The Langmuir probe modeling in ion-beam plasma

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It is known, that in transport space of intense ion beam the ion-beam plasma (IBP) appears as a result of the beam space charge neutralization by electrons [1]. The interest to research of IBP is stimulated by wide extension of different ion-beam technologies [2-6], where the charged, excited, chemically active particles and electromagnetic radiation hit the processed surface immediately from the ion-beam plasma.

One of the most attractive methods for measuring of the ion-beam plasma parameters is a Langmuir probe method, which permit to measure locally practically all essential plasma parameters in wide range of their change. But the probe measurements in IBP have specific peculiarities connected with complex composition of the plasma.

The aim of present work is construction of mathematical model, which describe dependence of ion current gathered by single cylindrical Langmuir probe on the basic parameters of ion-beam plasma, such as probe potential, cold and beam ion densities, relation between ion and electron temperatures. The model covers wide parameters range of the ion-beam plasma, particularly the whole range typical for technological ion-beam systems. The model is built for collisionless case, because transport of ion-beam plasma takes place at low pressures.

MODEL DESCRIPTION

A current of cold ions on infinite single cylindrical Langmuir probe is considered. It is assumed that the ion temperature $T_i$ is much less than electrons temperature $T_e$ and electron distribution is Maxwellian.

The following dimensionless variables and parameters are used in our model:

$$\eta_i = \frac{n_i}{n_e}, \eta_b = \frac{n_b}{n_e}, \eta_e = \frac{n_e}{n_e},$$

$$x_p = \frac{r_p}{\lambda_{i0}}, \psi_p = \frac{\varphi_p}{k T_e}, i = \frac{I_i}{2\pi r_p L n_e e}, \tau = \frac{T_i}{T_e}.$$  

Here $\varphi$ — potential in point with radius $r$, $n_i$, $n_b$, $n_e$ — densities of cold ions, beam ions and electrons respectively; $n_e0$, $n_i0$ — densities of electrons and cold ions at infinity, $r_p$, $\varphi_p$, $L$ — radius, potential and length of probe; $I_i$ — current of cold ions to the probe; $m_i$ — ion mass; $e$ — electron charge; $k$ — Boltzmann’s constant;

$$\lambda_{i0} = \sqrt{\frac{k T_i}{4\pi e n_{i0} e^2}}$$ — Debye length.

The analogous dimensionless parameters are used in the most of theoretical works dealing with computation of ion current to Langmuir probe [10–12]. But we have introduced additional parameter — "portion of ion beam in plasma": $\eta_{i0} = \frac{n_{i0}}{n_{e0}}$. The value of this parameter changes from 0 (case of “pure” ion-beam plasma without cold ions), to 1 (case of “classic” plasma without beam).

![Figure 1. Evolution of dimensionless ion current to voltage characteristic in dependence on parameter $\eta_{i0}$ at several $x_p$ values.](image)
Poisson's equation in dimensionless variables can thus be written as follows:

\[
\frac{d^2 \psi}{dx^2} + \frac{1}{x} \frac{d \psi}{dx} = \exp(\psi) - \left( \eta_i \frac{x_p}{x} \frac{1}{\sqrt{1 - \psi}} \right) \left( 1 - \eta_i \right),
\]

where the first term of right part is the dimensionless density of electrons, the second – the density of slow ions, the third – the density of beam of ions, which is constant.

For determination of initial conditions let us take a region in which the following conditions are satisfied: \( kT_i \gg kT, n_e \approx n_i + n_b \). In this region the current density of cold ions can be defined as \( \frac{2}{2\pi rL} \), and on the other hand, this density of cold ions is determined by potential \( \phi \):

\[
j = e n_0 \sqrt{\frac{2e}{m_i}(\phi - \psi)}.
\]

From this equation an analytic expression for radial distribution of potential can be found:

\[
\psi(x) = \frac{-x^2}{x^2}.
\]

The initial conditions for equation (1) are set on the cylindrical surface with dimensionless radius \( x_0 \), which is in the region defined above. The potential of this surface we define as \( \psi_0 = -br \), where \( b>1 \) (for example, \( b=5 \)).

Radius \( x_0 \) and derivative of potential at \( x = x_0 \) can thus be written as follows:

\[
x_0 = \frac{L_p}{\sqrt{b \tau}}, \quad \left( \frac{d\psi}{dx} \right)_{x_0} = \frac{2b\tau}{x_0}.
\]

We solved Poisson's equation using the fourth order Runge-Kutta method.

**RESULTS AND DISCUSSION**

As a result of numerical computations we have got a data array of cold ions current dependence on four parameters \( i(\psi, x_p, \eta_i, \tau) \). The values of dimensionless potential was changed from 0 to 50, the dimensionless probe radius \( x_p \) from 0.25 to 100, that corresponds the change of plasma density from \( 10^5 \) to \( 10^{13} \) cm\(^{-3}\). Temperatures ratio was 0.01.

As has been shown on fig. 3 ion density in ion-beam plasma is greater, than in “classic” plasma. Therefore, in IBP the sheath is more thin, and the current of cold ions on Langmuir probe is less, than in “classic” plasma with equivalent parameters as has been shown on fig. 1.

For the limiting case of the large probe (\( x_p=100 \)) on fig. 1 we can make a conclusion, that for any values of parameter \( \eta_i \), as in the case of “classic” plasma, saturation of ion current takes place. The saturation current density can be presented by formula, which is analogous to the well known Bohm’s formula for plasma without ion beam:

\[
j = C e n_0 \sqrt{\frac{2kT_i}{m_i}},
\]

where coefficient \( C \) changes from 0.44 (for plasma without beam), to 0.09 (for ion-beam plasma). Figure 2 shows, that the ion current depends from \( \eta_i \) practically linearly, and the coefficient \( C \) can be presented by formula:

\[
C = 0.09 + 0.35 \eta_i.
\]

For practical usage of numerical results the approximation formula for the current of cold ions on Langmuir probe in ion-beam plasma is constructed:

\[
i = (0.95 + 0.35\eta_i) \left\{ 1 + 1.85 \left(x - 0.05(1-\eta_i) \right) \right\}^{\frac{1}{2}} \left( \frac{\psi}{40} \right)^{\frac{1}{32-\psi}}.
\]

This equation is built in such way, that in the limiting case \( \eta_i = 1 \) the result is equal to classic ABR theory [12]. The difference between the results of this formula and the results of numeral computations is not more 4% in the range of parameters \( \psi=5-50, \eta_i=0-1, x=0.25-100 \). For \( x > 1 \) this equation can be simplified:

\[
i = (0.95 + \eta_0) \left\{ 1 + 1.85x \right\}^{\frac{1}{4}} \left( \frac{\psi}{40} \right)^{\frac{1}{32-\psi}}.
\]

**Figure 2. Ion current \( i \) as a function of \( \eta_i \) for the case of high probe potential.**
Figure 3. Radial distribution of densities of electrons $\eta_e$ and ions $\eta_i$ in comparison with potential distribution at several $\eta_{i0}$ values. $\eta_e$ — thin line, $\eta_i$ — bold line.

REFERENCES