WAKEFIELD EXCITATION BY A LASER PULSE IN A DIELECTRIC MEDIUM

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The process of Cherenkov electromagnetic field excitation by a laser pulse in the dielectric waveguide is investigated. Nonlinear electric polarization in an isotropic dielectric medium and, accordingly, polarization charges and currents induced by ponderomotive force of the laser pulse are determined. Spatial structure of the excited wakefield in a dielectric waveguide is obtained and investigated. It is shown, that the wakefield consists of polarization potential electric field, caused by the nonlinear polarization of medium, and a set of eigen electromagnetic waves of the dielectric waveguide.

PACS: 41.75.Lx, 41.85.Ja, 41.69.Bq

INTRODUCTION

A charged particle moving in a dielectric medium with a velocity above light speed radiates electromagnetic waves called Cherenkov radiation [1, 2]. The electric field of a moving charge polarizes the atoms (molecules) of the dielectric medium, which in turn coherently re-emit electromagnetic waves.

A similar effect occurs when a short high-power laser pulse propagates in a dielectric [3, 4]. In the linear approximation in the field, the effect of polarization of the medium at the field frequency only leads to a change in the phase and group velocities of the laser pulse. In the nonlinear approximation, the pulsed ponderomotive force that propagates in a medium with the velocity equal to the group velocity of the laser pulse also acts quadratically with respect to the field on the coupled electrons of the dielectric medium. This force, in turn, leads to the polarization of the dielectric medium. When the Cherenkov synchronism condition between the ponderomotive force of the laser pulse and the slow electromagnetic waves of the medium is satisfied, it causes the excitation of electromagnetic Cherenkov radiation.

Note that in [4] proposed interpretation of the Cerenkov radiation of the laser pulse in a dielectric medium as an effect of three-wave decay process. It's about decay of an electromagnetic wave belonging to a laser wave packet and having frequency $\omega(\vec{k}_0)$, ($\vec{k}_0$ is wave vector), to a satellite with a lower frequency $\omega(\vec{k}_0-\vec{k})$ and an electromagnetic Cerenkov wave with a frequency $\omega_{ch}(\vec{k})$. For such process, the condition of frequency synchronism of the indicated waves has the form

$$\omega(\vec{k}_0)=\omega(\vec{k}_0-\vec{k})+\omega_{ch}(\vec{k}).$$

Assuming that $|\vec{k}|<<|\vec{k}_0|$, this condition implies the condition for Cherenkov radiation of a laser pulse in a dielectric medium

$$\omega_{ch}(\vec{k})=\vec{k}\frac{d\omega(\vec{k}_0)}{d\vec{k}_0}=\vec{k}\vec{v}_g(\vec{k}_0).$$

Thus, the effect of Cherenkov radiation of a laser pulse is quite similar to the Cherenkov radiation of a charged particle with the only difference that the role of the electric field of a charged particle is played by the ponderomotive force of a laser pulse.

The wake Cherenkov radiation of a powerful ultrashort laser pulse in a dielectric medium can be used to accelerate charged particles like a analogous method of laser wakefield acceleration in a plasma [5].

In this paper we formulate a system of nonlinear equations of macroscopic electrodynamics that describes the process of excitation of Cerenkov radiation by a laser pulse in a dielectric medium. On the basis of these equations, the effect of Cherenkov radiation of a laser pulse in a dielectric waveguide (optical fiber) will be investigated.

1. FORMULATION OF THE PROBLEM. BASIC EQUATIONS

In a homogeneous dielectric medium, a laser pulse (wave packet) propagates with components of the electromagnetic field

$$\vec{E}_L(\vec{r},t)=\frac{1}{2}\vec{E}_0(\vec{r},t)e^{i\psi}+c.c.,$$

$$\vec{H}_L(\vec{r},t)=\frac{1}{2ik_0}\mathrm{rot}\left[\vec{E}_0(\vec{r},t)e^{i\psi}\right]+c.c., \quad (1)$$

$\psi=\vec{k}\vec{r}-\omega_L t$, $\vec{k}$ is wave vector, $k_0=\omega_L/c$, ω_L is the carrier frequency of the laser pulse, $\vec{E}_0(\vec{r},t)$ is slowly changing in the space and time envelope of the laser pulse.

Under the action of the ponderomotive force (the HF-pressure force) quadratic in the laser field (1), a polarization, slow in the scale of the carrier frequency, arises in the dielectric, which in turn will be the source of the electromagnetic field excited by the laser pulse (i.e. Cherenkov radiation).

The system of Maxwell equations describing the electromagnetic field excited by the polarization induced by the laser pulse has the form

$$\mathrm{rot}\vec{E}=-\frac{1}{c}\frac{\partial\vec{H}}{\partial t},\ \mathrm{rot}\vec{H}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\frac{\partial\vec{P}}{\partial t},$$

$$\mathrm{div}\vec{E}=-4\pi\,\mathrm{div}\vec{P},\ \mathrm{div}\vec{H}=0, \quad (2)$$

$\vec{P}$ is vector of electric polarization.

2. DETERMINATION OF NONLINEAR POLARIZATION

The next step of the theory is the determination of the polarization $\vec{P}$ caused by the action on certain atoms of the condensed dielectric medium both an electric field which is in the Maxwell equations (2) and the ponderomotive force from side of the laser pulse (1). For this, a simple but adequate model of an elementary dipole located in the crystal lattice point is necessary. Note that beginning from the microwave range of radiation wavelengths and moreover in the optical range, the orientational (dipole) and induced ionic polarization mechanisms do not play an appreciable role due to the high inertia of the ions. In these wavelength ranges, the induced electron polarization of atoms is dominant [6]. Electronic polarization is due to the displacement of the shell from the bound electrons of the atom relatively to the nucleus under the action of the electric field.

The induced electronic polarization can be described in the framework of the following model [6]. The atom is represented as a point nucleus in a charge Z|e|, surrounded by a smeared electron cloud with the charge –Z|e|. The electron cloud will be considered as a spherically symmetric homogeneous charged region of radius R_0. When the electron cloud is shifted as a whole with respect to the nucleus, the dipole moment of the atom $\vec{p} = -eZ\vec{r}$ arises, where $\vec{r}$ is the radius vector of the center of the electron cloud. Accordingly, the following dipole returning force will act on the electron cloud [7]

$$\vec{F}_e = -\frac{(Ze)^2}{R_0^3}\vec{r}\,, \qquad (3)$$

which leads to harmonic dipole oscillations of an atom with eigen frequency

$$\omega_0 = \sqrt{\frac{Ze^2}{mR_0^3}}\,. \qquad (4)$$

In a condensed medium, each atom is in a local (acting) electric field $\vec{E}_{loc}$, which can differ greatly from the macroscopic field $\vec{E}$ contained in Maxwell's equations (1). The local electric field includes both the external field and the total electric field of the induced dipoles surrounding the taken atom. In a crystalline medium with a cubic crystal lattice, the local electric field is described by the Lorentz formula [6 - 8]

$$\vec{E}_{loc} = \vec{E} + \frac{4\pi}{3}\vec{P}\,. \qquad (5)$$

We note that the Lorentz formula is exact for a condensed dielectric medium with a cubic lattice. However, it qualitatively correctly describes the local electric field for more complex crystal structures and even for liquid structures [9]. Under the influence of external high-frequency fields, dipole oscillators with eigen frequency (4) will perform forced oscillations. The excitation equation for the dipole oscillations of an atom can be written as follows. Under the action of a ponderomotive force quadratic in the laser field (1) (HF pressure force), a polarization, slow in the carrier frequency, arises in a dielectric medium, which in turn will be the source of the electromagnetic radiation of the laser pulse field, in particular, the Cherenkov radiation.

So the excitation equation for the dipole oscillations of an atom can be written as follows

$$\frac{d^2\vec{r}}{dt^2} + \omega_0^2\vec{r} = -\frac{e}{m}\left(\vec{E}_L^{loc}(\vec{r},t) + \frac{1}{c}\left[\vec{v}\vec{H}_L\right]\right) - \frac{e}{m}\left(\vec{E}_{loc} + \frac{1}{c}\left[\vec{v}\vec{H}\right]\right). \qquad (6)$$

Here

$$\vec{E}_L^{loc}(\vec{r},t) = \frac{1}{2}\vec{E}_0(\vec{r},t)e^{i\psi} + \text{c.c.} + \frac{4\pi}{3}\vec{P}_L$$

is the local electric field from the side of the laser pulse, $\vec{P}_L$ is HF-polarization of the dielectric on the carrier frequency, induced by the laser pulse, $\vec{E}_{loc}(\vec{r},t)$ is a local electric field (5), which includes a slow field which excited by the ponderomotive force and contained in Maxwell's equations (2).

An approximate solution of equation (6) will be found in the form of a sum of the displacement $\vec{r}_L(t)$ rapidly oscillating at the carrier frequency of the laser pulse and the slow displacement $\vec{r}_c(t)$ of the center of the electron cloud relatively to the atomic nucleus

$$\vec{r}(t) = \vec{r}_L(t) + \vec{r}_c(t)\,.$$

The rapidly oscillating displacement $\vec{r}_L(t)$ is described by the linear equation of motion of the oscillator

$$\frac{d^2\vec{r}_L}{dt^2} + \omega_0^2\vec{r}_L = -\frac{e}{2m}\vec{E}_0(\vec{r},t)e^{i(\vec{k}\vec{r}_L - \omega t)} + \text{c.c.} - \frac{e}{m}\frac{4\pi}{3}\vec{P}_L\,. \qquad (7)$$

First of all, we determine the HF polarization. Taking into account the definition the polarization of unit volume

$$\vec{P} = -ZeN\vec{r}\,, \qquad (8)$$

where N is the number of atoms per unit volume of the dielectric, from (7) we obtain the equation for the polarization $\vec{P}_L$ in the field of the laser pulse (1)

$$\frac{d^2\vec{P}_L}{dt^2} + \omega_d^2\vec{P}_L - = \frac{Ze^2}{m}N\frac{1}{2}\left[\vec{E}_0(\vec{r}_c,t)e^{i\psi_c} + \text{c.c.}\right], \qquad (9)$$

$\psi_c = \vec{k}\vec{r}_c - \omega t$, $\omega_d^2 = \omega_0^2 - \omega_p^2/3$, $\omega_p^2 = \frac{4\pi Ze^2N}{m}$ – plasma frequency. The solution of this equation is easily found

$$\vec{P}_L = \chi\vec{E}_L\,, \qquad (10)$$

$$\chi = \frac{\alpha_L N}{1 - \frac{4\pi}{3}\alpha_L N} \qquad (11)$$

is electrical susceptibility,

$$\alpha_L = \frac{Ze^2}{m}\frac{1}{\omega_o^2 - \omega_L^2}$$

is the polarizability of an individual atom. In the quasistatic approximation $\omega_0^2 >> \omega_L^2$, the expression for the polarizability is simplified and does not depend on the frequency

$$\alpha_L = \frac{Ze^2}{m\omega_0^2}\,. \qquad (12)$$

Taking into account that $\chi_L = (\varepsilon_L - 1)/2\pi$, where

$$\varepsilon_L \equiv \varepsilon(\omega_L),\ \ \varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_d^2 - \omega^2}$$

is the permittivity of the medium, the relation (11) implies the well-known Clausius-Mosotti formula [5, 6]

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi}{3}\alpha N\,, \qquad (13)$$

which establishes a relationship between the dielectric permittivity and the polarizability of an individual atom. For a dielectric medium with a cubic lattice, taking into account the expression obtained for the polarization vector (10), we find the following expression for the local electric field of the laser pulse.

$$\vec{E}_L^{loc} = \frac{\varepsilon_L + 2}{3}\frac{1}{2}\left(\vec{E}_0 e^{i\psi_c} + c.c.\right). \quad (14)$$

Accordingly, from the equation of motion (11) we obtain expressions for high-frequency displacement $\vec{r}_L(t)$ and velocity $\vec{v}_L(t)$

$$\vec{r}_L(t) = -a\frac{1}{2}\left[\vec{E}_0(\vec{r}_c, t)e^{i\psi_c} + c.c.\right], \quad (15)$$

$$\vec{v}_L(t) = a\omega\frac{1}{2}\left[i\vec{E}_0(\vec{r}_c, t)e^{i\psi_c} + c.c.\right],$$

$$a = \frac{e}{m}\frac{\varepsilon_L + 2}{3}\frac{1}{\omega_0^2 - \omega_L^2}.$$

Let us now formulate the equation for the slow displacement $\vec{r}_c(t)$ of a dipole oscillator. Preserving on the right-hand side of Eq. (6) only quadratic terms with respect to the laser field, we obtain the equation

$$\frac{d^2\vec{r}_c}{dt^2} + \omega_0^2\vec{r}_c = -\frac{e}{m}\left[\vec{E}(\vec{r}, t) + \frac{4\pi}{3}\vec{P}\right] -$$

$$-\frac{e}{m}\left\{\frac{\varepsilon_L + 2}{3}\left\langle(\vec{r}_L\nabla)\vec{E}_L\right\rangle + \frac{1}{c}\left\langle\left[\vec{v}_L\vec{H}_L\right]\right\rangle\right\}. \quad (16)$$

Angular brackets mean averaging over HF oscillations. Performing the averaging procedure with taking into account the expressions for the HF quantities (14), (15) entering in (16), we obtain the following equation

$$\frac{d^2\vec{r}_c}{dt^2} + \omega_0^2\vec{r}_c = -\frac{e}{m}\left[\vec{E} + \frac{4\pi}{3}\vec{P}\right] + \frac{\alpha}{4mZ}\frac{\varepsilon + 2}{3}\vec{F}, \quad (17)$$

where

$$\vec{F} = \nabla\left|\vec{E}_0\right|^2 + \frac{\varepsilon_L - 1}{3}\left[\left(\vec{E}_0^*\nabla\right)\vec{E}_0 + \left(\vec{E}_0\nabla\right)\vec{E}_0^*\right]. \quad (18)$$

The first term in (18) describes the gradient force of HF pressure. The second term occurs only in the case of a crystalline medium and is due to the difference between the local electric field $\vec{E}_L^{loc}$ in the crystal and the electric field of the laser pulse. In dielectric media, where the acting field coincides with the external field, for example, in a gas dielectric, this term is absent. Taking into account the expression for polarization (8), we obtain a material equation for the nonlinear polarization

$$\frac{d^2\vec{P}}{dt^2} + \omega_d^2\vec{P} = \frac{Ze^2N}{m}\vec{E} - \frac{eN\alpha}{4m}\frac{\varepsilon + 2}{3}\vec{F}. \quad (19)$$

The Maxwell equations (2) together with the equation for the polarization (19) describe the electromagnetic Cerenkov radiation of a laser pulse in a condensed dielectric medium.

We solve this system of equations by the Fourier transform

$$\vec{E}(\vec{r}, t) = \int_{-\infty}^{\infty}\vec{E}_\omega(r)e^{-i\omega t}d\omega, \ \vec{P}(\vec{r}, t) = \int_{-\infty}^{\infty}\vec{P}_\omega(r)e^{-i\omega t}d\omega,$$

where $\vec{E}_\omega(r)$, $\vec{P}_\omega(\vec{r})$ are Fourier components of the corresponding quantities. For example

$$\vec{E}_\omega(\vec{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{E}(r, t)e^{i\omega t}dt.$$

From the material equation (19) we find the expression for the Fourier component of the polarization vector

$$\vec{P}_\omega = \frac{\varepsilon_{ch}(\omega) - 1}{4\pi}\vec{E}_\omega - \mu\vec{F}_\omega,$$

where

$$\varepsilon_{ch}(\omega) = \frac{1 + \frac{8\pi}{3}N\alpha_{ch}(\omega)}{1 - \frac{4\pi}{3}N\alpha_{ch}(\omega)}, \quad (20)$$

$$\alpha_{ch}(\omega) = \frac{Ze^2}{m}\frac{1}{\omega_0^2 - \omega^2}, \ \mu = \frac{1}{64\pi^2 ZeN}(\varepsilon_L - 1)(\varepsilon_{ch}(\omega) - 1),$$

$\vec{F}_\omega$ – Fourier component of the quadratic dependence (18) of the ponderomotive force on the electric field strength of the laser pulse. Accordingly, the system of Maxwell equations for the Fourier components of the electromagnetic field can be represented in the form

$$rot\vec{H}_\omega = -ik_0\varepsilon_{ch}(\omega)\vec{E}_\omega + \frac{4\pi}{c}\vec{j}_{\omega pol}, \ rot\vec{E}_\omega = +ik_0\vec{H}_\omega$$

$$div\varepsilon_{ch}(\omega)\vec{E}_\omega = 4\pi\rho_{\omega pol}, \ div\vec{H}_\omega = 0. \quad (21)$$

The Fourier components of the polarization currents and charges induced in the dielectric by the ponderomotive force of the laser pulse are described by expressions

$$\vec{j}_{pol\omega} = i\omega\mu\vec{F}_\omega, \ \rho_{pol\omega} = \mu div\vec{F}_\omega. \quad (22)$$

The obtained working system of equations makes it possible to investigate Cherenkov radiation in a variety of physical situations: a model of an infinite dielectric medium, dielectric waveguides and resonators.

3. CHERENKOV RADIATION OF A LASER PULSE IN A DIELECTRIC WAVEGUIDE

Let us consider a dielectric waveguide made in the form of a homogeneous dielectric cylinder whose lateral surface is covered with an ideally conducting metal film. Along the axis of the waveguide, a circularly polarized laser pulse propagates with the components of the electric field

$$E_{0x} = \sqrt{\frac{I_0}{2}}\psi\left(r, t - z/v_g\right), \ E_{0y} = iE_{0x},$$

$$\psi = \left[R(r)T\left(t - z/v_g\right)\right]^{1/2}. \quad (23)$$

The function $R(r)$ describes the radial intensity profile of the laser pulse, $I = \left|\vec{E}_0\right|^2$, $R(0) = 1$, $R(r = b) = 0$, b is the radius of the waveguide, the function $T(\tau)$ describes the longitudinal profile, $\tau = t - z/v_g$, v_g is the group velocity, $maxT(\tau) = 1$, I_0 is the maximum intensity.

From the system of Maxwell's equations (21) follows the wave equation for the longitudinal Fourier component of the Cherenkov electric field

$$\Delta E_{z\omega} + k_0^2\varepsilon_{ch}(\omega)E_{z\omega} = 4\pi\left(\frac{1}{\varepsilon_{ch}(\omega)}\frac{\partial\rho_{pol\omega}}{\partial z} - i\frac{k_0}{c}j_{pol\omega}\right). (24)$$

The Fourier components of the polarization charges and currents are determined by expressions (22). For a circularly polarized laser pulse (23), these expressions take the forms

$$\rho_{pol\omega} = \mu\left[\Delta I_\omega(r, \tau) + \frac{\varepsilon_L - 1}{6}\Delta_\perp I_\omega(r, \tau)\right], \quad (25)$$

ISSN 1562-6016. ВАНТ. 2018. №4(116)

$$j_{zpol\omega} = i\omega\mu \frac{\partial}{\partial z} I_\omega(r,\tau) \cdot \qquad (26)$$

Let's introduce the function

$$D_{z\omega} = \varepsilon_{ch}(\omega) E_z - 4\pi\mu \frac{\partial I_\omega(r,\tau)}{\partial z}. \qquad (27)$$

For this function, instead of equation (24), taking into account the relations (25), (26), we obtain the equation

$$\Delta D_{z\omega} + k_0^2 \varepsilon_{ch}(\omega) D_{z\omega} = 4\pi\mu \frac{\varepsilon_L - 1}{6} \Delta_\perp \frac{\partial I_\omega(r,\tau)}{\partial z}. \quad (28)$$

The function $D_{z\omega}$ has a simple physical meaning and is a longitudinal Fourier component of the electric induction $D_z = E_z + 4\pi P_z$ with taking into account the polarization (20), caused by the action of the ponderomotive force of the laser pulse.

The longitudinal component of the electrical induction should be found in the form of a series of Bessel functions

$$D_{z\omega} = \sum_{n=0}^{\infty} C_n(z,t) J_0\left(\lambda_n \frac{r}{b}\right), \qquad (29)$$

where λ_n are roots of the Bessel function $J_0(x)$. For the coefficients C_n of the expansion from (28) we obtain the equation

$$\frac{d^2 C_n}{dz^2} + k_z^2(\omega) C_n = -4\pi i k_g \mu I_0 \frac{\varepsilon_L - 1}{6} \frac{\lambda_n^2}{b^2} \frac{\alpha_n}{N_n} T_\omega e^{ik_g z}. \quad (30)$$

Here $k_g = \dfrac{\omega}{v_g}$, $k_z^2(\omega) = k_0^2 \varepsilon_{ch}(\omega) - \dfrac{\lambda_n^2}{b^2}$, $N_n = \dfrac{b^2}{2} J_1^2(\lambda_n)$,

$$\alpha_n = \int_0^b R(r) J_0\left(\lambda_n \frac{r}{b}\right) r dr, \quad T_\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} T(\tau) e^{i\omega\tau} d\tau.$$

In the considered case of an infinite dielectric waveguide in the longitudinal direction, the forced solution of Eq. (30) has the form

$$C_n = -i\Pi\omega\left[\varepsilon_{ch}(\omega) - 1\right] \frac{\rho_n}{\Delta_n(\omega)} T_\omega e^{ik_g z},$$

where

$$\Pi = \frac{I_0(\varepsilon_L - 1)}{16\pi v_g ZeN}, \quad \rho_n = \frac{\lambda_n^2}{b^2} \frac{\alpha_n}{N_n},$$

$$\Delta_n(\omega) = k_0^2 \varepsilon_{ch}(\omega) - \frac{\lambda_n^2}{b^2} - k_g^2.$$

Taking relations (27) and (29) into account, we obtain the following expression for the Fourier component of the longitudinal component of the electric field

$$E_{z\omega}(r) = \Pi T(\omega) G(r,\omega).$$

Here

$$G(r,\omega) = i \frac{\left[\varepsilon_{ch}(\omega) - 1\right]}{\varepsilon_{ch}(\omega)} e^{ik_g z} \left[R(r) - \right.$$

$$\left. - \frac{\varepsilon_L - 1}{6} \sum_{n=1}^{\infty} \rho_n \frac{J_0(\lambda_n r / b)}{\Delta_n(\omega)}\right]. \qquad (31)$$

Accordingly, the longitudinal component of the excited electric field can be represented as a convolution

$$E_z(r,\tau) = \Pi \int_{-\infty}^{\infty} T(\tau_0) G(r, \tau - \tau_0) d\tau_0, \qquad (32)$$

where

$$G(r, \tau - \tau_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(r,\omega) e^{-i\omega(\tau - \tau_0)} d\omega \qquad (33)$$

is Green's function. The Green's function actually describes the structure of the wakefield in a dielectric medium excited by a laser pulse with a δ-shaped longitudinal profile of the intensity.

3.1. CALCULATION OF THE GREEN FUNCTION

The expression for the Green's function (33), taking into account expression (31), is conveniently written as follows

$$G(r,\vartheta) = G_1(r,\vartheta) + G_2(r,\vartheta), \qquad (34)$$

$$G_1(r,\vartheta) = R(r) S_0(\vartheta), \qquad (35)$$

$$G_2(r,\vartheta) = -\frac{\varepsilon_L - 1}{6} \sum_{n=1}^{\infty} \rho_n J_0(\lambda_n \frac{r}{b}) S_n(\vartheta), \qquad (36)$$

where

$$S_0(\vartheta) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \omega d\omega e^{-i\omega\vartheta} \frac{\varepsilon_{ch}(\omega) - 1}{\varepsilon_{ch}(\omega)}, \qquad (37)$$

$$S_n(\vartheta) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \omega d\omega e^{-i\omega\vartheta} \frac{\varepsilon_{ch}(\omega) - 1}{\varepsilon_{ch}(\omega) \Delta_n(\omega)}, \quad \vartheta = \tau - \tau_0. \quad (38)$$

The expression (21) for the dielectric permittivity $\varepsilon_{ch}(\omega)$ can be written in the form

$$\varepsilon_{ch}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_d^2} \equiv \frac{\varepsilon_0 - x^2}{1 - x^2},$$

where $x = \omega / \omega_d$ $\omega_d^2 = \omega_0^2 - \omega_p^2 / 3$, $\varepsilon_0 = 1 + \omega_p^2 / \omega_d^2$ is static permittivity $\varepsilon_0 = \varepsilon_{ch}(\omega \to 0)$.

The collective frequency of dipole oscillations ω_d is lower than the frequency of an individual atomic dipole oscillator. This effect is due to the screening of the field of an individual atomic dipole by the total field of the set of the similar dipoles surrounding it.

The integrand in (37) has simple poles $\omega = \pm\omega_n - i0$ located in the lower half-plane of the complex variable ω, $\omega_g = \sqrt{\omega_p^2 + \omega_d^2}$ – is the frequency of the eigen longitudinal oscillations of the dielectric medium, which nullifies the dielectric permittivity $\varepsilon_{ch}(\omega) = 0$.

Calculating the residues at these poles, we obtain an expression for the integral $S_0(\vartheta)$ and, accordingly, for the first term of the Green's function

$$G_1(r,\vartheta) = -R(r) \omega_p^2 \chi(\vartheta) \cos\omega_g\vartheta, \qquad (39)$$

here $\chi(\tau - \tau_0)$ is unit Heaviside function. As for the Fourier integral (38), its integrand, in addition to the poles listed above, has additional poles $\Delta_n(\omega) = 0$. This equation can be given in a more convenient form for analysis

$$\Delta_n(\omega) = -\frac{1}{v_g^2 \gamma_g^2} \frac{1}{\omega^2 - \omega_d^2} \left(\omega^2 - \omega_{chn}^2\right)\left(\omega^2 + \nu_{stn}^2\right),$$

where

$\gamma_g^2 = (1 - \beta_g^2)^{-1}, \beta_g = v_g / c, \ \omega_{chn} = \omega_d x_{chn}, \ \nu_{stn} = \omega_d x_{stn},$

$$x_{chn}^2 = -\frac{1}{2}(b_0 + y_n^2) + \sqrt{\frac{1}{4}(b_0 + y_n^2)^2 + y_n^2},$$

$$x_{stn}^2 = \frac{1}{2}(b_0 + y_n^2) + \sqrt{\frac{1}{4}(b_0 + y_n^2)^2 + y_n^2},$$

$$b_0 = \gamma_g^2(\beta_g^2 \varepsilon_0 - 1), \quad y_n^2 = \frac{\lambda_n^2 c^2}{\omega_d^2 b^2} \beta_g^2 \gamma_g^2.$$

The poles $\omega = \pm\omega_{chn} - i0$ are also located in the lower half-plane of the complex variable ω near the real axis. They correspond to the eigen electromagnetic waves of the dielectric waveguide, which are in Cerenkov synchronism with the laser pulse. Since these poles are always real, the cutoff radiation of the laser pulse in the dielectric waveguide takes place for all values of the group velocity of the laser pulse and the parameters of the dielectric waveguide (in our case, the values of the static dielectric constant ε_0 and the radius of the waveguide b). The frequency range, in which the discrete frequencies ω_{chn} of the eigen waves there are, depends on the sign of the parameter b_0. If $b_0 > 0$, what mean s $v_g > c/\sqrt{\varepsilon_0}$, then the Cherenkov frequencies are in the range $\omega_d > \omega_{chn} > 0$. If $b_0 < 0$ (this corresponds to $v_g < c/\sqrt{\varepsilon_0}$), then for the Cherenkov frequencies it is rightly $\omega_d > \omega_{chn} > \omega_d\sqrt{|b_0|}$. Note that for $b_0 < 0$, it is always $|b_0| < 1$.

Besides of the real Cherenkov poles, there is also a pair of complex conjugate poles $\omega = \pm i\nu_{stn}$ in the integrand (38) located on the imaginary axis. These poles correspond to a quasi-static electromagnetic field localized in the laser pulse region. Calculating the residues at all poles of the integrand (38), we find the second term of the Green's function

$$G_2(r,\tau-\tau_0) = -\frac{\varepsilon_L - 1}{6}\omega_p^2\Big\{\Phi(r)\chi(\vartheta)\cos\omega_g\vartheta + v_g^2\gamma_g^2\sum_{n=1}^{\infty}\frac{\rho_n J_0(\frac{\lambda_n}{b}r)}{\omega_{chn}^2+\nu_{stn}^2}\left[\frac{1}{\varepsilon_{chn}}\chi(\vartheta)\cos\omega_{chn}\vartheta - \frac{1}{2\varepsilon_{stn}}sign(\vartheta)e^{-\nu_{stn}|\vartheta|}\right]\Big\}, \quad (40)$$

where

$$\Phi(r) = \sum_{n=1}^{\infty}\frac{\rho_n J_n(\lambda_n r/b)}{k_g^2+\lambda_n^2/b^2},\ k_g = \omega_g/v_g, \varepsilon_{chn} \equiv \varepsilon_{ch}(\omega_{chn}),$$

$$\varepsilon_{stn} \equiv \varepsilon_{ch}(i\nu_{stn}).$$

The series $\Phi(r)$ can be exactly summed

$$\Phi(r) = k_g^2\int_0^b \Gamma(r,r_0)R(r_0)r_0 dr_0, \quad (41)$$

$$\Gamma(r,r_0) = \frac{1}{I_0(k_g r)}\begin{cases}\Delta_0(k_g r, k_g b)I_0(k_g r_0), & r > r_0,\\ I_0(k_g r)\Delta_0(k_g r_0, k_g b), & r < r_0.\end{cases} \quad (42)$$

Summing the expressions (39), (40), we obtain the final expression for the Green's function

$$G(r,\vartheta) = -\omega_p^2\left[R(r) + \frac{\varepsilon_L - 1}{6}\Phi(r)\right]\chi(\vartheta)\cos\omega_g\vartheta - \sigma\sum_{n=1}^{\infty}\frac{\rho_n J_0(\frac{\lambda_n}{b}r)}{\omega_{chn}^2+\nu_{stn}^2}\left[\frac{1}{\varepsilon_{chn}}\chi(\vartheta)\cos\omega_{chn}\vartheta - \frac{1}{2\varepsilon_{stn}}sign(\vartheta)e^{-\nu_{stn}|\vartheta|}\right],\ \sigma = \frac{\varepsilon_L - 1}{6}\omega_p^2 v_g^2\gamma_g^2. \quad (43)$$

The Green's function contains potential monochromatic wakefield, caused by the excitation of polarization oscillations of the dielectric medium. The electromagnetic part of the Green's function contains a wakefield in the form of a superposition of eigenmodes of a dielectric waveguide, as well as a set of bipolar electromagnetic pulses.

3.2. PICTURE OF THE WAKEFIELD EXCITED BY A LASER PULSE

After substituting the Green's function (43) into the expression for the total field of the laser pulse in the dielectric waveguide (32), we obtain the following expression for the longitudinal component of the wakefield of a laser pulse with an arbitrary profile

$$E_z(r,\tau) = E_z^{(pol)}(r,\tau) + E_z^{(em)}(r,\tau), \quad (44)$$

where

$$E_z^{(pol)}(r,\tau) = -\omega_p^2\Pi F(r)Z(\omega_g\tau), \quad (45)$$

$$F(r) = R(r) + \frac{\varepsilon_L - 1}{6}\Phi(r),$$

$$E_z^{(em)}(r,\tau) = -\sigma\Pi\sum_{n=1}^{\infty}\frac{\rho_n J_0(\lambda_n r/b)}{\omega_{chn}^2+\nu_{stn}^2}\left[\frac{1}{\varepsilon_{chn}}Z(\omega_{chn}\tau) - \frac{1}{\varepsilon_{stn}}X(\nu_{stn}\tau)\right]. \quad (46)$$

Function

$$Z(\omega\tau) = \int_{-\infty}^{\tau} T(\tau_0)\cos\omega(\tau-\tau_0)d\tau_0 \quad (47)$$

describes the longitudinal distribution of the wakefield polarization field $\omega = \omega_g$, as well as the wakefield electromagnetic field of the corresponding radial harmonic $\omega = \omega_{chn}$ of the dielectric waveguide.

Function

$$X(\nu_{stn}\tau) = \int_{-\infty}^{\infty} T(\tau_0)sign(\tau-\tau_0)e^{-\nu_{stn}|\tau-\tau_0|}d\tau_0 \quad (48)$$

describes the longitudinal component of a set of bipolar quasi-static electromagnetic pulses.

For a symmetric laser pulse $T(\tau_0) = T(-\tau_0), T(\tau_0 \to \pm\infty) \to 0$, the expression for the function (46) can be written as follows

$$Z(\omega\tau) = 2\widehat{T}(\omega)\cos\omega\tau + Z_{st}(\omega\tau), \quad (49)$$

where

$$\widehat{T}(\omega) = \int_0^{\infty} T(\tau_0)\cos\omega\tau_0 d\tau_0.$$

Is an amplitude of the Fourier component at the frequency of a function describing the longitudinal profile of the intensity of the laser pulse,

$$Z_{st}(\omega\tau) = -\int_{\tau}^{\infty} T(\tau_0)\cos\omega(\tau-\tau_0)d\tau_0. \quad (50)$$

Behind the laser pulse $\tau > 0$, the electric field is the superposition of electromagnetic monochromatic waves and polarization oscillations, and the quasistatic field that disappears with distance from the laser pulse $\tau \to \infty$. Before the pulse, there is only a quasistatic field, that also decreases with distance from the pulse.

Let us investigate the expression for the electric field (44) - (46) for a number of model transverse and longitudinal intensity profiles of the laser pulse. Firstly we consider the model longitudinal profile of the laser pulse

$$T(\tau_0) = e^{-\frac{|\tau_0|}{t_L}},$$

where t_L is the laser pulse duration.

For such an impulse, we have

$$Z(\omega\tau) = \frac{t_L}{\omega^2 t_L^2 + 1}\left[2\chi(\tau)\cos\omega\tau + t_L\frac{dT(\tau)}{d\tau}\right],$$

$$\omega = \omega_g, \omega_{chn},$$

$$X(\nu_{stn}\tau) = \frac{2t_L}{\nu_{stn}^2 t_L^2 + 1} sign(\tau)\left[T(\tau) - e^{-\nu_{stn}|\tau|}\right].$$

The electromagnetic pulse is bipolar, and taking into account these expressions for the longitudinal component of the electric field excited by the laser pulse, we obtain the expression

$$E_z(r,\tau) = -\Pi\frac{\omega_p^2 t_L}{\omega_g^2 t_L^2 + 1}F(r)\left[2\chi(\tau)\cos\omega_g\tau + t_L\frac{dT(\tau)}{d\tau}\right] -$$

$$-\sigma\Pi\sum_{n=1}^{\infty}\frac{\rho_n J_0(\lambda_n r/b)}{\omega_{chn}^2 + \nu_{stn}^2}\left\{\frac{1}{\varepsilon_{chn}}\frac{t_L}{\omega_{chn}^2 t_L^2 + 1}\left[2\chi(\tau)\cos\omega_{chn}\tau +\right.\right.$$

$$\left.\left. + t_L\frac{dT(\tau)}{d\tau}\right] + \frac{1}{\varepsilon_{stn}}\frac{2t_L}{\nu_{stn}^2 t_L^2 - 1}\left[e^{-\nu_{stn}|\tau|} + t_L\frac{dT(\tau)}{d\tau}\right]\right\}. \quad (51)$$

It follows from this expression that the polarization electric field, along with a monochromatic wakefield wave, contains a solitary pulse whose longitudinal profile completely repeats the electrical polarization profile of the dielectric and, accordingly, of the ponderomotive force. The width of this pulse $\Delta\tau_g \sim t_L$ is close to the width of the laser pulse and does not depend on the parameters of the dielectric waveguide. Each radial electromagnetic harmonic of the dielectric waveguide has an analogous structure. In addition, there is a set of electromagnetic pulses. The width of each of them $\Delta\tau_{stn} \sim 1/\nu_{stn}$ is determined by the parameters of the dielectric waveguide and does not depend on the duration of the laser pulse.

Let us investigate the expression for the electromagnetic field (44) in the quasistatic approximation $\omega_d^2 >> \omega^2$. In this approximation, the permittivity $\varepsilon_{ch} \sim \varepsilon_0$ is independent of frequency.

Consider the most interesting case $b_0 > 0$ or $v_g > c/\sqrt{\varepsilon_0}$. In this case we have

$$\omega_{chn} = \frac{\omega_d}{\sqrt{b_0}} = \frac{\lambda_n v_g}{b\sqrt{\beta_g^2\varepsilon_0 - 1}},$$

$$\nu_{stn} = \omega_d\sqrt{b_0} = \omega_d\gamma_g\sqrt{\beta_g^2\varepsilon_0 - 1} \equiv \nu_{st}. \quad (52)$$

Accordingly, the expression for the electric field (51) becomes

$$E_z(r,\tau) = -\Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0}q_g F(r)\left[2\chi(\tau)\cos\omega_g\tau +\right.$$

$$\left. + t_L\frac{dT(\tau)}{d\tau}\right] -$$

$$-\Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0}\frac{\varepsilon_L - 1}{6}\sum_{n=1}^{\infty}\frac{\alpha_n}{N_n}q_n J_0(\lambda_n r/b)\left[2\chi(\tau)\cos\omega_{chn}\tau +\right.$$

$$\left. + t_L\frac{dT(\tau)}{d\tau}\right] -$$

$$-\Pi\frac{1}{t_L}\beta_g^2\frac{(\varepsilon_0 - 1)(\varepsilon_L - 1)}{3}\left[e^{-\nu_{st}|\tau|} + t_L\frac{dT(\tau)}{d\tau}\right]S(r), \quad (53)$$

where

$$q_g = \frac{\omega_g^2 t_L^2}{\omega_g^2 t_L^2 + 1}, q_n = \frac{\omega_{chn}^2 t_L^2}{\omega_{chn}^2 t_L^2 + 1},$$

$$S(r) = \sum_{n=1}^{\infty}\frac{\omega_{chn}^2}{\nu_{st}^2}\alpha_n J_0(\lambda_n b/r).$$

The width of all electromagnetic pulses is the same $\Delta\tau_{stn} \sim 1/\nu_{st}$ and does not depend on the number of the radial harmonic. The level of the field of these pulses is small.

Let us now consider the case $b_0 < 0$ or $v_g < c/\sqrt{\varepsilon_0}$, when the Cherenkov radiation condition is not satisfied. In this case, instead of (51), we have

$$\omega_{chn} = \omega_d\sqrt{-b_0} = \omega_d\gamma_g\sqrt{1 - \beta_g^2\varepsilon_0},$$

$$\nu_{stn} = \frac{\omega_d y_n}{\sqrt{-b_0}} = \frac{\lambda_n v_g}{b\sqrt{1 - \beta_g^2\varepsilon_0}} \equiv \nu_{st}.$$

Then for the longitudinal component of the electric field we obtain the following expression

$$E_z(r,\tau) = -\Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0}q_g F(r)\left[2\chi(\tau)\cos\omega_g\tau +\right.$$

$$\left. + t_L\frac{dT(\tau)}{d\tau}\right] -$$

$$-\Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0}\frac{\varepsilon_L - 1}{3}\sum_{n=1}^{\infty}\frac{\alpha_n}{N_n}q_{stn}J_0(\lambda nr/b)\left[sign\tau e^{-\nu_{st}|\tau|} +\right.$$

$$\left. + t_L\frac{dT(\tau)}{d\tau}\right] -$$

$$-\Pi\frac{1}{t_L}\frac{\varepsilon_0 - 1}{\varepsilon_0}\frac{\varepsilon_L - 1}{6}\left[2\chi(\tau)\cos\omega_{chn}\tau + \frac{dT(\tau)}{d\tau}\right] \times$$

$$\times\sum_{n=1}^{\infty}\frac{\alpha_n}{N_n}q_n J_0(\lambda_n r/b). \quad (54)$$

The electromagnetic part of the field (53) contains a sequence of bipolar antisymmetric pulses, as well as a wake monochromatic electromagnetic wave of a small amplitude.

Let us investigate the expression for the electric field (52) for a number of model transverse profiles of the laser pulse intensity. First of all, let us consider the Gaussian transverse profile

$$R(r) = e^{-r^2/r_L^2}.$$

In the most interesting limiting case $b >> r_L$ we have for the coefficients α_n

$$\alpha_n = \frac{r_L^2}{4}\exp\left(-\frac{\lambda_n^2}{4}\frac{r_L^2}{b^2}\right).$$

For a model radial profile of a laser pulse

$$R(\tau) = \begin{cases} J_0\left(\lambda_1\frac{r}{r_L}\right), r \le r_L, \\ 0, r > r_L, b \ge r \ge r_L, \end{cases}$$

$$\lambda_1 = 2.405,$$

we obtain

$$\alpha_n = \frac{r_L^2 \lambda_1 J_0\left(\lambda_n \frac{r_L}{b}\right) J_1(\lambda_n)}{\lambda_1^2 - \lambda_n^2 \frac{r_L^2}{b^2}}.$$

There are efficiently excited a finite number of radial harmonics for which on the one hand $\lambda_n r_b / 2b << 1$ and on the other hand $\omega_{chn} t_L / 2 << 1$.

CONCLUSIONS

In this paper the process of excitation of the wake Cherenkov radiation by a laser pulse in a dielectric waveguide is investigated. The nonlinear polarization of the dielectric medium, induced by the ponderomotive force from the laser pulse, is determined.

In the case when the local electric field acting on a separate atom (molecule) of the medium coincides with the applied external electric field of the laser pulse, the ponderomotive force is purely potential $\bar{F}_{pon} \propto \nabla I$. This situation is realized, for example, in a gas dielectric medium, in which the dipole electric fields of neighboring atoms can be ignored. In this case, the electric field excited by the laser pulse is purely potential and contains a bipolar solitary pulse, as well as a monochromatic wakefield wave at the frequency of the polarization oscillations.

The situation changes radically for condensed matter: solids or liquids. In this case, the effective (local) electric field acting on separate atom can differ substantially from the applied electric field, in our case the field of the laser pulse, since it is necessary to take into account the electric field caused with the polarization of all other atoms of the sample. In condensed dielectric media, a non-potential (vortex) part $\mathrm{rot}\bar{F}_{pon} \neq 0$ of the ponderomotive force arises.

On the example of a dielectric medium with a cubic crystal lattice, a solution is obtained for the excitation of Cerenkov wake radiation of a laser pulse in a dielectric waveguide. It is shown that the excited electric field consists of a potential field of polarization oscillations excited by a potential component of a ponderomotive force and a set of eigen wakefield electromagnetic waves of a dielectric waveguide. The latter are excited by polarization charges and currents induced by the vortex component of the ponderomotive force.

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Article received 26.06.2018

ВОЗБУЖДЕНИЕ КИЛЬВАТЕРНЫХ ПОЛЕЙ ЛАЗЕРНЫМ ИМПУЛЬСОМ В ДИЭЛЕКТРИЧЕСКОЙ СРЕДЕ

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Исследован процесс возбуждения черенковского электромагнитного излучения лазерным импульсом в диэлектрическом волноводе. Определена нелинейная электрическая поляризация в изотропной диэлектрической среде и, соответственно, поляризационные заряды и токи, индуцированные пондеромоторной силой со стороны лазерного импульса. Получена и исследована пространственно-временная структура кильватерного поля в диэлектрическом волноводе. Показано, что возбуждаемое поле состоит из потенциального поляризационного электрического поля, вызванного нелинейной поляризацией среды, и набора собственных электромагнитных волн диэлектрического волновода.

ЗБУДЖЕННЯ КІЛЬВАТЕРНИХ ПОЛІВ ЛАЗЕРНИМ ІМПУЛЬСОМ У ДІЕЛЕКТРИЧНОМУ СЕРЕДОВИЩІ

В.А. Балакірєв, І.М. Оніщенко

Досліджено процес збудження черенковського електромагнітного випромінювання лазерним імпульсом у діелектричному хвилеводі. Визначена нелінійна електрична полярізація в ізотропному діелектричному середовищі та, відповідно, полярізаційні заряди і струми, індуковані пондеромоторною силою з боку лазерного імпульсу. Отримана та досліджена просторово-часова структура кільватерного поля в діелектричному хвилеводі. Показано, що збуджуване поле складається з потенціального полярізаційного електричного поля, викликаного нелінійною поляризацією середовища, та набору власних електромагнітних хвиль діелектричного хвилеводу.