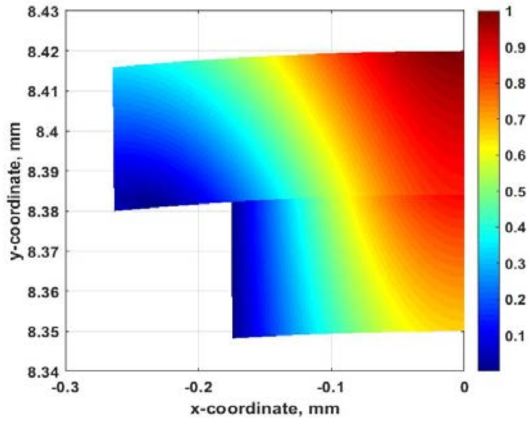
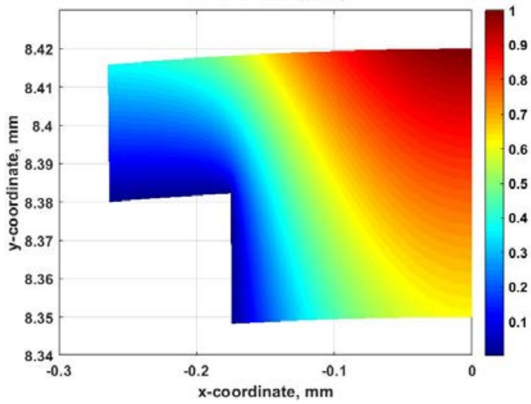


Fig. 5. The absolute value of the longitudinal electric field in the cavity entrance for the $TM_{34,19}$ mode



a



b

Fig. 6. The electromagnetic field distribution in the cavity entrance near the corrugation edge: a – GP method (0, 2, 2); b – GP method (10, 100, 100)

CONCLUSIONS

The improved method has been applied for calculating the eigenvalues and the eigenfields of TM modes of coaxial gyrotron cavity with corrugated inner conductor. It is well-known that such calculations require matching the fields of the waveguide channel to those of the grooves. The former and the later fields are expressed as series of the Bloch and the Fourier harmonics, respectively. The improvement consists in the re-expansion of the matched fields in terms of the Gegenbauer polynomials, which, unlike the Bloch and the Fourier harmonics, satisfy identically the Meixner's condition on the corrugation edges.

Owing to this, compared to the Bloch and the Fourier harmonics, the number of the Gegenbauer polynomials can be selected much smaller to guarantee the same computational accuracy. As a consequence, the size of the resultant determinant involved in the dispersion equation can be reduced. It has been found that such reduction ranges up to 30 times for the $TM_{34,19}$ mode of the 2.2-MW 170-GHz coaxial cavity gyrotron. This in turn alleviates the problem of large-size determinants and diminishes heavily the computation time required for solving the eigenvalue problem. It has been shown that for the $TM_{34,19}$ mode the eigenvalue and the eigenfields converge rapidly with a number of Gegenbauer polynomials in use.

REFERENCES

1. G.S. Nusinovich, M.K.A. Thumm, M.I. Petelin. The Gyrotron at 50: Historical Overview // *J Infrared Milli Terahz Waves*. 2014, v. 35, № 4, p. 325-381.
2. R. Ikeda, K. Kajiwara, Y. Oda, K. Takahashi, K. Sakamoto. High-power and long pulse operation of $TE_{31,11}$ mode gyrotron // *Fusion Eng. and Design.* 2015, v. 96-97, p. 482-487.
3. Z.C. Ioannidis, K.A. Avramides, G.P. Latsas, I.G. Tigelis. Azimuthal mode coupling in coaxial waveguides and cavities with longitudinally corrugated insert // *IEEE Trans. Plasma Sci.* 2011, v. 39, № 5, p. 1213-1221.
4. C.T. Iatrou, S. Kern, A.B. Pavelyev. Coaxial cavities with corrugated inner conductor for gyrotrons // *IEEE Trans. Microw. Theory Tech.* 1996, v. 44, № 1, p. 56-64.
5. Y.V. Gandel, G.I. Zaginaylov, S.A. Steshenko. Rigorous electrodynamic analysis of resonator systems of coaxial gyrotrons // *Tech. Phys.* 2004, v. 49, № 7, p. 887-894.
6. O.S. Kononenko, Y.V. Gandel. Singular and hypersingular integral equations techniques for gyrotron coaxial resonators with a corrugated insert // *Int J Infrared Milli Waves.* 2007, v. 28, № 4, p. 267-274.
7. Z.C. Ioannidis, G.P. Latsas, I.G. Tigelis, O. Dumbrajs. TM modes in coaxial cavities with inner surface corrugations // *IEEE Trans. Plasma Sci.* 2008, v. 36, № 5, p. 2613-2617.
8. G.I. Zaginaylov, S.S. Iaremenko. Efficient method for analysis of gyrodevices with slotted cavities // *IEEE Trans. Plasma Sci.* 2013, v. 41, № 10, p. 3005-3011.
9. G.F. Zargano, V.P. Lyapin, V.S. Michalevsky, Y.M. Sinelnikov, G.P. Sinyavsky, I.M. Chekrigina. *Waveguides of complex cross-section*. Moscow.: "Radio and communication", 1986, 124 p. (in Russian).
10. V.P. Lyapin, V.S. Mikhalevsky, G.P. Sinyavsky. Taking into account the edge condition in the problem of diffraction waves on step discontinuity in plate waveguide // *IEEE Trans. Microw. Theory Tech.* 1976, v. 30, № 7, p. 1107-1109.
11. F.F. Dubrovka, S.I. Piltyay. Eigenmodes of coaxial quad-ridged waveguides. Theory // *Radioelectronics and Communications Systems.* 2014, v. 57, № 1, p. 1-30.
12. D. Homentcovschi, R.N. Miles. A re-expansion method for determining the acoustical impedance and the scattering matrix for the waveguide discontinuity problem // *J Acoust Soc Am.* 2010, v 128, № 2, p. 628-638.
13. D. Homentcovschi, R.N. Miles. Re-expansion method for circular waveguide discontinuities: application to concentric expansion chambers // *J Acoust Soc Am.* 2012, v. 131, № 2, p. 1158-1171.
14. A.V. Maksimenko, G.I. Zaginaylov, V.I. Shcherbinin. On the theory of longitudinally inhomogeneous waveguides with impedance walls // *Physics of Particles and Nuclei Letters*. 2015, v. 12, № 2, p. 362-370.
15. V.I. Shcherbinin, V.I. Tkachenko. Cylindrical cavity with distributed longitudinal corrugations for second harmonic gyrotron // *J. Infrared Millim. Terahertz Waves.* 2017, v. 38, № 7, p. 838-852.
16. R.E. Collin. *Field Theory of Guided Waves.* New York: "IEEE Press", 1991, 852 p.

Article received 16.05.2018

ЭФФЕКТИВНЫЙ МЕТОД ЭЛЕКТРОМАГНИТНОГО АНАЛИЗА ДЛЯ ТМ МОД РЕЗОНАТОРА КОАКСИАЛЬНОГО ГИРОТРОНА С ГОФРИРОВАННОЙ ВСТАВКОЙ

Т.И. Ткачева, В.И. Щербинин, В.И. Ткаченко

Исследованы электромагнитные характеристики ТМ мод в резонаторе коаксиального гиротрона с гофрированным внутренним проводником. Метод связанных мод, основанный на переразложении поля в резонаторе по полиномам Гегенбауэра, применен с целью снижения времени расчета этих характеристик с требуемой точностью. Получено дисперсионное уравнение для собственных значений ТМ мод. Для этих мод исследовано распределение поля в поперечном сечении резонатора. Показана сходимость вычислений для собственных значений и собственных полей резонатора.

ЕФЕКТИВНИЙ МЕТОД ЕЛЕКТРОМАГНІТНОГО АНАЛІЗУ ДЛЯ ТМ МОД РЕЗОНАТОРА КОАКСІАЛЬНОГО ГІРОТРОНА З ГОФРОВАНОЮ ВСТАВКОЮ

Т.І. Ткачова, В.І. Щербінін, В.І. Ткаченко

Досліджено електромагнітні характеристики ТМ мод в резонаторі коаксіального гіротрона з гофрованим внутрішнім провідником. Метод зв'язаних мод, заснований на перерозкладанні поля в резонаторі за поліномами Гегенбауера, застосовано з метою зменшення розрахункового часу цих характеристик з необхідною точністю. Отримано дисперсійне рівняння для власних значень ТМ мод. Для цих мод досліджено розподіл поля в поперечному перерізі резонатора. Показана збіжність розрахунків для власних значень та власних полів резонатора.